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Research on the prediction of elderly population structure in Shanghai based on GM(1,1)

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Abstract. The issue of population aging has long been a key focus in the field of demography, involving multiple factors such as social development, economy, and policy. This study utilizes the grey prediction GM(1,1) model, calculating residuals between actual data and predicted values, then applying grey prediction to the residuals. By superimposing the initial forecast with the residuals, the model corrects errors and verifies that the resulting calculations exhibit high accuracy. Focusing on Shanghai's elderly population aged 60 and above, subdivided into young-old, middle-old, and old-old age groups, the error-corrected GM(1,1) method is separately applied to forecast changes in the proportion of elderly population across these cohorts. From the perspectives of economic and social sustainability, the results are analyzed to address population-related issues, and relevant recommendations are provided for policymakers' consideration.

Keywords:population prediction, population aging, differential equations, grey prediction, GM(1,1), error correction

1. Introduction

In recent years, with China’s modernization process and development, the issue of population aging has become increasingly severe [1]. Particularly in first-tier and new first-tier cities, the number of elderly individuals (aged 60 and above) has been rising annually (as shown in Figure 1), and most urbanized areas in China have entered the “silver age” era [2]. To adequately prepare for the challenges and issues brought about by population aging, numerous scholars have conducted research addressing the economic pressures, urban development planning, and elderly care demands resulting from the growing elderly population. Early studies, such as those by Mo Long, forecasted the population aging problem, identified the economic and demographic pressures it entails, and proposed relevant countermeasures [4]. Scholar Li He similarly pointed out the fiscal burden population aging imposes on China’s government expenditure and innovatively suggested measures such as encouraging the reemployment of retired elderly individuals [5]. Li Chenghua proposed an “age-friendly” urban development model against the backdrop of aging, exploring a sustainable development path [6]. Scholars including Sang Tong have focused on the cultural needs of elderly groups and promoted cultural elderly care at the community level [7]. Additionally, research has been conducted on strategies concerning the adequacy of elderly service facilities and specialized elderly care models [8]. Overall, academic research on population aging has become relatively extensive.

Research on population structure within the domain of aging issues also holds practical significance. Scholars such as Ni Xuanming have employed the Leslie matrix as a tool to derive population dynamic systems based on demographic assumptions, predicting the future convergence of China’s elderly population proportion [11]. Su Changgui and colleagues utilized elderly population structure data from Chenzhou to forecast future structural information of the elderly population group [12]. Generally, population studies related to aging focus on the elderly population group in relation to the total population size. However, addressing the severe aging trend anticipated in the future requires more in-depth research on the internal age distribution proportions within the elderly population. At present, there is a scarcity of academic research on the changing trends in age structure within the elderly group, representing a research gap in the field of population aging. Therefore, this paper stratifies the elderly population proportion, examining the issue by analyzing the proportions of different elderly age segments alongside the future development trends of the total population, and puts forward relevant recommendations.

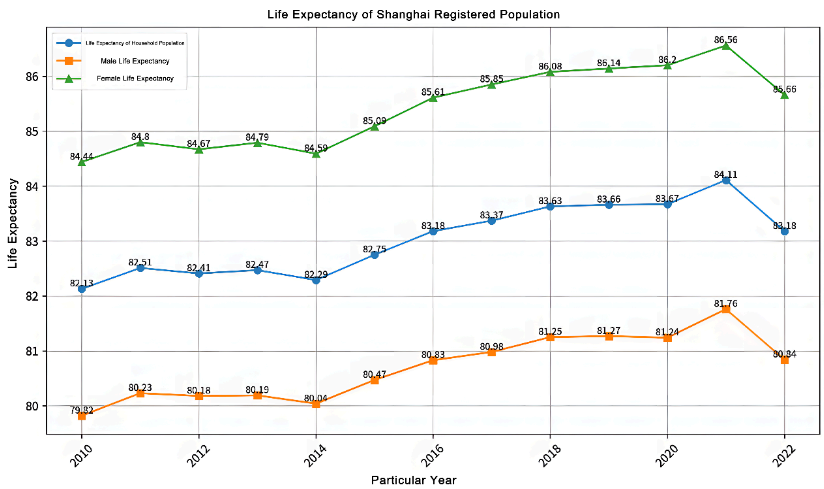

Figure 1. Life expectancy of registered population in Shanghai, 2010–2022

2. Elderly population quantity prediction model

Predictions related to the economic data and developmental trends of the elderly population are relatively rich and comprehensive; however, studies forecasting the population size within different age structures of the elderly group remain scarce. In this paper, the data is drawn from the elderly population statistics of Shanghai, sourced from the Shanghai Municipal Bureau of Statistics website, covering the period from 2015 to 2022. The population numbers of three elderly age groups—60 to 64 years old, 65 to 79 years old, and 80 years and above—are separately predicted, enabling a more detailed exploration of the depth of the aging issue. According to the age stratification of the elderly, we classify them into three categories:

1. Aged 60–64 (Young-old population): Characterized by relatively good health and certain labor capabilities; some individuals remain employed. Attention is focused on the issue of reemployment for this group.

2. Aged 65–79 (Middle-old population): Characterized by declining health but retaining a certain degree of self-care ability; most are retired and primarily rely on pensions. Key concerns for this group include medical security, community services, and elderly care.

3. Aged 80 and above (Old-old population): Characterized by significant decline in physical abilities, with many requiring care; the family caregiving burden is substantial. Critical issues include medical care and psychological support for the elderly within communities.

Given that elderly populations in different age groups have distinct care needs, social roles, and corresponding policy responses, researching the population size within each age segment holds practical significance. Such research can assist relevant stakeholders in making adequate preparations to address the severe aging challenges anticipated in the future.

Regarding the aforementioned age stratification, different elderly age groups correspond to distinct government policy responses. By statistically analyzing and forecasting the proportion of the elderly population aged 65 and above relative to the total elderly population, it is possible to evaluate the current and future elderly population size and the age distribution within a given region. For first-tier cities in China, such as Shanghai, which face severe aging challenges, a significant future increase in the population aged 65 and above would imply substantial elderly care demands and economic growth pressures in the coming years. Conversely, forecasting the proportion of the elderly aged 60 to 64 provides insight into potential solutions for aging-related issues; if the number of elderly individuals in this age group continues to rise, it could effectively promote the reemployment of the "young-old" elderly population and create new economic growth poles within the elderly demographic.

In this study, the forecasting for each elderly age group is based on recent data from the past several years. Since the emergence of the aging issue occurred approximately around the year 2000, the demographic data related to the aging population have experienced rapid changes, with a significant increase in the elderly population size. Therefore, short-term forecasting for the coming years is the primary focus of this research. Using the recent data from 2015 to 2022 as the known actual population baseline, this study predicts relevant elderly population data for the near future five years (2023–2027). Employing recent short-term datasets to forecast mid- to long-term elderly population trends constitutes the core of the elderly population prediction model developed in this paper.

3. Construction of the GM(1,1) prediction model

Using short-term data to predict mid- to long-term trends requires specialized forecasting methods. The grey prediction GM(1,1) model is a computationally simple yet highly accurate forecasting approach, commonly employed for estimating future values over the mid- to long-term horizon. This model and its variants have been widely applied in data prediction studies across various fields.

To improve prediction accuracy, this study employs an enhanced grey prediction GM(1,1) model. First, using actual data, the grey prediction model is applied to derive the prediction equation and obtain preliminary forecast results. By comparing these predicted values with the actual data, residuals $\varepsilon(k)$ are calculated, where $k = 2,3,...,8$ (with the initial statistical year 2015 counted as 1). Based on data from 2015 to 2022, the model forecasts elderly population numbers for the next five years (2023–2027). Simultaneously, grey prediction is also performed on the residuals, and the predicted residuals are combined with the initial forecast to compute the final predicted values. The detailed process flow is illustrated in Figure 2.

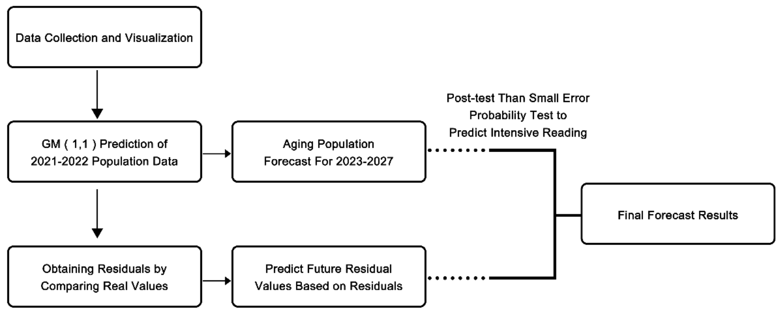

Figure 2. Process flow of the improved GM(1,1) prediction model

3.1. Data preparation

Based on the model described above, the analysis utilizes data from Shanghai [13]. First, the necessary information is obtained from the relevant Shanghai government statistical websites to form the sample set required for predicting the age structure of the elderly population in Shanghai (as shown in Table 1). This data serves as the foundation for constructing the GM(1,1) prediction model, providing a reference for future strategies to address the serious aging issue.

Table 1. Population numbers of elderly in Shanghai aged 60–64, 65–79, and 80+ from 2015 to 2022

Elderly Population\Year	2015	2016	2017	2018	2019	2020	2021	2022
Total Elderly Population	435.95	457.79	483.60	503.28	518.12	533.49	542.22	553.66
Age 60–64	152.57	158.77	165.93	166.38	156.46	151.05	139.85	129.26
Age 65–79	205.33	219.36	237.09	255.23	279.68	299.92	318.49	341.25
Age 80 and above	78.05	79.66	80.58	81.67	81.98	82.53	83.88	83.15

The above population data were cleaned and visualized to provide an intuitive representation of the actual dataset, as shown in Figure 3. The data from the table indicate a significant increase in Shanghai's elderly population over the past seven years, while the number of "young-old" elderly (aged 60–64) has markedly declined. This pattern is characteristic of a region experiencing severe population aging. Therefore, Shanghai serves as an authentic case study for analysis in this research.

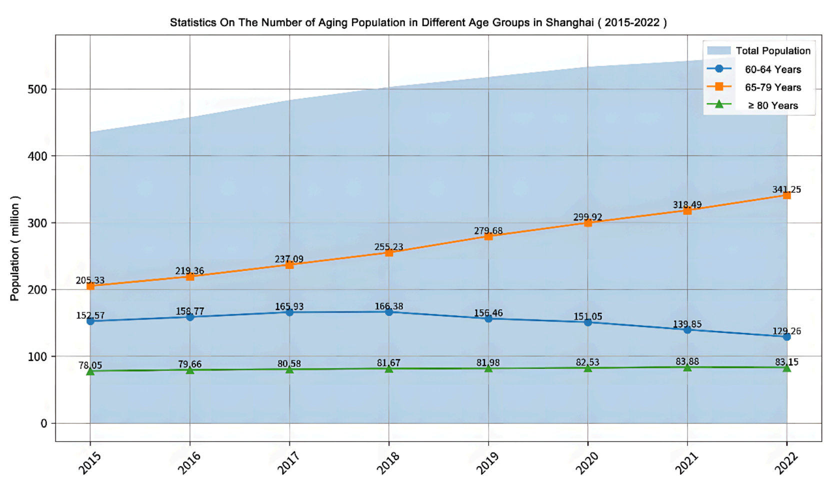

Figure 3. Age composition of the elderly population in Shanghai

3.2. Initial time series and testing

Taking the total population prediction as an example, the initial time series $X^{(0)} = \{x^{(0)}1, x^{(0)}2, \ldots, x^{(0)}n\}$ is obtained. In this study, the initial time series is $X^{(0)} = \{435.95, 457.79, 483.60, 503.28, 518.12, 533.49, 542.22, 553.66\}$, from which the initial sequence is derived.

First, the sequence undergoes a ratio test, denoted as the ratio $\mu(k) = \frac{x^{(0)}(k-1)}{x^{(0)}k}, k = 2,3,\ldots,n$. Typically, the acceptable range for the ratio $\mu(k)$ is $\left(e^{-\frac{2}{n+1}}, e^{\frac{2}{n+1}}\right)$; when $\mathrm{n} = 8$, the range is $\left(e^{-\frac{2}{9}}, e^{\frac{2}{9}}\right) \approx (0.8007, 1.2488)$. By applying the above formula, the data are preliminarily tested—if the ratio falls within the specified range, the sequence passes the test. The specific results are presented as shown in Table 2.

Table 2. Ratio test statistics

Year	2016	2017	2018	2019	2020	2021	2022
Ratio	0.9523	0.9466	0.9609	0.9714	0.9712	0.9839	0.9793

After passing the ratio test, to verify that data fluctuations fall within an acceptable range, the smoothness ratio is calculated. If the majority of the smoothness ratios are less than 0.5, the data are considered relatively stable. The smoothness ratio for the k-th data point is $\sigma(k) = \frac{x^{(0)}k}{\sum_{i=1}^{k-1} x^{(0)}i}$. Substituting the data into the formula, the calculated smoothness ratios are presented in Table 3.

Table 3. Smoothness ratio statistics

Year	2016	2017	2018	2019	2020	2021	2022
Ratio	1.0501	0.5411	0.3654	0.2755	0.2224	0.1849	0.1594

3.3. Residual calculation for initial data

Using the sequence $X^{(0)}$ as the baseline, the population data for 2023–2027 are predicted. The GM(1,1) model is employed here, and the steps are as follows:

(1) Generate the 1-AGO Sequence and Establish the ModelBy cumulatively summing the first k data points, the first-order accumulated generation operation (1-AGO) sequence $X^{(1)}$ is obtained as follows.

$$X^{(1)} = \{435.95, 893.74, 1377.34, 1880.62, 2398.74, 2932.23, 3474.45, 4028.11\}$$

Generated Adjacent Mean Sequence $X^{(1)}$:

$$z^{(1)}k = \frac{1}{2}\left(x_1^{(1)}k + x_1^{(1)}(k-1)\right), k = 2,3,\ldots,n$$

The adjacent mean sequence is obtained as follows:

$$Z^{(1)} = \{664.845, 1135.54, 1628.98, 2139.68, 2665.48, 3203.34, 3751.28\}$$

Establish the GM(1,1) differential whitening equation, where a, b represent the development coefficient and grey input, respectively:

$$\frac{dX_1^{(1)}}{dt} + aX_1^{(1)} = b$$

(2) Construct the Matrix to Solve the Differential Equation

Construct the Matrix $B = \left[Z^{(1)}(K) \quad 1\right] = \begin{bmatrix} -664.845 & 1 \\ -1135.54 & 1 \\ -1628.98 & 1 \\ -2139.68 & 1 \\ -2665.48 & 1 \\ -3203.34 & 1 \\ -3751.28 & 1 \end{bmatrix}$, $Y = \left[x^{(0)}k\right] = \begin{bmatrix} 457.79 \\ 483.60 \\ 503.28 \\ 518.12 \\ 533.49 \\ 542.22 \\ 553.66 \end{bmatrix}$, and solve for $\hat{l} = (a,b)^T = \left(B^TB\right)^{-1}B^TY$,thereby obtaining the coefficient solution of the differential whitening equation as follows:

$$a = -0.0338, b = 433.862$$

With $a = -0.0338$, the model equation is suitable for mid- to long-term forecasting. After solving for $\hat{l}$, the time response sequence $\widehat{x}^{(1)}(k+1) = \left(x^{(0)}1 - \frac{b}{a}\right)e^{-ak} + \frac{b}{a},\ k = 0,1,\ldots,n$ is obtained, from which the predicted values $\widehat{x}^{(0)}(k+1) = \widehat{x}^{(1)}(k+1) - \widehat{x}^{(1)}(k)$ are restored.

The predicted values obtained in this study are $\widehat{x}^{(0)}(k), k = 1,2,3,\ldots,13$, with a relative error of $\varphi(k) = \frac{\left|x^{(0)}(k) - \widehat{x}^{(0)}(k)\right|}{x^{(0)}(k)} = \frac{|\varepsilon(k)|}{x^{(0)}(k)}$ 。. The forecasted data for the years 2015 to 2022 are presented in Table 4.

Table 4. Comparison of actual and predicted values from 2015 to 2022

Year k	Actual Value $x^{(0)}$	Predicted Value $\widehat{x}^{(0)}$	Residual $\varepsilon(k)$	Relative Error $\varphi(k)$ %
2015	435.95	435.95	0	0
2016	457.79	447.54	10.25	2.24
2017	483.60	463.23	20.37	4.21
2018	503.28	479.63	23.65	4.70
2019	518.12	496.73	21.39	4.13
2020	533.49	514.56	18.93	3.55
2021	542.22	533.12	9.10	1.68
2022	553.66	552.43	1.23	0.22

3.4. Residual prediction and model validation

Building upon the previous application of the GM(1,1) prediction model, an initial time series ε of the residuals is constructed. The initial residual time series is as follows:

$$\varepsilon^{(0)} = \{0,10.25,20.37,23.65,21.39,18.93,9.10,1.23\}$$

The GM(1,1) grey prediction method is similarly applied to the residuals, with computational steps analogous to those described above. Solving yields $\hat{l} = (a', b')^T$, and the parameters are determined as $a' = -0.252, b' = 22.51$. The residuals are then statistically analyzed and combined with the initial predicted values to obtain the final forecast results.

Table 5. Final forecast results

Year	Original Predicted Value	Residual Predicted Value	Corrected Predicted Value
2015	435.95	0	435.95
2016	447.54	9.53	457.07
2017	463.23	18.23	481.46
2018	479.63	23.57	503.20
2019	496.73	25.54	522.27
2020	514.56	24.35	538.91
2021	533.12	20.32	553.44
2022	552.43	13.83	566.26
2023	572.47	5.44	577.91
2024	593.24	-3.78	589.46
2025	614.80	-12.58	602.22
2026	637.18	-21.04	616.14
2027	660.42	-29.17	631.25

Based on the data presented in Table 5, a line chart was plotted, as shown in Figure 4.

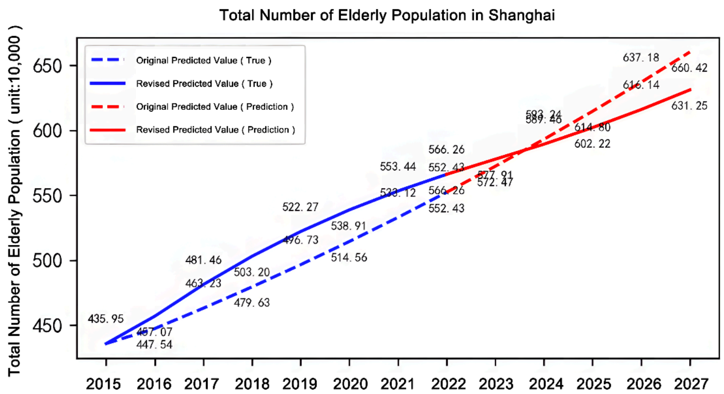

Figure 4. Predicted and corrected elderly population forecast

The uncorrected forecast data are evaluated due to the presence of prediction errors. A comparison is made between the actual and predicted values from 2015 to 2022. In this study, two methods are used to assess the accuracy of the prediction: the posterior error ratio and the small error probability test.

First, the standard deviation S_k of the initial actual value sequence and the standard deviation S_ε of the resulting residual sequence are calculated. Their respective formulas are $S_k^2 = \frac{1}{n}\left(x^{(0)}(k) - \overline{x}^{(0)}\right)^2$, $S_\varepsilon^2 = \frac{1}{n}\left(\varepsilon(k) - \overline{\varepsilon}\right)^2$, where $\overline{x}^{(0)}$ represents the mean of the initial data sequence, and $\overline{\varepsilon}$ is the mean of the residuals. The posterior error ratio is denoted as $C_k = \frac{S_\varepsilon}{S_k}$. Substituting the data into the formula yields $C_k = 0.27 < 0.35$. When $C < 0.35$, the model is considered to have high accuracy. Therefore, although the value here exceeds the threshold, the forecasted data still offer some reference value.

Next, the small error probability is calculated to further verify the prediction accuracy. For the residual values, the small error probability $P = P\left\{\left|\varepsilon(k) - \overline{\varepsilon}\right| < 0.6475 \times S_k\right\}$ is computed. Substituting the data into the formula yields $P = 0.9997 > 0.95$, indicating a high level of accuracy.

4. Forecast and structural analysis of the elderly population in Shanghai

4.1. Population forecast results

Based on the overall forecast results presented above, the predicted population figures for the elderly in the age groups 60–64, 65–79, and 80 and above are as follows (see Tables 6(a), 6(b), and 6(c)).

Table 6(a). Forecast of the elderly population aged 60–64 in Shanghai

Year	Original Predicted Value	Residual Predicted Value	Corrected Predicted Value
2015	152.57	0	152.57
2016	154.35	3.89	158.24
2017	157.45	7.19	164.64
2018	160.70	8.92	169.62
2019	164.12	8.26	172.38
2020	167.71	5.52	173.23
2021	171.47	0.09	171.56
2022	175.40	-5.47	170.03
2023	179.51	-11.77	167.74
2024	183.80	-18.65	165.15
2025	188.29	-25.94	162.35
2026	192.97	-33.54	159.43
2027	197.87	-41.34	156.53

Table 6(b). Forecast of the elderly population aged 65–79 in Shanghai

Year	Original Predicted Value	Residual Predicted Value	Corrected Predicted Value
2015	205.33	0	205.33
2016	218.11	1.23	219.34
2017	233.08	3.92	237.00
2018	249.19	5.94	255.13
2019	266.48	12.99	279.47
2020	284.99	14.67	299.66
2021	304.76	13.48	318.24
2022	325.83	15.14	340.97
2023	348.23	14.67	362.90
2024	372.04	12.95	384.99
2025	397.35	10.14	407.49
2026	424.24	6.41	430.65
2027	452.78	1.05	453.83

Table 6(c). Forecast of the elderly population aged 80 and above in Shanghai

Year	Original Predicted Value	Residual Predicted Value	Corrected Predicted Value
2015	78.05	0	78.05
2016	79.42	0.23	79.65
2017	80.81	-0.22	80.59
2018	82.21	-0.53	81.68
2019	83.63	-1.61	82.02
2020	85.06	-2.48	82.58
2021	86.51	-2.57	83.94
2022	87.98	-4.73	83.25
2023	89.47	-4.43	85.04
2024	90.98	-3.35	87.63
2025	92.51	-2.07	90.44
2026	94.06	-0.72	93.34
2027	95.64	0.87	96.51

Summarizing the above work, the predicted data are visualized alongside the actual population figures (as shown in Figure 5). In the figure, the light blue shaded area represents the actual and forecasted total elderly population in Shanghai (2015–2022 actual values and 2023–2027 predictions), while the three colored lines correspond to the population numbers for each elderly age group. It should be noted that the total population forecast does not equal the sum of the forecasts for the three age groups, as the total population and each age segment were predicted separately using the error-corrected GM(1,1) models described above.

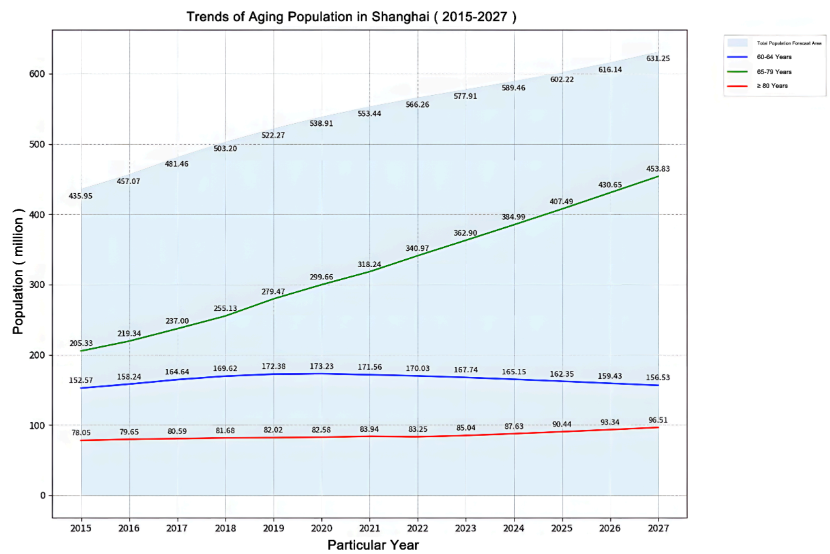

Figure 5. Forecast of total elderly population and population by age groups in Shanghai

To gain a deeper understanding of the future development trends of Shanghai's elderly population and to provide reference suggestions for relevant policy-making departments, it is necessary to study the proportions of elderly population across different age groups. The following discussion focuses primarily on the age group proportions within the elderly population. The proportions of each age segment in the elderly group are shown in Figures 6 and 7.

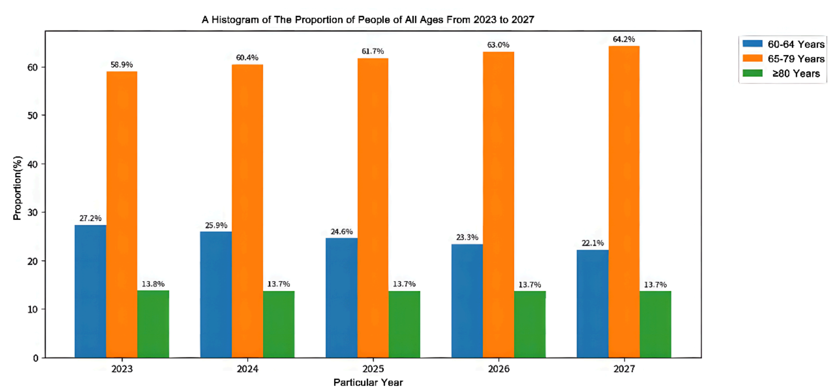

Figure 6. Proportion of elderly population by age group in Shanghai, 2023–2027

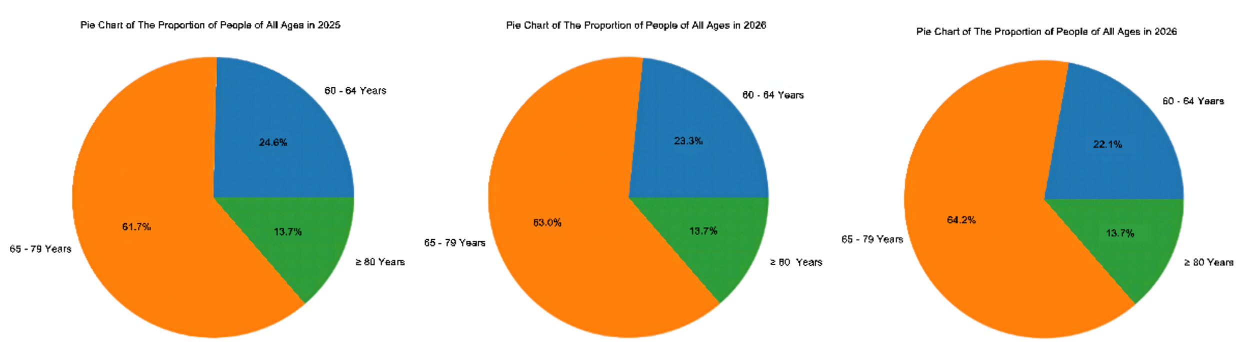

Figure 7. Pie chart of elderly population proportions (example: 2025–2027)

4.2. Future analysis and policy recommendations

4.2.1. Intensification of the aging population problem

1. Increased Longevity and Pressure on Healthcare and Nursing Systems

The proportion of the population aged 80 and above is projected to rise from 12.3% (850,400 individuals) in 2023 to 13.4% (965,100 individuals) in 2027, with an average annual growth rate of 3.2%. Nearly half of this group experiences disability or partial disability, leading to a surge in demand for long-term medical and nursing care services.

2. Continued Contraction of the Labor Supply

The proportion of the "young-old" aged 60–64 is expected to decline from 24.2% in 2023 to 22.1% in 2027. Coupled with the slowdown in the growth of the migrant labor force, the shrinking number of "young-old" elderly workers will pose challenges for economic growth. Although flexible retirement policies allow employees to extend their working years beyond the statutory retirement age, labor shortages persist in sectors such as manufacturing and services.

4.2.2. Countermeasures and policies

1. Promote the Construction of "15-Minute Elderly Care Service Circles" (Targeting the 80+ Age Group)

Fully utilize community-embedded elderly care by integrating community comprehensive elderly service centers with health centers and cultural centers to provide one-stop services including meal assistance, rehabilitation, and recreational activities. Promote home care beds equipped with smart mattresses and fall detection devices, with service fees covered under long-term care insurance.

2. Develop the Smart Elderly Care Industry (Targeting the 65–79 Age Group)

Actively promote the "Smart Home Elderly Care" platform, integrating telemedicine, health monitoring, and other functions to cover as many home-based elderly as possible throughout the city. Leverage Shanghai's suburban landscapes to attract elderly tourists from the Yangtze River Delta region—for example, develop a "Health + Cultural Tourism" demonstration zone on Chongming Island, featuring elderly-friendly accommodations and traditional Chinese medical therapies to attract senior visitors. Provide income tax exemptions for companies engaged in the research and development of medical, nursing, and health products for the elderly, and establish an industrial fund for the silver economy.

3. Implement Flexible Retirement Policies (Targeting the 60–64 Age Group)

Based on the predicted proportions of elderly population in each age group, flexibly adjust retirement ages for different job positions, with incentives for rehiring in certain roles. Provide additional pension benefits for "young-old" retirees who delay retirement and allow continued contributions to housing provident funds.

In summary, first-tier cities such as Shanghai and Guangzhou will face significant challenges related to population aging, which will also extend to new first-tier cities and inland regions of China. To address this issue, this study proposes a forecasting method that effectively analyzes the development trends of the elderly population over the coming years. The proportions of different age groups within the elderly population are also predicted and analyzed to gain a deeper understanding of the population structure, thereby enabling the flexible formulation of policies to respond accordingly.

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Appendix

The following is the core code snippet for the improved grey prediction model used to forecast the total population:

```
import numpy as np
def grey_model(X0):
n = len(X0)
# Ratio Test
lamb = X0[:-1] / X0[1:]
if not all(0.798 < l < 1.253 for l in lamb):
print("Ratio Test Not Passed, Model May Not Be Applicable")
else:
print("Ratio Test Passed, Model Applicable")
# First-Order Accumulated Generation
X1 = X0.cumsum()
# Adjacent Mean Generation
Z1 = (X1[:-1] + X1[1:]) / 2
# Construct Matrices B and Y
B = np.column_stack((-Z1, np.ones(n - 1)))
Y = X0[1:].reshape(-1, 1)
# Parameter Estimation
A = np.linalg.inv(B.T @ B) @ B.T @ Y
a, b = A.flatten()
# Prediction Function
def predict(X0, a, b, steps=5):
X1_pred = (X0[0] - b / a) * np.exp(-a * np.arange(len(X0) + steps)) + b / a
X0_pred = np.diff(X1_pred)
X0_pred = np.insert(X0_pred, 0, X0[0])
return X0_pred[:len(X0) + steps]
# Original Prediction
X0_pred = predict(X0, a, b)
# Residual Calculation
residual = X0 - X0_pred[:len(X0)]
# Grey Prediction of Residuals
a_res, b_res = grey_model_params(residual)
residual_pred = predict(residual, a_res, b_res)
# Corrected Prediction Values
X0_corrected = X0_pred + residual_pred
# Model Validation
C_original, P_original = model_evaluation(X0, X0_pred[:len(X0)])
C_corrected, P_corrected = model_evaluation(X0, X0_corrected[:len(X0)])
return X0_pred, residual_pred, X0_corrected, a, b, C_original, P_original, C_corrected, P_corrected
def grey_model_params(X0):
n = len(X0)
X1 = X0.cumsum()
Z1 = (X1[:-1] + X1[1:]) / 2
B = np.column_stack((-Z1, np.ones(n - 1)))
Y = X0[1:].reshape(-1, 1)
A = np.linalg.inv(B.T @ B) @ B.T @ Y
a, b = A.flatten()
return a, b
def model_evaluation(real, pred):
S1 = np.std(real)
residuals = real - pred
S2 = np.std(residuals)
C = S2 / S1
mean_res = np.mean(residuals)
threshold = 0.6745 * S1
P = np.mean(np.abs(residuals - mean_res) < threshold)
return C, P
if __name__ == "__main__":
X0 = np.array([78.05, 79.66, 80.58, 81.67, 81.98, 82.53, 83.88, 83.15])
X0_pred, residual_pred, X0_corrected, a, b, C_original, P_original, C_corrected, P_corrected = grey_model(X0)
```

```
print(f"Development Coefficient a: {a:.4f}")
print(f"Grey Action Quantity b: {b:.4f}")
print("Original Predictions and Residuals from 2015 to 2022：")
for i in range(len(X0)):
print(f"Year: {2015 + i}, Original Predicted Value: {X0_pred[i]:.2f}, Residual: {X0[i] - X0_pred[i]:.2f}, "
f"Relative Error: {((X0[i] - X0_pred[i]) / X0[i] * 100):.2f}%")
print("Corrected Predictions for 2023 to 2027：")
target_residual_pred = [0, 0.23, -0.22, -0.53, -1.61, -2.48, -2.57, -4.73, -4.43, -3.35, -2.07, -0.72, 0.87]
for i in range(8, 13):
X0_corrected[i] = X0_pred[i] + target_residual_pred[i]
print(f"Year: {2015 + i}, Corrected Prediction Values: {X0_corrected[i]:.2f}")
print(f"Posterior Error Ratio of Original Model C: {C_original:.4f}, Small Error Probability P: {P_original:.4f}")
print(f"Posterior Error Ratio of Corrected Model C: {C_corrected:.4f}, Small Error Probability P: {P_corrected:.4f}")
```