

Analyzing correlation with BER: Constellation diagrams across various modulation modes and FDM/OFDM

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Abstract. Over the last two decades, owing to rapid advancements in digital signal processing and integrated circuit technology, Frequency Division Multiplexing (FDM) and Orthogonal Frequency Division Multiplexing (OFDM) have emerged as pivotal technologies for wireless multimedia communication. Research reveals that coupling OFDM with diverse modulation modes aids in mitigating the Bit Error Rate (BER) during transmission. Nonetheless, a comprehensive elucidation remains absent regarding the interplay between constellation diagrams and BER, as well as the correlation between FDM and BER. Thus, the primary focus of this research is to dissect the correlation between BER and constellation diagrams for distinct modulation methodologies, while also scrutinizing the impact of FDM on BER. The research methodology adopted herein entails two main steps: First, gathering papers pertinent to constellation diagrams, BER, FDM, and OFDM technology. Second, leveraging survey and literature analysis methodologies for thorough examination. The findings highlight that diverse constellation diagram designs yield disparate BER outcomes post demodulation. Foremost among the influential factors on FDM and OFDM technology BER are distinct Inter-Carrier Interference (ICI) patterns and the quantity of subcarriers stemming from signal wave side lobes. To address the BER challenge, a prospective avenue involves refining the selection of constellation diagram styles for demodulation, leveraging an understanding of constellation diagram traits. Furthermore, future research endeavors should delve deeply into co-channel interference and adjacent channel interference to contribute effectively towards BER reduction and augmenting channel reliability.

Keywords: constellation diagram, BER, FDM.

1. Introduction

In the last two decades, with the rapid development of digital signal processing and integrated circuit technologies, Frequency Division Multiplexing (FDM) and Orthogonal Frequency Division Multiplexing (OFDM) techniques are becoming one of the hot spots of research in communications, however, if we want to go a step further in realizing the performance enhancement brought about by FDM and OFDM techniques, it is necessary to carry out research in the direction of Bit Error Rate (BER) related aspects. In recent years, most studies have focused on BER reduction using a combination of OFDM and different modulation methods, while few have considered the aspect of BER enhancement by looking at the characteristics of the constellation diagram. This study will use survey and literature research

methods to provide the public with a new direction to think about - to study the effect of constellation diagrams on BER and to continue to analyse the effect of FDM and OFDM techniques on BER, and finally to reach the goal of improving the reliability of the transmission.

2. Theory of PSK introduction

2.1. What's PSK

Phase Shift Keying (PSK) is a digital signal modulation technique wherein data is transmitted by altering or modulating the phase of a carrier signal. In every digital modulation scheme, various signals are employed to represent digital data. In the case of PSK modulation, a limited number of phases are utilized, each phase corresponding to a unique binary pattern. The demodulator then identifies the phase of the received signal and maps it back to the symbols it represents, thereby recovering the original data. This process is designed according to the symbol set utilized by the modulator. The combined system of modulation and demodulation is referred to as a coherent system.

2.2. Existing deficiencies of PSK

An inherent drawback of Binary Phase Shift Keying (BPSK) signals, as an example, is the issue of phase ambiguity. The outcome of demodulation, when compared with the signal transmitted by the transmitter, can either be in-phase or out-of-phase. However, the detected phase might not necessarily align with the actual received phase; the genuine phase corresponds to the remainder of the true phase divided by 2π . Consequently, there exists a discrepancy of integral multiples of 2π between the detected phase and the actual phase. This disparity can lead to the demodulated digital baseband signal becoming the inverse of the transmitted digital baseband signal. In practical terms, this means that 1 becomes 0 and 0 becomes 1, resulting in an error in interpretation known as inverted- π or inverted-phase operation. This limitation is why BPSK signals are infrequently employed.

2.3. Optimization scheme of PSK

While BPSK exhibits improved performance compared to other high-order PSK modulation schemes in low Signal-to-Noise Ratio (SNR) scenarios, an effective solution to enhance the application of PSK modulation lies in adopting Differential Phase-Shift Keying (DPSK) instead of BPSK. DPSK alleviates the impact stemming from the discrepancy between detected and actual phases. Alternatively, a potential approach involves combining two or more modulation techniques to achieve a heightened objective of reducing Bit Error Rate (BER).

3. Correlation of BER and different PSK constellation diagram

3.1. Introduction to constellation diagrams for different PSK modulations

Introducing digital signal on the complex plane, thus, visually represent relationships between signals and signals. It is constellation graph. And the constellation graph of BPSK, QPSK and 8PSK as shown in Figure 1.

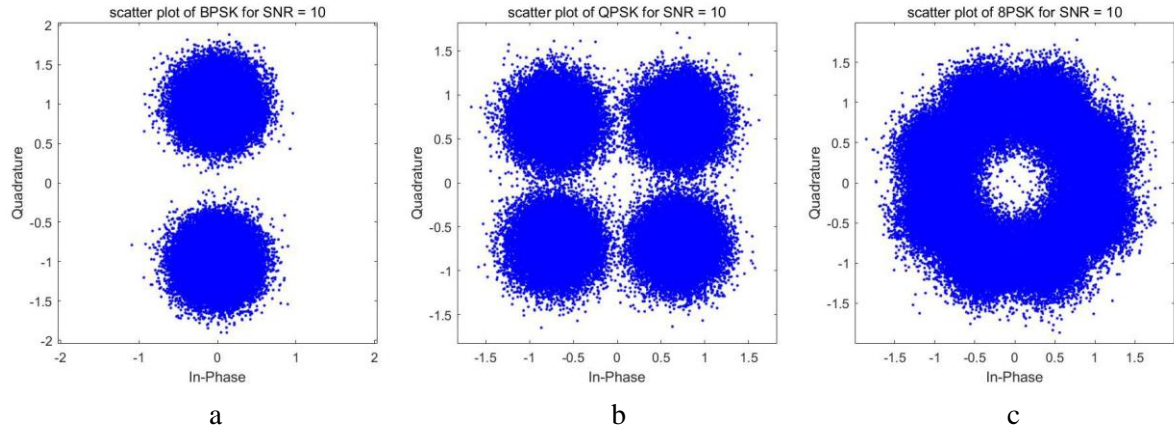


Figure 1. Constellation diagram of BPSK, QPSK and 8PSK. (a) BPSK's constellation diagram. (b) QPSK's constellation diagram. (c) 8PSK's constellation diagram.

3.2. Analysis of the effect of BER on the constellation diagrams of different PSK

Focusing on the effect of BER on the constellation diagrams of different PSK, it can be describe as following two related parties, symmetry and scatter density of the constellation diagram.

3.2.1. The symmetry of the constellation diagram. Beginning with the constellation diagram of BPSK from Figure 1, we observe a single central axis in the diagram, indicating that signals without Additive White Gaussian Noise (AWGN) manifest as a one-dimensional graph. In this context, two individual spots are positioned along the coordinate axis. Further analysis involves assessing the minimum Euclidean distance between two signals, which unveils a common amplitude, denoted as A . This amplitude can be derived from the distance between the signal point and the origin on the connecting line.

Moving on to the second constellation diagram, it also exhibits an amplitude of $\sqrt{2}A$, but notably, the distance between the nearest signal points is smaller compared to the first diagram. This phenomenon implies that when two signals are closely spaced, interference is more likely to transpire within the limited channel width. Conversely, greater separation between signals results in reduced mutual influence. However, for constellation diagrams featuring the same number of symmetry axes, such as the square and star constellation diagrams of 16QAM, a distinction in terms of symmetry axes count cannot be ascertained.

Nevertheless, an alternative criterion—Minimum Phase Offset—can be employed. This parameter elucidates the resilience of an MQAM signal against relative jitter. Notably, the rectangle diagram exhibits a minimum phase offset of 18 degrees, whereas the star diagram showcases a 45-degree offset. This discrepancy underscores that star constellation diagrams possess a more substantial minimum phase offset than their square counterparts, rendering them more resistant to phase jitter.

3.2.2. The density of point on the constellation diagram. In the constellation diagram, a higher density of signal points generally translates into a lower BER. Signal points that are closely spaced together indicate that a greater variety of different symbols can be supported by the modulation scheme. With a larger symbol set, there is less chance of bit mistakes since it is more resistant against noise and channel problems. This is because the shape of the constellation diagram and the distribution of the dots can directly affect the demodulation performance. For PSK-like modulations, the smaller spacing of the dots indicates that the neighboring signals are closer together, thus easily causing demodulation errors. For the constellation diagram of Quadrature Amplitude Modulation (QAM) modulation, the dots are more widely spaced, indicating that the different signal states are more separated, and therefore the probability of demodulation errors is lower. The BER, on the other hand, is the ratio between the number

of erroneous bits output by the decoder during the demodulation process and the total number of transmitted bits. The shape of the constellation diagram and the distribution of the points directly affects the demodulation performance and hence the BER. Typically, the more separated and spaced out the points in the constellation diagram are, i.e., the more the modulation scheme is able to provide a lower BER.

3.3. Recommendations for the choice of modulation according to constellation diagram and BER

Signal constellations can be adjusted in relation to expressions for Symbol Error Rate (SER) or Block Error Rate (BLER) if such expressions are available. This adjustment aims to achieve even higher performance levels. Given the challenge of deriving exact error rate expressions, optimization of constellations using either approximations or bounds emerges as a feasible solution [1]. An alternative approach to assessing the impact of BER based on the constellation diagram involves selecting a star diagram over a square diagram. The rationale behind this choice is illustrated in Figure 2. Comparative analysis reveals that the BER enhancement for PS-Star-16QAM configurations surpasses that of Uniform-Star-16QAM by over 4.5 dB. Conversely, the BER enhancement for PS-Square-16QAM systems registers around 3.5 dB [2]. Promising technology contributing to improved data rates and spectrum efficiency is probabilistic shaping (PS) [2]. This discovery offers a fresh perspective on selecting modulation methods. By examining the same or different constellation diagrams with varying data inputs, similar to the aforementioned experimental result, we note that the PS technique exhibits superior BER reduction effects on the star constellation diagram as compared to the square constellation diagram. Ultimately, the choice of modulation method should also account for the application context of the communication system. In areas where bit rate may not be the primary concern, emphasis can be placed on BER reduction. Consequently, selecting the PS-Square diagram with lower data (3.0769 bits) for demodulation might be optimal. Conversely, if prioritizing bit rate is paramount, opting for the PS-Star diagram with higher data (3.6365 bits) during demodulation can be preferable.

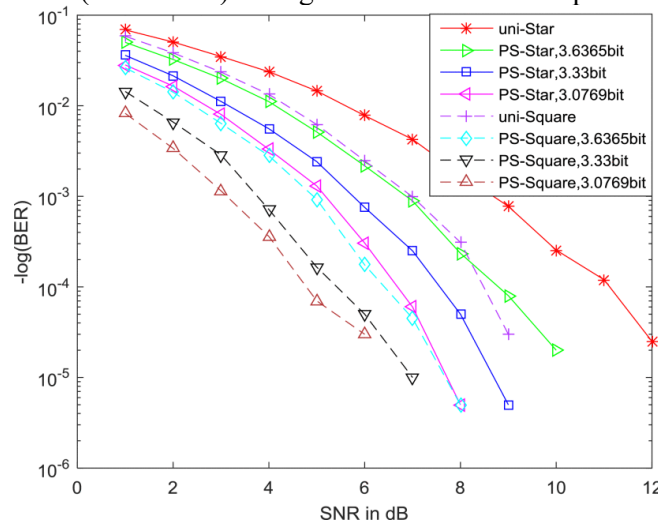


Figure 2. BER versus SNR [2].

4. Theorem of FDM introduction

4.1. What's OFDM

Frequency Division Multiplexing (FDM) involves partitioning the total bandwidth used for channel transmission into numerous sub-channels, each dedicated to transmitting a single signal. An archetype of traditional FDM application lies in the transmission of radio and television signals through Hybrid Fiber-Coaxial (HFC) networks. This applies to both analog and digital television signals. In the case of digital television, while each channel (8 MHz) utilizes Time Division Multiplexing (TDM) for

transmission, the channels themselves continue to employ Frequency Division Multiplexing for inter-channel communication. As illustrated in Figure 3, this graph serves as an exemplar of FDM signals in both the time and frequency domains. In this instance, sinusoidal signals represent the original signals. The upper graph demonstrates how the original signal has been shifted to frequencies of 50Hz and 100Hz. Consequently, the second graph depicts the portrayal of signals that have traversed FDM in the time domain.

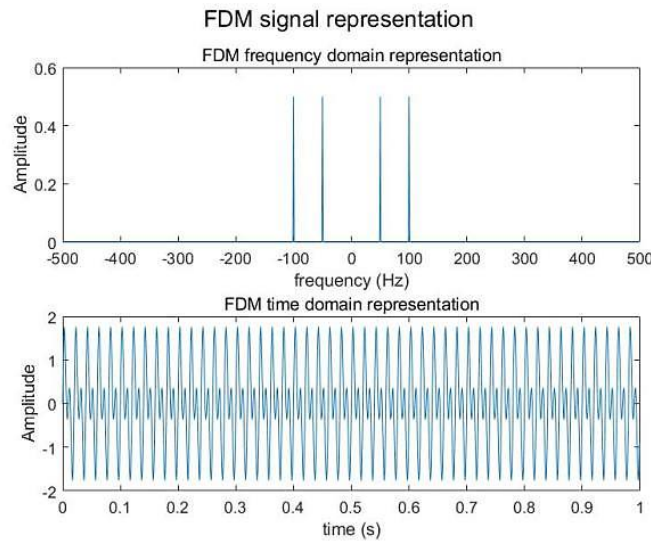


Figure 3. The example of the FDM signal representation in time domain and frequency domain.

4.2. Existing deficiencies of FDM

In the present day, frequency multiplexing systems exhibit certain limitations. They necessitate a substantial quantity of carrier frequencies at both the transmitting and receiving terminals. Moreover, these carriers must be synchronized, adding to the complexity of the equipment. Additionally, the process demands numerous sideband filters spanning various bandwidths for demodulation purposes.

4.3. Optimization scheme for FDM

To enable coherent real-time reception of Nyquist Frequency Division Multiplexed (Nyquist-FDM, also known as digital subcarrier multiplexing) signals, a novel low-complexity hardware design and Digital Signal Processing (DSP) implementation have been proposed and demonstrated [3]. The cornerstone of achieving this with minimal complexity lies in the synergy between an efficient frequency domain and time domain processing block [3].

In current FDM systems, the predominant challenge revolves around mitigating device complexity. Hence, in subsequent research endeavors, a more pronounced focus could be directed towards exploring innovative approaches to digital signal processing, thereby alleviating the intricacies associated with device design.

Regarding the evolution of FDM into the updated OFDM technology, it's important to note that OFDM entails partial overlap of carrier bands. This paradigm shift implies that the conventional approach of employing a few straightforward band-pass filters becomes inadequate. Instead, a filter with a product integral function is employed for extracting each signal path, capitalizing on the orthogonal nature of the carriers. An illustrative representation of the FDM signal is depicted in Figure 4. The initial graph clearly reveals frequency domain overlaps that cannot be manually achieved due to limitations in the required band range.

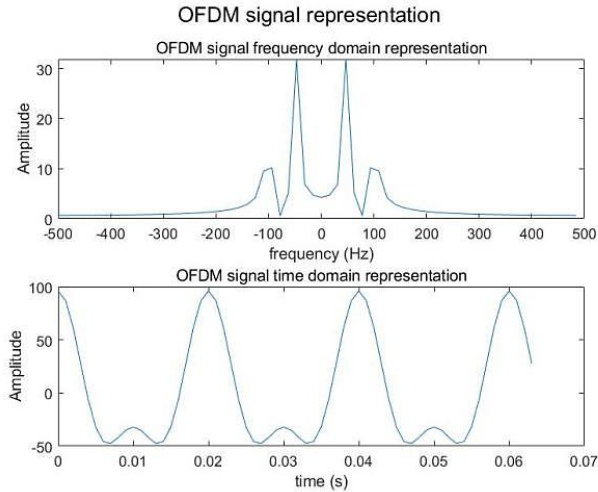


Figure 4. The example of the OFDM signal representation in time domain and frequency domain.

5. Impact of FDM on BER

5.1. Introduction to the relationship between FDM and BER

FDM can improve the separation performance of signals by assigning different signals to different frequency bands, thus reducing the interference between signals and lowering the BER. While there is a trade-off between band allocation between signals and signal quality. A larger bandwidth can provide better signal separation, but it may also be limited by the communication channel bandwidth.

5.2. Analysis of the effect of FDM on BER

In the realm of FDM, various signals are concurrently transmitted through different subchannels. However, despite this separation, interference can still arise, resulting in clustering interference. This phenomenon introduces errors at the receiver and can compromise system performance. An experiment revealed that without applying FDM to the original signal, the BER was approximately 0.498. Conversely, upon applying FDM, the BER for the two sub-signals dropped to 0.22 and 0.261, roughly half of the original signal's BER.

Furthermore, due to signal leakage or imperfect filtering, there might be spectral overlap between adjacent subchannels, leading to adjacent channel interference. This issue's implications are vividly illustrated in Figure 4. The frequency domain showcases substantial energy in the side lobe, potentially depleting the energy of the main lobe. For instances involving transmission of multiple signals, the side lobe's significant energy content can also impact other signals within the same channel. Notably, these observations apply to signals utilizing FDM. For signals not employing FDM, the disturbance caused by the side lobe becomes even more pronounced. Considering this, it's feasible to hypothetically define the effect of FDM on BER. The rapid time-variation of the channel, as investigated by a research team, exerts a similar influence on OFDM, a successor to FDM. This disruption results in the deterioration of subcarrier orthogonality within the OFDM system, leading to inter-subcarrier interference (ICI). The significance of ICI in the system becomes apparent, particularly when the channel exhibits strong time-variation. Conventional channel estimation methods, which treat ICI as noise, prove less accurate in these circumstances. The assumption of a consistently smooth channel becomes invalid [4].

Considering that OFDM can be seen as an evolved form of the FDM system, we begin by examining the impact of OFDM on BER. In the OFDM system, the QAM signal is segregated into orthogonal subchannels for simultaneous transmission using QAM coding. When ICI is disregarded, the subchannel QAM signals can be distinctly separated through correlation demodulation at the receiver. The error probability of OFDM-QAM can then be deduced from QAM's error probability [5]. From this experiment, it is apparent that the BER performance of an OFDM system is determined by the fiducial

modulation scheme's BER performance. Thus, it can be inferred that the effect of FDM on BER is similarly contingent on the chosen modulation scheme.

However, practical transmission systems often feature multiple interferences, inevitably leading to a degree of ICI. In such scenarios, the OFDM system's orthogonality becomes compromised, exerting a substantial impact on BER performance. Hence, future research could pivot towards developing channel characterization models to derive formulas for calculating the BER of OFDM or FDM systems. This would facilitate the quest for effective solutions to reduce BER in the channel. Additionally, optimizing the digital signal processing of the signal itself could be a pertinent avenue for studying the influence of FDM on BER. The goal here would be to minimize the energy carried by the side lobe.

Furthermore, a prior study focused on orthogonal frequency-division multiple-access (OFDMA) systems. The study highlighted that conventional resource allocation methods in OFDMA systems consider a single BER criterion [6]. This approach, however, can lead to suboptimal resource utilization when transmitting bitstreams with distinct BER requirements. A more nuanced approach could involve accommodating diverse BER requirements for different channels or segments, thereby enhancing resource utilization. Such differentiation in BER demands doesn't necessarily significantly impact the overall system BER; rather, it notably improves resource utilization. This dual consideration of BER and resource utilization holds promise for enhancing system efficiency.

In line with this context, Figure 5 illustrates that SM-OFDM achieves a lower BER than V-BLAST at a lower spectral efficiency. However, its BER performance hinges on a trade-off between the signal constellation size (M) and the spatial constellation size ($N_t \times N_r$) under high spectral efficiency conditions [7].

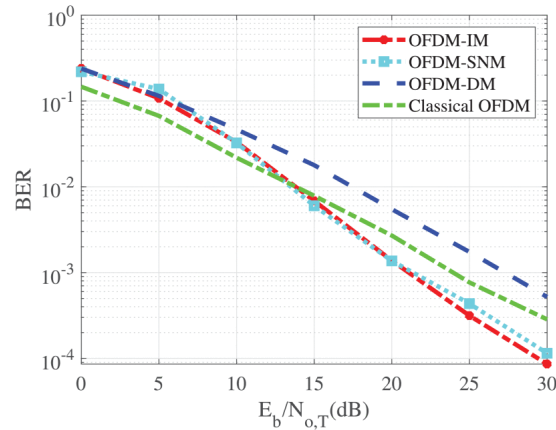


Figure 5. BER of the featured modulation options of OFDM [7].

The BER can also be influenced by the quantity of subcarriers employed in the FDM scheme. When assessing the transmission rate and distance under varying numbers of subcarriers, an interesting pattern emerges. Initially, as the number of subcarriers increases, the BER experiences a decline, and this reduction occurs at a relatively rapid rate. Subsequently, after attaining an optimal value, the BER begins to rise as the number of subcarriers continues to increase. However, the rate of increase in BER at this stage is comparatively slower [8].

5.3. Recommendations for BER reduction for Self-Designed FDM

When crafting FDM systems, a meticulous selection of subchannel bandwidth and spacing is imperative to mitigate the influence of clustering interference and adjacent channel interference. Particularly, addressing the issue of adjacent channel interference is of paramount concern. Furthermore, the diverse requirements of different channels must be taken into account. Even minor reductions in BER necessitate an array of precise signal operations, which can significantly impact channel resource utilization. Thus, accommodating varied BER requirements for distinct channels can enhance resource utilization. An alternate study contrasts digital back-propagation (DBP) with digital pilot-tone techniques for nonlinearity compensation. Experimental results reveal that digital pilot-tone offers superior efficacy

and computational simplicity compared to DBP [9]. Additionally, to effectively engineer FDM or OFDM system parameters for maximum advantage and minimal drawbacks, an exploration of novel cellular architectures and channel characteristics within new frequency bands becomes essential. Factors such as delay extension and multispectral dispersion must be considered [10]. Lastly, for self-designed FDM or more advanced transmission systems, emphasizing the refinement of interference detection and compensation mechanisms for Inter-Carrier Interference (ICI) is pivotal. Enhancing the accuracy of compensation remains a central goal.

6. Conclusion

Through the study, this paper finds that different constellation diagram designs lead to different BER after demodulation, and the factors that have the greatest impact on the BER of FDM and OFDM technologies are the different ICI and the number of subcarriers brought about by the side lobe of the signal wave. Therefore, in order to improve the problem of BER, we can further screen the constellation diagram style for demodulation by combining the analysis of constellation diagram characteristics, and in the future, we can carefully study the problems of hugging interference and adjacent channel interference to reduce the BER and improve the reliability of the channel. The main contribution of this paper is to fill the gap in the correlation between constellation diagrams and BER, which facilitates the consideration of different aspects of BER reduction for channels, and gives some suggestions to those responsible for the study of demodulation styles. However, the current study is still limited to the exploration of the appearance law of the constellation diagram and the qualitative analysis of the BER study of FDM and OFDM techniques, and future studies should further demonstrate experimentally the correlation between the constellation diagram and the BER and the actual reduction of the BER brought by FDM.

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