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Application of matrix in signal processing

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Abstract. Signal processing, a foundational discipline in modern technology, encompasses a diverse array of applications, ranging from audio and image processing to communication systems and medical imaging. This review investigates how matrix-based techniques are widely used to advance signal processing methodologies. In order to discretize continuous-time signals for digital processing, which occurs in the first section of the paper, matrices play a crucial role in signal sampling. A key principle, the Nyquist-Shannon Sampling Theorem, directs appropriate sampling rates to prevent aliasing, with matrices permitting effective signal representation. The effectiveness of matrix-based filtering methods for frequency modulation and noise reduction, such as convolution and correlation, is then investigated. By utilising matrix operations, these methods enable real-time signal processing. The Fourier Transform and Wavelet Transform are also featured in matrix-driven signal transformation, providing insights into frequency analysis and non-stationary signal characterization. By reducing noise components, matrix-based approaches, particularly Singular Value Decomposition (SVD) denoising, are essential for improving signal quality. Additionally, image compression employs SVD. Matrix-based compressive sensing revolutionises signal recovery from sparse data and results in data-efficient reconstruction. Signal processing has been transformed by matrix-based approaches, which have enabled previously unheard-of levels of efficiency, accuracy, and adaptability. The review highlights their significant influence on several signal processing fields.

Keywords: signal processing, matrix-based techniques, signal sampling, signal transformation.

1. Introduction

Matrices, rectangular arrays of numbers, symbols, or expressions organized in rows and columns. Although the idea of matrices has been around since antiquity, it wasn't until the 19th and 20th centuries that matrix theory was formally developed as a separate field of mathematics. Persian mathematician al-Khwarizmi employed them during the Islamic Golden Age [1]. Cayley introduced matrix algebra in the nineteenth century, describing operations such as addition, scalar multiplication, and matrix multiplication [2]. The notion of determinants was introduced by Gauss [3], and matrix rank was defined by Sylvester [4]. Łukasiewicz, Gelfond, and subsequent mathematicians furthered the formalization and applications of matrices. As the foundation of linear algebra today, matrix theory offers a strong framework for data representation and manipulation, enabling effective algorithms and techniques in many areas, including signal processing.

Signal processing is an interdisciplinary field that encompasses a wide range of techniques for manipulating and analyzing signals to extract meaningful information. These signals can come from different sources such as images, audio, video, radar, sonar, biomedical sensors, etc. In order to process

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and interpret these signals effectively, a rigorous mathematical foundation is essential. With its versatility and well-defined operations, matrices have become an indispensable tool in the field of signal processing. The use of matrices in signal processing dates back to the early developments in the field. In the mid-20th century, with the advent of computers and digital signal processing technology, the processing of signals became more efficient and accurate. Matrices provide a natural representation of discrete signals and allow efficient computation of signal processing operations [5]. The pioneering work of researchers such as Wiener laid the foundation for the application of matrix-based techniques in signal processing [6].

The capacity of matrices to effectively describe and alter signals in both the time and frequency domains is one of the fundamental benefits of employing them in signal processing [7]. In the time domain, signals are represented as sequences of samples, and matrices enable the concise representation and processing of these samples. The employment of matrices in addition, subtraction, and multiplication operations allows for the implementation of both linear and nonlinear signal transformations. Matrix-based signal analysis and transformation are essential in the frequency domain. For instance, the Fourier transform, which enables the breakdown of a signal into its individual frequency components, can be visualised as a matrix operation. This frequency representation provides valuable insights into the spectral content of a signal, allowing for various applications such as spectral analysis, filtering, and modulation [7].

The intrinsic capability of matrices to handle vast volumes of data efficiently is another benefit of employing them in signal processing. Matrices allow for parallel computation and enable the exploitation of hardware acceleration techniques, such as GPUs, to process signals in real-time or near real-time scenarios [8]. Matrix-based signal processing techniques are appropriate for a variety of applications due to their scalability, including small embedded systems and large-scale data processing and communication systems. Additionally, matrices provide a structured framework for expressing and resolving signal processing issues. By formulating signal processing tasks as matrix operations, researchers and practitioners can leverage well-established mathematical techniques and algorithms. This results in effective implementations and the possibility of optimisation, enabling signal processing algorithms to operate more quickly and accurately [9].

In conclusion, the invention and use of matrices are integral to the history of signal processing. The use of matrices in signal processing has become quite popular as a result of its benefits, including effective representation, manipulation in the time and frequency domains, scalability, and methodical problem-solving. By harnessing the power of matrices, researchers and practitioners continue to advance signal processing techniques, enabling innovative applications across various domains. Then, start to elaborate on the "Signal Sampling" portion, referencing the Nyquist-Shannon Sampling Theorem, pertinent works, and correctly notating the equation in matrix form.

2. Method

2.1. Signal sampling

In order to process continuous-time signals digitally, a crucial step in signal processing is signal sampling. Due to their ability to effectively represent and reconstruct signals from discrete samples, matrices are essential to this process [10]. The Nyquist-Shannon sampling theorem serves as a guiding principle for proper sampling rates to prevent aliasing.

A key conclusion in signal processing is the Nyquist-Shannon theorem, sometimes referred to as the Nyquist Sampling Theorem or the Sampling Theorem. It demonstrates that a continuous-time signal may be accurately reconstructed from its discrete samples if the sampling frequency is at least twice the highest frequency present in the signal. The sampling rate must be greater than or equal to double the signal's bandwidth in order to prevent aliasing.

From a series of real numbers, a continuous-time bandlimited function can be created using the Whittaker-Shannon interpolation formula or sinc interpolation [11].

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$$x(t) = \sum_{n=-\infty}^{\infty} x[n] sinc\left(\frac{t-nT}{T}\right) \quad (1)$$

Mathematically, this theorem can be represented using matrices as follows:

$$x(t) \to x[n] = x(nT_s) = (T_s \cdot n)\ where\ T_s = \frac{1}{2f_s} \quad (2)$$

Here, $x(t)$ is the continuous-time signal, $x[n]$ is the discrete-time signal obtained through sampling, T_s is the sampling interval, f_s is the sampling frequency, and n represents the discrete time index.

In this equation, we can represent the signal sampling process using matrices. The continuous-time signal $x(t)$ can be converted into its discrete-time counterpart $x[n]$ by sampling at regular intervals T_s [12]. The matrix notation $[x[0]x[1]x[2]\cdots]$ represents the discrete samples of the signal $x(t)$ at discrete time indices $n = 0, 1, 2, \cdots$.

2.2. Signal filtering

Matrix-based filtering techniques are widely used to process signals by modifying their frequency content or attenuating unwanted noise and interference [13]. Commonly used techniques include convolution, correlation, and adaptive filtering, all of which may be properly expressed and put into practise using matrices.

Techniques for filtering and analysing signals that are essential include convolution and correlation. One signal (the input) is convolved or correlated with another (the kernel or template) in these procedures, which are theoretically described as matrix operations. These procedures may be carried out more effectively thanks to matrices, which makes them computationally possible for real-time signal processing [14].

2.3. Signal transformation

A potent tool for examining the frequency content of signals is the Fourier Transform. It breaks down a signal into its individual sinusoidal parts. Modern signal processing heavily relies on the Fast Fourier Transform (FFT), an effective approach for performing the Discrete Fourier Transform (DFT), which uses matrix operations [15]. Noise reduction is essential in signal processing to improve the quality and accuracy of signals in noisy environments. In order to reduce undesired noise while maintaining signal integrity, matrix-based approaches offer effective denoising algorithms and filters [16]. SVD is a matrix factorization technique widely used in denoising applications. Singular values, right singular vectors, and left singular vectors are the three components that make up a signal matrix. To effectively minimise noise components, small isolated values can be removed or attenuated [17]. The technique can also be used to compress and simplify signals.

2.4. Signal recovery

Signal recovery involves retrieving signals from incomplete or degraded observations . By taking advantage of the inherent sparsity of signals, matrix-based approaches, particularly in the area of compressive sensing, have revolutionised signal recovery [18].

3. Process and result

Signal sampling is a critical process that converts continuous-time signals into discrete-time representations, enabling digital processing. The Nyquist-Shannon Sampling Theorem, a cornerstone of signal processing, establishes the necessary conditions for accurate signal sampling. The importance of proper signal sampling cannot be overstated. It forms the basis for the accurate representation and faithful reconstruction of continuous-time signals in the discrete domain, forming the foundation for various digital signal processing tasks. Proper sampling is particularly crucial in applications such as audio and image processing, communication systems, and medical imaging, where the preservation of signal integrity is paramount. Numerous literature achievements have contributed to the exploration of optimal sampling strategies, extending beyond the traditional uniform sampling.

3.1. Seminal works in signal sampling

The Nyquist-Shannon Sampling Theorem, which forms the bedrock of signal sampling, can be traced back to Claude Shannon's groundbreaking paper "Communication in the Presence of Noise" published in 1949 [12]. Shannon's work established the concepts of the theorem, showing that continuous signals may be precisely reconstructed from their discrete samples if the sampling frequency is at least twice the highest frequency contained in the signal. This work served as the cornerstone for information theory. With its solid structure for signal capture and reconstruction, this essential article revolutionised communication engineering and digital signal processing.

Another key figure in the development of sampling theory is the Swedish-American engineer Harry Nyquist. Nyquist's work in the 1920s laid the theoretical groundwork for Shannon's theorem, contributing the concept of the Nyquist rate and underscoring the need for adequate sampling to avoid aliasing [19]. Nyquist's work on signal-to-noise ratio and minimum sampling rates has been pivotal in shaping modern signal processing practices.

Beyond the Nyquist-Shannon theorem, modern signal sampling has been significantly influenced by breakthroughs in compressed sensing. David Donoho, a prominent mathematician, and statistician, has made remarkable contributions to compressed sensing theory. Donoho's work on sparsity-based signal reconstruction algorithms and the concept of compressed sensing in high-dimensional data recovery has opened new possibilities for efficient sampling and signal processing [20]. Compressed sensing leverages the sparsity of signals in certain domains, and it involves the use of measurement matrices to acquire fewer samples and reconstruct the original signal. The matrix formulation of compressed sensing algorithms allows for the efficient and accurate recovery of signals from a reduced set of non-uniform samples.

Moreover, the contributions of Tao and Candes are notable in advancing the understanding of the potential of sparsity-based sampling. Their work on mathematical theory and the design of robust sparse signal recovery algorithms has enabled compressed sensing to be applied successfully in various fields [21].

3.2. Practical applications of proper signal sampling

The concept of proper signal sampling finds ubiquitous applications in numerous practical domains. In audio processing, for instance, digital audio signals are sampled at rates higher than the Nyquist rate to capture the complete audio spectrum without distortion, the process involves matrix multiplications to obtain the discrete samples. This ensures that high-frequency components, essential for audio fidelity, are preserved during digital conversion. The principles of proper sampling are fundamental to the success of modern digital audio formats and high-quality sound reproduction [22].

In image processing, pixel sampling plays a pivotal role in preserving the original image details during digitization. Pixel sampling is achieved through matrix operations, ensuring that the original image details are preserved during the digitization process. Properly sampling image pixels is crucial in applications such as digital photography, medical imaging, and video processing [23]. Image sensors in digital cameras and medical imaging devices use proper sampling techniques to capture high-resolution images that accurately represent the visual information.

3.3. Thechinque used for signal filtering

Signal filtering is a fundamental aspect of signal processing, matrix-based techniques have proven to be highly effective in various filtering applications due to their efficiency and versatility. Two common filtering methods, convolution and correlation, are expressed as matrix operations, where one signal is convolved or correlated with another.

Convolution is a widely used technique for smoothing or modifying a signal's frequency characteristics. It involves sliding a filter kernel over the input signal and computing the weighted sum of the overlapping elements. The operation can be efficiently represented using matrix multiplication, making it suitable for real-time signal processing applications [24]. Correlation, on the other hand, is essential for signal matching and pattern recognition. It measures the similarity between two signals by

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sliding one signal over another and computing the dot product at each position. Like convolution, correlation can be efficiently implemented using matrix operations, enabling rapid and accurate signal analysis [24].

To illustrate the effectiveness of matrix-based filtering, Figure 1 displays a wave graph show the comparison. The noisy signal contains random noise that obscures the underlying pattern. However, after convolving the signal with a properly designed filter, the noise is significantly reduced, and the original signal is more discernible [25].

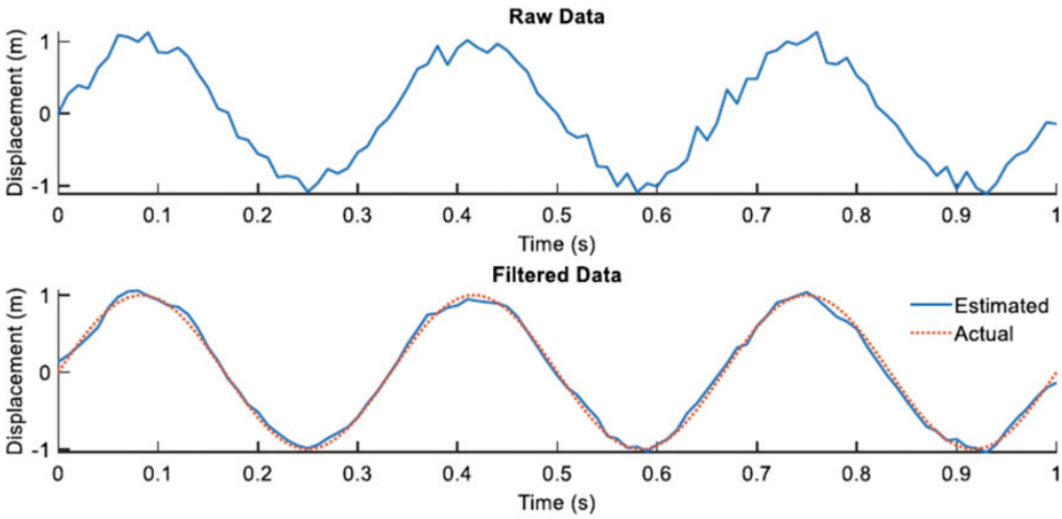

Figure 1. A 3 Hz sine wave that has been contaminated by noise in the top graph, followed by a comparison of the output from a three-point moving average filter with the original (actual) signal.

For instance, consider a scenario where a continuous-time signal represents a sound wave variation over time. The raw data might contain various sources of noise, including measurement errors, interference, or random fluctuations. As shown in Figure 1, the raw audio data appears to have a discernible pattern, but the noise makes it challenging to analyze and extract meaningful information from the signal.

To enhance the quality of the data and highlight the underlying pattern, a convolution-based filtering technique can be applied. The filtering process involves using a convolution kernel, designed to smooth the signal while preserving essential features. In matrix-based convolution, the filter kernel is represented as a matrix, and the convolution operation is performed through matrix multiplication.

As a result, the noisy sound signal is convolved with the filter kernel, effectively reducing the impact of noise and enhancing the clarity of the original pattern. The resulting filtered signal displays a smoother and more accurate representation of the sound variation, making it easier for analysts to interpret and extract valuable insights from the data.

Matrix-based filtering techniques, such as convolution, offer computational efficiency, making them suitable for real-time signal processing applications, including weather monitoring systems, audio processing and image enhancement. The ability to represent filtering operations as matrix multiplications contributes to the versatility and effectiveness of matrix-based signal processing techniques in a wide range of applications.

3.4. Signal transformation

Signal transformation techniques enable the conversion of signals between several domains, such as from time to frequency or from spatial to transform domains. A crucial technique in frequency analysis, the Fourier Transform may break down a signal into its individual sinusoidal components. It delivers useful details on the frequency content of a signal and is used in a range of industries, including audio

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processing, image analysis, and communication systems. The Fast Fourier Transform (FFT), a matrix-based technique that quickly computes the Discrete Fourier Transform (DFT), is a key component of modern signal processing [26]. By employing matrices for computation, the FFT significantly reduces the computational complexity of the DFT, making it applicable for real-time and large-scale signal analysis tasks. Another powerful transformation technique is the Wavelet Transform, which has gained widespread popularity in signal processing. The Wavelet Transform represents signals as a combination of wavelets of varying scales and positions, providing both frequency and time localization. This property makes it particularly useful for analyzing non-stationary signals and detecting transient events. Both the Discrete Wavelet Transform (DWT) and Continuous Wavelet Transform (CWT) leverage matrix-based operations for wavelet decomposition and reconstruction [27]. These matrix representations allow efficient computation of the transform coefficients and facilitate various applications, such as image compression, signal denoising, and feature extraction.

Noise reduction techniques are indispensable for enhancing signal quality and accuracy, especially in noisy environments or during signal acquisition processes. Matrix-based methods offer efficient denoising algorithms and filters that are widely used in various applications. Singular Value Decomposition (SVD) denoising is one such technique that has gained prominence. SVD is a matrix factorization method that decomposes a signal matrix into its singular components, namely the left singular vectors, singular values, and right singular vectors. In the context of noise reduction, SVD is used to identify and attenuate small singular values, which often correspond to noise components in the signal. By removing or dampening these noise-related singular values, SVD denoising effectively reduces the impact of noise while preserving the important signal features [16].

The SVD of a matrix can be used for image compression. Modern technology frequently uses digitised images, which are comparable to a matrix representing the value of each pixel's level of grey. Pictures made from natural sources typically have fairly huge size. For instance, the image in Figure 2 corresponds to a picture with large matrix size [28].

Figure 2. A picture with a matrix on the order of $204 \times 290 = 59610$ pixels.

We wish to represent the image with less numerical values than the original number of pixels in order to avoid the issue of transmitting or storing the numerical values of such large images. Calculating the singular value decomposition of an image and then reconstructing it using an estimate of reduced rank is one method of compressing an image. This method is seen in Figure 3, which a rank 25 approximation rebuilt almost exactly (to the human sight). This results in an 80% compression ratio [28].

Figure 3. The image in Figure 2 originated using 25 pairs of singular vectors with 80% compression rate.

3.5. Signal recovery

Signal recovery involves the retrieval of signals from incomplete or degraded observations. Matrix-based techniques, particularly those used in compressive sensing, have revolutionized signal recovery by exploiting the inherent sparsity of signals. Compressive sensing is based on the principle that a signal can be accurately recovered from a small number of non-uniform samples, provided the signal is sparse or compressible in some domain [29]. This breakthrough technique has significant implications for data acquisition and storage, enabling the reconstruction of signals from a fraction of the original data.

The matrix-based approach in compressive sensing involves the use of sparse matrices for signal representation and recovery. The signal is first represented using a sparse matrix, which captures the essential information while reducing redundancy [30]. Then, the recovery process utilizes matrix operations to reconstruct the original signal from the sparse measurements efficiently. Compressive sensing has found applications in various fields, including imaging systems, wireless communication, and sensor networks, where it offers significant advantages in terms of data reduction, energy efficiency, and robustness to noise and data loss. By leveraging matrices in signal transformation, noise reduction, and signal recovery, modern signal processing techniques have attained unprecedented efficiency and accuracy. The versatility and power of matrix-based methods have enabled their widespread adoption in numerous applications, revolutionizing the way signals are analyzed, processed, and utilized in various domains.

4. Conclusion

In the realm of signal processing, the utilization of matrix-based techniques has played an instrumental role in transforming the landscape of signal analysis, manipulation, and application. This review has delved into the profound impact of matrices across various signal processing domains, highlighting their pervasive presence and pivotal contributions.

The inception of matrix-based signal sampling has fundamentally shaped the way we acquire and represent signals in digital form. Guided by the Nyquist-Shannon Sampling Theorem, proper signal sampling has become a cornerstone, preventing aliasing and ensuring accurate signal reconstruction. Matrix-based signal filtering techniques, exemplified by convolution and correlation, have demonstrated exceptional efficacy in frequency modification and noise reduction. Signal transformation, a process facilitated by matrices, has empowered the shift between diverse signal domains, uncovering essential information vital to various industries. The pinnacle of matrix-based contributions lies in signal recovery, where compressive sensing has revolutionized the reconstruction of signals from sparse data.

In conclusion, the matrix-driven paradigm in signal processing has ushered in an era of unprecedented precision, efficiency, and adaptability. From sampling to filtering, transformation to noise reduction, and recovery to application, matrices have indelibly transformed signal processing into a dynamic, data-driven discipline. The marriage of mathematical elegance and practical utility positions matrix-based techniques as a cornerstone of modern signal processing, offering a myriad of tools that

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empower signal analysts and practitioners to unlock the full potential of signals across diverse applications.

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