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Research on closed ecosystems

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Abstract. In a closed ecosystem, the correlation between different species and the specific evolution process is important research subject, and the relevant experimental research and field investigations have attracted attention in recent years. In view of the complexity of chaotic systems, this paper aims to introduce and summarize the mathematical description of the aquarium project from multiple aspects, and try to propose a more concise scheme to describe the movement relationship between entities in the ecosystem. The artificial fish school algorithm was used in this research to simulate the response relationship of two species, and the rules of this aquarium were explored. The results show that this ecosystem still evolves to the same result even if the initial parameters are slightly different. This research simplifies the complex calculations in the process and provides new ideas and supplements for understanding the dynamic balance mechanism of closed ecosystems.

Keywords: aquarium, closed ecosystem, mathematical modeling, artificial fish swarm algorithm, ecological box

1. Introduction

The ecosystem box is an important concept, typically referring to a closed environment used to simulate and study ecosystems. Such studies help scientists understand the interactions within ecosystems, the relationships between species, and the impact of environmental changes on ecosystems.

Early ecological--ecosystem box can be traced back to the mid-20th century, when scientists began using closed containers to observe the growth and changes of plants and animals under specific conditions. These experiments helped people understand the complexity and interdependence of ecosystems.

In recent years, with the intensification of global climate change and environmental issues, the study of the ecosystem box has become increasingly important. Because this seems to be a common problem on Earth. This leads to myriad problems in terms of climate, for example: the Guernino phenomenon [1-4]. The food shortage it brings, the disruption of marine ecological balance, the impact on the living environment of fish, climate change in some areas, and other extreme weather events [5, 6]and the threat it poses to human life safety The study of certain types of weather has become a hot topic in the field of meteorology, and many renowned scholars often use methods such as micro disturbances for research, but the results are not satisfactory [7]. There is a further problem that for large-scale weather, it is a chaotic system [8-11], and its mathematical description is very difficult [12-14]. These experiments about ecosystems can provide data on the impact of various environmental factors, such as temperature, light, and water on ecosystems, thereby aiding in the development of strategies for environmental protection. In this study, the author will attempt to use a simple producer-consumer model [7, 15] to describe a simple closed ecosystem.

2. Literature review

2.1. Research on closed systems

There are many examples related to the study of closed systems.

Let's first take a look at the Malte Ecological Indicator Experiment. In 2020, ecologists studied the patterns of nutrient cycling and energy flow in ecosystems through the construction of closed ecological boxes, the interactions between different types of plants and animals. Research found that biodiversity contributes to the stability of ecosystems [16].

Also, Transient response of forests to CO2-induced climate change: simulation modeling experiments in eastern North America. It is pointed out that regulating substances such as carbon dioxide can change the entire ecosystem. Some scientists use ecological boxes to simulate the impact of future climate change. For example, by adjusting the temperature and carbon dioxide concentration, the growth of plants under these conditions is studied, and then the possible impact on the entire ecosystem is analyzed [17].

And Water Eco Tank, in aquatic ecosystems, scientists construct closed tanks to simulate small lake or river environments, in order to study the interactions among aquatic organisms, such as algae, fish, microorganisms, etc. By adjusting the concentration of nutrients, the impact of eutrophication on aquatic ecosystems can be observed [16].

Without them, scientists also have Urban Eco Box, and with the process of urbanization, the study of urban ecology has gradually risen. Scientific research institutions designed ecological boxes simulating urban environments, studying factors such as the urban heat island effect and vegetation impact, which help us understand the evolution of urban ecosystems and their impact on residents' lives.

These experiments on closed systems explore the impact of biodiversity, nutrient concentrations, and environmental factors on ecosystem stability. These small and medium-sized closed experiments provide an important foundation for understanding the intrinsic mechanisms of ecosystems and lay the groundwork for larger ecosystems in the future. Based on these valuable experiences, some scientists have begun to conduct larger-scale ecological experiments to explore more complex ecological processes and long-term changes.

2.2. Larger-scale experiments

There are also many larger-scale experiments.

The first one is the Prairie Project, this experiment was conducted in the grassland region of the central United States, studying the effects of different management practices such as burning, grazing, and planting on the ecosystem. By monitoring the changes over a long period, researchers were able to obtain large-scale ecological data, analyzing the variations in plant diversity and ecological functions. This study is considered highly representative in grassland ecosystem research [18].

Also, ecological research in the Brazilian rainforest, in the Amazon rainforest, scientists have conducted extensive studies to assess the impact of deforestation and environmental changes on biodiversity and ecosystem functions. These studies not only observe the response of individual species but also analyze the dynamic changes of the entire ecosystem [19].

What else is the Arctic Ecological Change Project? In this study, scientists have carried out large-scale ecological research in the Arctic region, focusing on the impact of climate warming on the Arctic ecosystem. These studies examine the responses of plants, animals, and microorganisms through long-term field monitoring and ecological box experiments, thereby providing data support for ecological changes in the Arctic region [20].

The Global Change Experiment, the experiment was conducted in different ecosystems across multiple countries, studying the combined effects of climate change, increasing atmospheric concentration, and land-use on ecosystems. By conducting controlled experiments at different locations, researchers were able to assess the impacts of global change on biodiversity, productivity, and ecosystem services. This study is notable for its universality and broad application value in global ecological research [21].

In summary, these large-scale ecological experiments provide important data and a theoretical basis for understanding the long-term impacts of environmental changes on different ecosystems. From regional research to global projects, scientists have explored the impact of various ecological factors and human activities on biodiversity, ecosystem stability, and ecological functions.

2.3. A review of key studies

There are also some key studies on this subject, and here are some representative examples:

First is "Evolution: The Modern Synthesis." [22]. While this book primarily focuses on the foundations of evolutionary biology, it also delves into experimental observations of the interactions within ecosystems and the adaptations of species, highlighting the roles of different species within ecosystems, such as the producer-consumer model. The book mainly discusses evolutionary theory and does not provide specific data modeling models. However, it lays the groundwork for understanding species adaptations and ecosystem dynamics, providing theoretical support for subsequent ecological models.

Building on this theoretical groundwork, "Planetary boundaries: Guiding human development on a changing planet." [23]. In the paper, the researchers defined nine planetary boundaries related to climate change, biodiversity, and the water cycle, among other areas. The study uses a combination of extensive data analysis and model simulations to explore the potential impacts on ecosystems when human activities exceed these boundaries. System Dynamics Models were used to model the effects of change on ecosystems. These models predict future ecological changes by integrating multiple environmental variables, such as CO_2 concentrations, biodiversity, land use, etc., and assess the impact of human activities on the Earth system. The mathematical description is presented as follows:

$$\frac{dC}{dt} = E - kC \tag{1}$$

where C is the CO_2 concentration, E the emission rate, and K the natural absorption rate of CO_2.

As well as "The science of ecology.[24]". This review article provides a summary of different fields of ecology, including the design and implementation of ecological experiments. Although no specific experimental cases are presented, it provides a basis for the importance and methodology of ecological experiments, and it discusses the models and methods commonly used in ecology, including population dynamics models and ecosystem models. Models are usually based on differential equations or difference equations and are used to describe the temporal variation in the number of species and the Lotka-Volterra equations formulate the predator-prey interaction:

$$\frac{dN}{dt} = \mathrm{rN} - \mathrm{aNP} \tag{2}$$

And

$$\frac{dP}{dt} = baNP - mP \tag{3}$$

In which N is the number of prey, P is the number of predators, r is the natural growth rate of the prey, a is the predation rate, b is the conversion efficiency of the predator, and m is the death rate of the predator.

Further expanding on ecosystem modeling, "Resource heterogeneity increases the number of species and stabilizes ecosystem functioning [25]". In this study, the authors investigated how resource heterogeneity, such as different nutrient compositions and structural complexity, affects species diversity and stability through simulation experiments. The results showed that resource diversity can promote species coexistence and functional stability of the ecosystem. The study used network models and stochastic simulations to analyze the resource heterogeneity on species diversity and ecosystem functioning. By constructing a species-resource interaction network, researchers were able to model the dynamics of the ecosystem under different allocation scenarios. The growth of biological populations can be expressed by the following equation:

$$\frac{dN}{dt} = \mathrm{rN}(1 - \frac{\mathrm{N}}{\mathrm{K}}) \tag{4}$$

Here, N is the population size, r is the population growth rate, and K is the environmental carrying capacity.

The dynamic changes in resource utilization can also be expressed in differential equations:

$$\frac{dR}{dt} = R_0 - c \cdot N \tag{5}$$

where R is the amount of the resource, $\mathrm{R_0}$ is the initial amount available of the resource, and c is the consumption rate.

Finally, "Human land use in the tropics reduces the resilience of rain forest.[26]"The study evaluated the impact of human land use (such as agriculture and urbanization) in tropical regions on the restoration capacity of rainforest ecosystems through field surveys and data analysis. The experimental results showed that land use changes significantly reduced the biodiversity and ecological restoration capacity of rainforests. The study used spatial explicit models and regression analysis to assess the impact of land use changes on tropical rainforest ecosystems. Spatial explicit models can simulate the spatial distribution of the impact of different land use patterns on the structure and function of ecosystems. Using spatial explicit models and regression analysis, there are:

$$\mathrm{D} = \beta_0 + \beta_1 \mathrm{L} + \beta_2 \mathrm{C} + \epsilon \tag{6}$$

where D is the biodiversity index, L is the land-use intensity, C is the climate variable, $\beta_0, \beta_1, \beta_2$ are regression coefficients, and ϵ is the error term.

Overall, most existing research focuses on macro ecological patterns or complex models, lacking simplified descriptions and simulations of the dynamic relationships between producers and consumers in simple closed ecosystems. Therefore, based on previous research, this article attempts to construct a simple producer-consumer model to describe energy flow and population changes in closed ecosystems, providing new ideas and supplements for understanding the dynamic balance mechanism within ecosystems.

3. Methodology

In this study, the author used a model that is more cost-effective and environmentally friendly; this model is designed to observe the patterns of a small ecological box. The basic logic of my model is shown in Figure 1. The 'world model' and 'Monolithic interaction model' are the basic models. On top of them, the author constructed the 'producer reproduction model' and 'consumer movement model', and built the 'Population change model' and common attack tactic 'Elder chain attack model', to integrate the entire 'history'. The author summarized the 'DFA system evolution model' used to describe history.

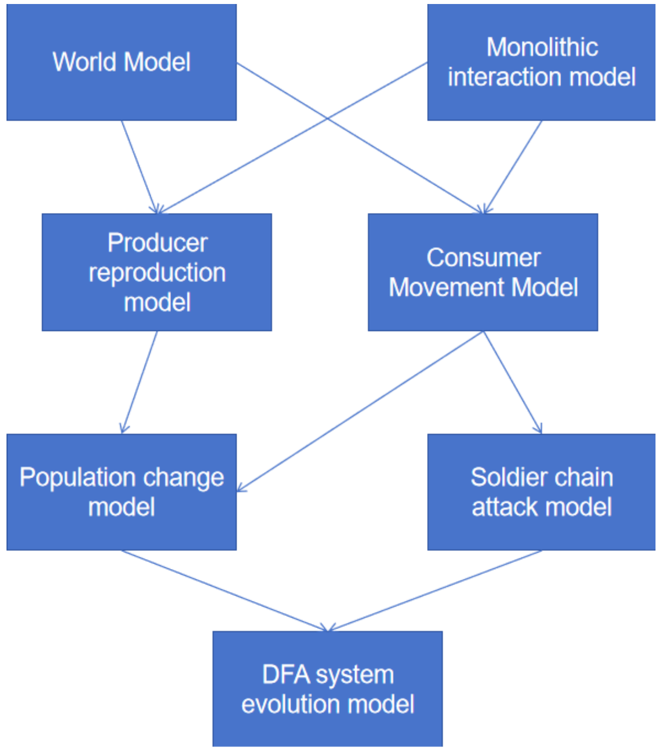

Figure 1. Relationships and interactions between the models

The author has a project in this regard for study [27]. Here, the author describes the producer and consumer agents as $\mathrm{Produce}(\varphi_p, \omega_p, o_p)$, $\mathrm{Consumer}(\varphi_c, \omega_c, o_c)$, t, Oxygen. Where $\mathrm{Produce}$ is the producer class, described by φ_p (perception radius), v_p (velocity), o_p (current individual's oxygen amount), $\mathrm{Consumer}$ for consumer class, described using φ_c (sensing radius), v_c (speed), o_c (current individual's amount), and world time t, world oxygen total Oxygen describe this ecological box.

And it is stipulated that both $\mathrm{Produce}$ and $\mathrm{Consumer}$ move according to the fish school algorithm and random walk [7].

4. Results

4.1. Modeling 1: world model and monolithic interaction model

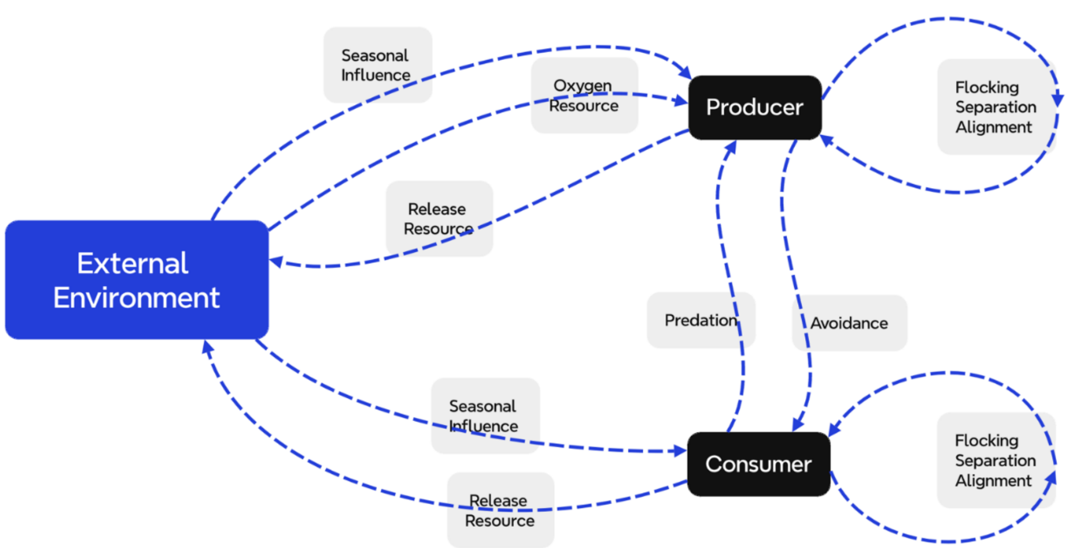

Figure 2. Framework of world model and monolithic interaction model

This model is implemented based on the fish school algorithm and innovatively incorporates the constraints of season and lifespan, as shown in Figure 2. It is the foundation for all the models in the subsequent text. The middle fish school algorithm is described as follows for the producer class:

In each time step, the position of Produce is updated based on its velocity and direction. The position update formula is:

$$x(t+1) = x(t) + v\cos\theta \cdot \Delta t \; y(t+1) = y(t) + v \cdot \sin\theta \cdot \Delta t \tag{7}$$

where $x(t)$ and $y(t)$ are the positions at the current moment, v is the velocity, θ is the moving direction, Δt is the time step.

The speed is affected by the seasonal factor, which varies periodically with time, as described below:

$$\text{season} = 1 + A \cdot \sin(\frac{t}{T} \cdot 2\pi) \tag{8}$$

where A is the seasonal amplitude, t is the time passed, T is the seasonal cycle, and it will act directly on the velocity, with:

$$v = v \cdot \text{season} \tag{9}$$

And, each time step, the Producer has a probability of obtaining oxygen, which can be described as follows:

$$\text{if Random Number} < q \text{ then } o_p + \Delta o; \text{Oxygen} - \Delta o \tag{10}$$

where $\text{RandomNumber} \in [1,100]$ is a random number, $\Delta o \in [0, \text{Oxygen}]$ is the amount oxygen obtained, and q is the probability of obtaining oxygen.

Moreover, when

$$\sqrt{(x_p - x_c)^2 + (y_p - y_c)^2} < \varphi_p \left((x_p, y_p)\text{coordinates of the current Producer}, (x_c, y_c)\text{Coordinates for the consumer's}\right) \tag{11}$$

Calculate the avoidance deflection angle to avoid collision $\theta = \tan^{-1}\frac{x_p - x_c}{y_p - y_c}$,It will act directly on the original angle:θ_{old},have:

$$\theta_{new} = \theta + \Delta\theta \tag{12}$$

Where, $\Delta\theta$ is a random angle, usually $\Delta\theta \in [0.5,1.5]$, This can enhance the randomness of its evasive behavior, causing the producer to not escape in a straight line but to change direction.

Also, the boundary check of Produce is as follows:

$$\text{if } x < r \text{ or } x > W - r \text{ then } \theta = \pi - \theta \tag{13}$$

corresponding to

$$\text{if } y < r \text{ or } y > H - r \text{ then } \theta = -\theta \tag{14}$$

where r is the radius of Produce, and W and H are the width and height of the screen.

To sum up, Modeling 1 describes an agent that can accept information from other entities and make behavioral changes, modeling the creatures in this ecosystem. This model is the basis for all that follows.

4.2. Modeling 2: producer reproduction model

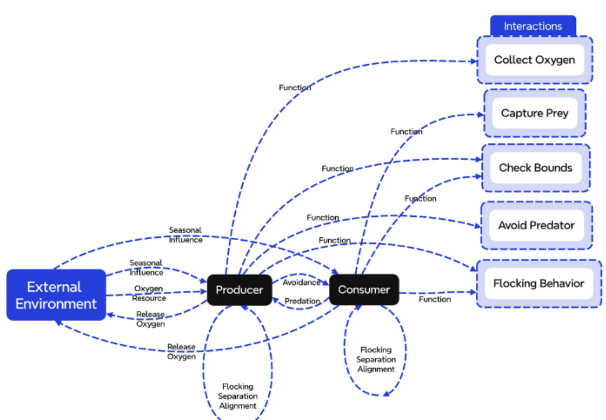

Figure 3. Framework of producer reproduction and consumer movement

Based on the foundation of Modeling 1, in order to describe the motion of the Producer, this section also constructs a model suitable for the Producer based on the fish algorithm, and innovatively adds the limitation of oxygen as shown in Figure 3.

To counter Consumer, Produce has the ability to reproduce, with description:

The conditions for breeding depend on the oxygen store and the cooldown time (i.e. the breeding cooldown timer). Defined:

C: Currently breeding cooldown timer.

O: The amount of oxygen currently stored by the producer.

Producer P can perform reproduction if the following conditions are met:

$$\text{Can spawn} \underset{\text{always}}{\Longleftrightarrow} \mathrm{O} > 0 \text{ and } \mathrm{C} < 1 \tag{15}$$

If two producers P_1 and P_2 come into contact (meet) within their sensing radius r_p, the contact distance d can defined as:

$$d = \sqrt{(x_{p_1} - x_{p_2})^2 + (y_{p_1} - y_{p_2})^2} \tag{16}$$

if the distance satisfies:

$$d < r_{p_1} + r_{p_2} + \Delta \tag{17}$$

where r_{p_1} and r_{p_2} are the radii of the two producers, and the empirical constant Δ is a contact threshold (e.g., 10 pixels), the breeding procedure can be started as follows:

If both producers P_1 and P_2 meet the conditions for reproduction, they will generate a new producer near the contact point. The initial position (x_{new}, y_{new}) of the new producer can be defined as the average position of the two plus a random offset:

$$x_{new} = \frac{x_{p_1} + x_{p_2}}{2} + \Delta x$$

$$y_{new} = \frac{y_{p_1} + y_{p_2}}{2} + \Delta y \tag{18}$$

where Δx, Δy are random offsets.

The properties of the newly generated producer P_{new} can be set as follows:

1. Oxygen volume: $O_{new} = \frac{O_{p_1} + O_{p_2}}{2}$
2. Sensing radius (take the average of the two sensing radii plus a random offset, and ensure a maximum value of 10 pixels): $r_{p,new} = \max(10, \frac{r_{p_1} + r_{p_2}}{2} + \Delta r)$
3. Halve the amount of oxygen in each parent: $O_{P_1} \leftarrow \frac{O_{P_1}}{2}$,and $O_{P_2} \leftarrow \frac{O_{P_2}}{2}$
4. Reproduction Cooling Time Settings: $C_{new} = 100$ (Set the new producer's cooling timer)

In summary, by focusing on the study of the motion description of the Producer, this model implements a Producer that can accept information from other entities and make behavioral changes to simulate the producers in this ecosystem. It has achieved good results and laid a good foundation for the model in the following text.

4.3. Modeling 3: consumer movement model

Also based on Modeling 1, in order to focus on the Consumer, which is an adversarial object of the Producer,a model for describing Consumer is also constructed the fish-school algorithm. The focus is on his relationship with the Producer is show in Figure 3.

The Consumer, who is in active opposition to Produce, can have his movements summed up in the following steps and expressed in mathematical language:

In each time step, the Consumer will choose the nearest prey (producer) from the Produce_list to chase, as described below

$$\text{closest\_prey} = \arg\min_{p \in prey} \sqrt{(x_c - x_p)^2 + (y_c - y_p)^2} \tag{19}$$

where (x_c, y_c) is the location of the predator and (x_p, y_p) is the location of the prey.

Then calculate:

$$\text{direction\_x} = x_{\text{closest\_prey}} - x_c \tag{20}$$

and

$$\text{direction\_y} = y_{\text{closest\_prey}} - y_c \tag{21}$$

Compute the distance between the two by the direction vector, and perform normalization to ensure that the unit direction vector has:

$$d = \sqrt{\text{direction\_x}^2 + \text{direction\_}y^2} \text{ if } \text{normalized\_direction\_x} = \frac{\text{direction\_x}}{d} ; \text{normalized\_direction\_y} = \frac{\text{direction\_y}}{d}$$

Then the position can be updated according to its velocity, with:

$$x_c += \text{normalized\_direction\_x} \cdot v \cdot \Delta t \tag{22}$$

and

$$y_c += \text{normalized\_direction\_y} \cdot v \cdot \Delta t \tag{23}$$

where v is the predator's speed and Δt is the time step.

Likewise, the predator checks if its position is outside the screen boundaries and adjusts its coordinates if it is:

$$\begin{gathered} \text{if } x_c < r \text{ then } x_c = r \\ \text{if } x_c > W - r \text{ then } x_c = W - r \end{gathered} \tag{24}$$

And similar checks are made for the y coordinate Moreover.

Consumer group behaviors are implemented among predators, avoiding overly dense movements through operations such as alignment and.

Suppose there are C_1 and C_2 with positions (x_{c_1}, y_{c_1}) and (x_{c2}, y_{c_2}), respectively. Then the distance between them is obtained as:

$$d = \sqrt{(x_{c_1} - x_{c_2})^2 + (y_{c_1} - y_{c_2})^2} \tag{25}$$

Then judge whether the two are too dense according to if $d < r$, where r is an empirical constant (e.g.,10 pixels).

When predators are too close to each other, predator C_1 makes a location adjustment based on the location of C_2. The corresponding separation $\vec{s}$ of the predator C_1 can be defined as:

$$\vec{s} = (x_{c_1} - x_{c_2}, y_{c_1} - y_{c_2}) \tag{26}$$

To prevent the application of the separating vector from causing a sudden change in position, it must be standardized:

$$s_{norm} = \sqrt{(x_{c_1} - x_{c_2})^2 + (y_{c_1} - y_{c_2})^2} \ \ (\text{if } s_{norm} > 0) \tag{27}$$

Normalizing the direction gives:

$$\hat{s} = \left(\frac{\vec{s}_x}{s_{norm}}, \frac{\vec{s}_y}{s_{norm}}\right) \tag{28}$$

Finally, the Predator C_1 will update its position based on the separation vector:

$$\begin{gathered} x_{c_1,new} = x_{c_1} + \hat{s}_x \cdot k \\ y_{c_1,new} = y_{c_1} + \hat{s}_y \cdot k \end{gathered} \tag{29}$$

where k is a small constant, representing the strength of separation.

This section builds a motion model for the Consumer, adding the action of implementing the chase Producer. This model is also the basis for the analysis in the following section.

4.4. Modeling 4: population change model at the start

The following discussion is based on the empirical constants given in Table 1.

Table 1. Empirical constants in population change model

Constant Name	Value	Description
screen_dimensions	(1000, 600)	The size of the Pygame window, defining the simulation scene's range.
fps	120	Frames per second, determining the smoothness of the simulation.
all_oxygen	1000	The total initial amount of oxygen in the ecosystem, a key resource for the survival and reproduction of organisms.
season_cycle	60	The duration of a complete seasonal cycle in seconds, affecting the movement speed of organisms.

Table 1. Continued

season_amplitude	2	Determines the amplitude of the speed change caused by seasonal variations.
base_speed_blue	1	The base movement speed of blue balls (students) without considering seasonal effects.
base_speed_predator	2	The base movement speed of red balls (predators) without considering seasonal effects.
oxygen_gain_probability	0.01	The probability that a blue ball can obtain a portion of the remaining oxygen per frame.
radius (in Student and Teacher classes)	5	The radius of the organisms, used for collision detection and graphical rendering.
perception_radius (in Student class)	Random between 10 and 50	The range within which a blue ball can perceive other organisms, influencing its behavior decisions.
perception_radius (in Teacher class)	20	The range within which a predator can perceive other organisms, influencing its behavior decisions.
10 (used in multiple conditions)	-	A distance threshold used in separation behavior and reproduction judgment, affecting group behavior and reproduction mechanisms.
2 (in blue ball reproduction condition)	-	The oxygen threshold for a blue ball to reproduce.
9 and 10 (in blue ball reproduction judgment)	-	Used in a random integer generation to introduce randomness in blue ball reproduction.
6 (in predator reproduction condition)	-	The capture count threshold for a predator to reproduce.

These constants play a crucial role in constructing the simulation environment, defining the rules of biological behavior, and so on. These constants cooperate with each other and jointly an ecological simulation system with certain rules and dynamic changes. Based on the first three models, Modeling 4 starts to run. After a large number of runs, a large amount of data with good repeatability is collected, and some general rules are summarized. The rules with a large number of repetitions and reproducibility in the first 400 frames of opening are of particular concern, and an attempt is made to fit this rule here.

In this experiment, the fitting algorithm was used in the environment of these data. A cubic equation was fitted to this ecosystem tank, and some of them had good, reproduced data fitting.

So that's a basic model of the behavior of the organism, but don't forget, we're interested in the group of that satisfies:

$$\mathrm{creature} \in \{\mathrm{Consumer}, \mathrm{Produce}\} \tag{30}$$

Running, it can be seen that at the beginning, the number of Produce rises rapidly and then gradually declines, and Consumer follows closely behind, consuming all the oxygen, its mathematical description is roughly a cubic function, with:

$$\mathrm{count} = \mathrm{at}^3 + \mathrm{bt}^2 + \mathrm{ct} + \mathrm{d} \tag{31}$$

where count is the current number of organisms, and a, b, c, d are constants associated with Oxygen, the following are the fitting for a representative example:

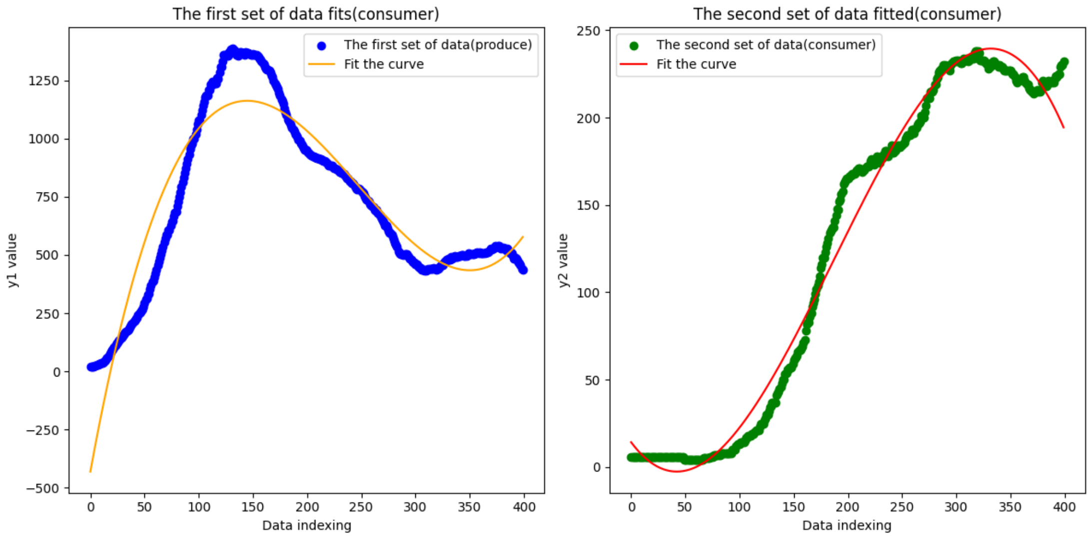

Figure 4. Fitting results of the population change model

It is not difficult to see that the growth of Consumer increases with the growth of Produce, roughly of cubic function type, it is worth noting the extreme value of Produce 1386 appears at t=131 frames, and then the extreme value of Consumer 238 first appears t=318 frames, then continues for 2 frames, falling at t=320 frames, and then, Produce and Consumer's number are roughly in a 1:2 ratio, which is a relatively stable ratio after running is show in Figure 4.

To sum up, it is concluded that the quantity of Produce always increases over time and the final maximum value is between [1000,250], followed by Consumer, with the final maximum value between [200,400], which provides support for further research.

4.5. Modeling 5: soldier chain attack model

This chapter will attempt to summarize the Consumer′s spontaneous and commonly used offensive maneuvers – Soldier chain attack and provide corresponding descriptions.

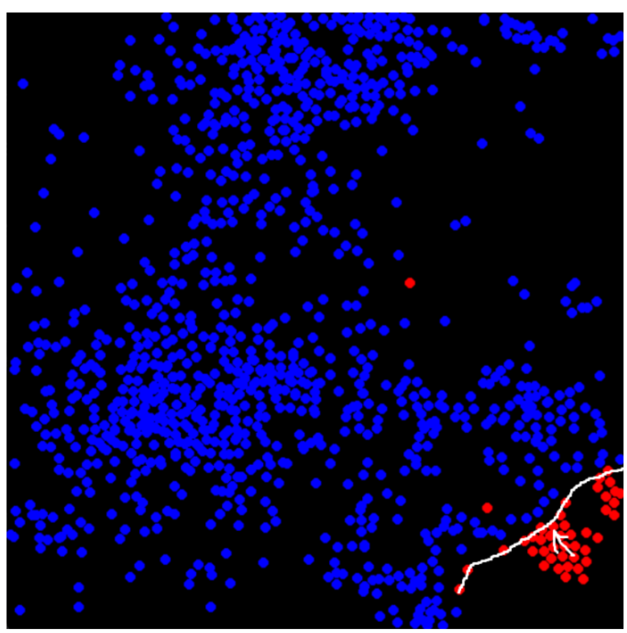

Figure 5. Evolution of the soldier chain structure

In addition, the author also found that the main way of Consumer's attack is to form a pawn chain, with the evolution as shown in Figure 5. This is one way: pawn chain The direction of the attack is indicated by the arrow in the middle, which is described as follows:

Firstly, it is stipulated that: $\{(x\_i, y\_i)|i \in I\}$ where I is a set of indices indicating all points within this area.

Secondly, the function $f(x, y)$ is used to represent the color of the points.

So the boundary B of the region to which the white arrows point, can be described by a parametric equation: $B(t) = (x(t), y(t)), t \in [a, b]$ where B is defined in terms of the direction and shape of the arrows, and the value this function can be used to analyze the degree of clustering of points.

This study uses Gaussian mixture model and K-means clustering to describe the aggregation of points, in the form of:

$$\rho(x, y) = \sum_{j=1}^{K} \pi_j \cdot N(\begin{bmatrix} x \\ y \end{bmatrix}; [\begin{matrix} \mu_{yj} \\ \mu_{x_j} \end{matrix}]; [\begin{matrix} \sigma_{xj}^2 & 0 \\ 0 & \sigma_{yj}^2 \end{matrix}]) \tag{32}$$

where π_j is the weight, $\mathcal{N}$ is the Gaussian distribution, (μ_{yj}, μ_{x_j}) is the mean of the j-th Gaussian distribution, and σ_{xj}^2 and σ_{yj}^2 are the variances.

$$J = \sum_{k=1}^{K} \sum_{x_1 \epsilon C_k} ||x_i - \mu_k||^2 \tag{33}$$

where C_k is the kth cluster, μ_k is the center of the kth cluster, and it is verified that the above equations can be used to describe the method of the battle chain in the offensive behavior of Consumer.

Of course, this method is not always invariable, and when it is difficult to summarize the cluster centers or there are multiple obviously unrelated evolution regions, other methods should be tried to describe it, so this study will discuss its variants next.

4.6. Modeling 6: military chain variant

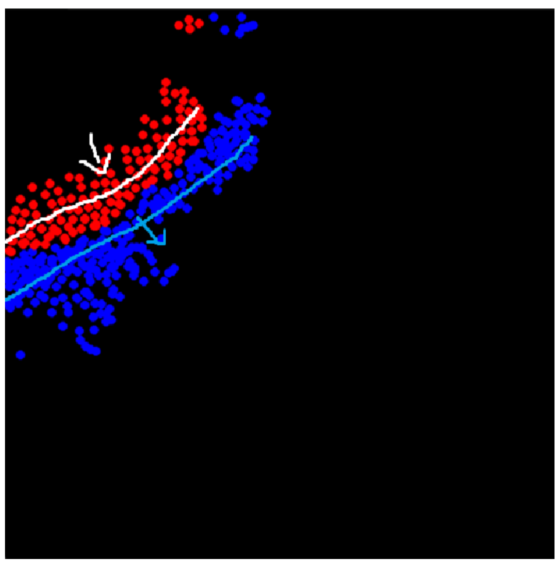

Figure 6. A Variant of the soldier chain structure

There is also a variant, as shown in Figure 6. In the diagram, it is obvious that the Consumer at this time does not satisfy the definition of the cluster center in the above equation. Moreover, the at the back of the queue can hardly eat the Produce located in front, so the above equation must be replaced with:

Definition:

Red area R: $\{(x_i, y_i)|i \in I_R, f(x_i, y_i) = red\}$

Blue area B: $\{(x_i, y_i)|i \in I_B, f(x_i, y_i) = blue\}$

The boundary can be defined as B_R and B_B, which can be represented as smooth curves, parameterized as:

Red area boundary B_R: $B_R(t) = (x_R(t), y_R(t)), t \in [a_R, b_R]$

Blue area boundary B_B: $B_B(t) = (x_B(t), y_B(t)), t \in [a_B, b_B]$

Then the density function can be used to describe the clustering of points in these two regions, there is:

Red region density function: $\rho_R(x,y): \rho_R(x,y) = \frac{1}{|R|}\sum_{(x_i,y_i)\in R}\delta(x-x_i)\delta(y-y_i)$

Blue region density function: $\rho_B(x,y): \rho_B(x,y) = \frac{1}{|B|}\sum_{(x_i,y_i)\in B}\delta(x-x_i)\delta(y-y_i)$

Then the features of each area can be analyzed, such as the center, variance, etc., and only the calculation method of the center is given:

Centroid of the red area $(\bar{x}_R, \bar{y}_R): \bar{x}_R = \frac{1}{|R|}\Sigma_{(x_i,y_i)\in R}x_i$, $\bar{y}_R = \frac{1}{|R|}\Sigma_{(x_i,y_i)\in R}y_i$

Centroid of the blue area $(\bar{x}_B, \bar{y}_B): \bar{x}_B = \frac{1}{|B|}\Sigma_{(x_i,y_i)\in B}x_i$, $\bar{y}_B = \frac{1}{|B|}\Sigma_{(x_i,y_i)\in B}y_i$

In addition, if there is a clear dividing line or boundary, such as the white curve shown in the picture, it can be expressed by a function

Split curve: $f(x): y = f(x)$

Such a curve can be obtained through a fitting algorithm (such as linear regression or other regression methods).

Also, if the rate at which the monomorphic parent produces descendants exceeds the expectation of the frontrunner of the team, it is easy to see that the descendant rate exceeds the expectation of the frontrunner of the team: q^2.

In these two sections, the soldier chain and its variants. The clustering rules are summarized. The innovation point, the steps are optimized for the complex equations of the chaotic system The two models pave the way for the later text.

4.7. Modeling 7: other attack model

This section describes the attack of monomer c-monomer p, and the defense of monomer p-monomer c. The movement law and the evasion are discussed.

4.7.1. Modeling 7.1: monomer C to monomer P

So that's a model of an attack methodology for a Consumer group. Next, the method of attack on Consumer entities are modeled.

First, let there be a Consumer agent C, and a group P of Producers, positioned from (x_1, y_1) to (x_n, y_n), with velocities V_i and orientation angles θ_i.

Then the position of P_i in the next frame can be predicted as:

$$x_i' = x_i + V_i \cdot \cos\theta_i \cdot \Delta t Y_i' = Y_i + V_i \cdot \sin\theta_i \cdot \Delta t \tag{34}$$

where Δt is the time interval per frame.

It can be seen from the above text that C needs to select the nearest Produce as the target each time. Let the position of the predator (x_c, y_c), then the nearest P can be expressed as:

$$\text{ClosestPrey} = \min_i \sqrt{(x_i - x_c)^2 + (y_i - y_c)^2} \tag{35}$$

Predators adjust their movement direction based on the predicted location of the nearest Produce. In this case, the movement of the Consumer can be described as where the Produce will be in the next frame.

To avoid collisions with other predators, the predator needs to detect the positions of other predators in the vicinity and adjust its own direction. Suppose that the position of predator j is (x_j, y_j), then the predator needs to avoid entering the following range:

$$\text{distance}(x_c, y_c, x_j, x_j) < \text{perception\_radius}$$

Taking all the above factors into consideration, the best attack route for C can be described mathematically as:

Predict P position:

$$x_i' = x_i + V_i \cdot \cos\theta_i \cdot \Delta t Y_i' = Y_i + V_i \cdot \sin\theta_i \cdot \Delta t \tag{36}$$

Select the nearest Produce:

$$\text{ClosestPrey} = \min_i \sqrt{(x_i - x_c)^2 + (y_i - y_c)^2} \tag{37}$$

Calculate the speed and direction of C: $V_c = V_{\text{BaseProduce}} \cdot S$, after $S = 1 + \text{SeasonAmplitude}$, and $\theta_c = \arctan 2(y_i' - y_c, x_i' - x_c)$

Update the Predator's position:

$$x_c' = x_c + V_c \cdot \cos\theta_c \cdot \Delta t Y_c' = Y_c + V_c \cdot \sin\theta_c \cdot \Delta t \tag{38}$$

Obstacle avoidance: for each $j \neq c$: $\text{if}\sqrt{(x_i - x_c)^2 + (y_i - y_c)^2} < \text{PerceptionRadius}: \theta_c = \theta_c + \Delta\theta$, $x_i' = x_i + V_i \cdot \cos\theta_i \cdot \Delta t Y_i' = Y_i + V_i \cdot \sin\theta_i \cdot \Delta t$
where θ_c is an increment of the adjusted angle, determined by the number of P.

4.7.2. Modeling 7.2: monomer P to monomer C

Having said the equation of motion of Consumer, let me describe the equation of motion of Produce.

Suppose there are n number of Produce, and m number of Consumer, then the hiding behavior of the nth Produce is as:

To describe them, it can be assumed that the position of the predator is (x_j, y_j) and the velocity is V_j and the direction angle is θ_j. The position of the predator in the next frame can be predicted as:

$$x_i' = x_i + V_i \cdot \cos\theta_i \cdot \Delta t Y_i' = Y_i + V_i \cdot \sin\theta_i \cdot \Delta t \tag{39}$$

where Δt is the time interval per frame.

Produce can only sense predators within its sensing radius. Let the sensing radius of Produce be R_p, predators within a distance R_p Produce need to be screened:

$$\text{dist}(x_i, x_j, x_j', y_j') = \sqrt{(x_j' - x_i)^2 + (y_j' - y_i)^2} \tag{40}$$

If:

$$\text{dist}(x_i, y_i, x_j', y_j') < R_p \tag{41}$$

then Predator_j is in the sensing range of Produce.

Of all the Predators in the sensory range, select the closest one as the Consumer to avoid. Let the position of the Produce be denoted as (x_i, y_i), the closest predator can be expressed as:

$$\text{Closest Consumer} = \min_i \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2} \tag{42}$$

The Produce needs to move away from the nearest Predator, setting the deflection angle to θ_{avoid}, such that:

$$\theta_{avoid} = \arctan 2\,(Y_{closest} - Y_i, X_{closest} - X_i)\theta_{new} = \theta_{avoid} + \pi \tag{43}$$

Based on these two models, c-p discovers some rules, p-c is also regular. The innovation point, the steps optimize the complex equations for chaotic, reducing the amount of computation, but the research on individual data still needs to continue.

4.8. Modeling 8: an $\boldsymbol{\varepsilon - DFA}$ system evolution model

To describe the historical general laws of this system, the author uses an $\varepsilon - \text{DFA}$ to discuss it.

Next let's look at the historical regularity of this ecosystem, the running program, it is not difficult to find that the running of organisms in the ecosystem always jumps in several stable States, so we can construct a deterministic finite automaton: DFA $(\Sigma, Q, \text{start}, F, \delta)$ where Σ is the character set, which is the state of the ecosystem here, Q is the set of states which will be constructed step by step in the following text, start is the start state and maintains $\text{start} \in Q$, F is the set of accepting states and maintains $F \subseteq Q$, is a set of special states, and δ is the transition function.

Here, the automaton can be represented in Figure 7.

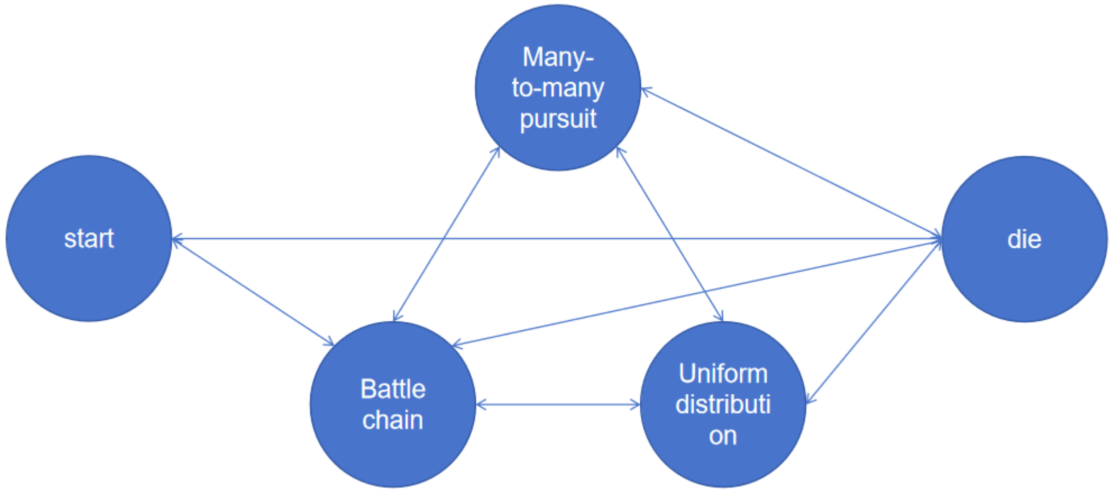

Figure 7. The deterministic finite automaton

It is to be noted that all the transitions here are $\varepsilon -$ transitions, and the right-hand side is changed to an undirected, the mathematical description of which has already been mentioned above. But only one state is running at a time. Also note that this state machine may self-replicate, i.e., each of the evo-lutionary regions of the ecosystem will be described by a copy of this state machine, and they may not be in the same state.

Finally, these states will regularly migrate to other state sheets, and there will only be one state. This model can be further generalized to describe the transition relations between these states and the reasons why the entities in the ecosystem can replicate the state machine.

5. Conclusion

Above is a description of some of the details, and the mathematical description of this ecosystem is partially completed.

Model 1 sets the basis for the following, 2 and 3 develop the motion of the two roles, 4 the general rules at the start of simulation; 5 and 6 the analysis of the battle chain and its variations; 7 c-p/p-c describes the individual in relation to the individual; attempts to generalize the historical laws of this ecosystem.

This research presents clear innovations. First, there is a methodological innovation. The study introduces the artificial fish swarm algorithm and DFA theory into modeling, providing new tools for the analysis of complex systems. Secondly, there is the simplification of the model. The research proposes a more understandable and computable producer-consumer model suitable for simple closed ecosystems, filling gaps in existing research. Additionally, the study reveals the synergistic predation strategy of consumer groups and the predator-avoidance mechanism of producers, deepening the understanding of the micro-dynamics of ecosystems. The author integrates mathematical, computer science, and ecological theories, providing a comprehensive framework for the study of closed ecosystems.

This study only considers the influence of seasonal changes on the speed of organisms and the limiting effect of total oxygen on the survival of organisms, without delving into other important environmental factors, such as the specific impact of temperature changes, the influence of light intensity and spectrum distribution on the photosynthesis of producers, and the effect of water quality changes (such as pH) on the survival of organisms. In real ecosystems, these environmental factors interact with each other and jointly affect the growth, reproduction, and survival of organisms. Furthermore, the model simplifies the closed ecosystem into a binary relationship between producers and consumers, neglecting other potential biological roles within the ecosystem, such as decomposers and parasites. At the same time, the model assumes that organisms are only limited by factors like oxygen and food, without considering the complex interspecies relationships such as symbiosis and competition.

For further research, other researchers should explore the comprehensive effects of environmental factors, key biological roles such as microorganisms, and validate the ecological adaptability of the model through experiments. Specifically, future studies can focus on the following directions: first, when establishing ecosystems, future research should incorporate the dynamic interactions of variables like temperature fluctuations, light intensity, and water quality (e.g., pH and nutrient concentration). Introducing microbial groups to simulate their role in organic matter decomposition and nutrient regeneration, and analyzing their interactions with producers and consumers, would provide deeper insights. Further exploration of interspecies relationships, such as symbiosis, competition, and parasitism, is also needed.

Additionally, current research mainly focuses on the short-term evolution of ecosystems. Future studies should extend the simulation time scale to analyze the cumulative effects of long-term climate change on ecosystems. Furthermore, extreme disturbance experiments, such as sudden pollution events or species invasions, can be designed to assess the model's response mechanisms in non-equilibrium states, providing theoretical support for ecological risk management.

Finally, the algorithm and computational efficiency of the model need optimization. The current model is based on the artificial fish swarm algorithm, and there is potential to explore integration with other intelligent algorithms, such as particle swarm optimization or reinforcement learning, to enhance the decision-making process of biological behaviors.

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