

Use wavelet analysis and other algorithms for the process of image denoising in the mathematical field

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Abstract. By using sophisticated mathematical methods in MATLAB, the investigation into the complex field of image noise reduction presented in this paper is thorough. Like threshold denoising, correlation denoising, and modal maxima denoising. The use of both manual questionnaires and machine-based evaluation measures to fully evaluate the effectiveness of the denoising algorithms is a key component of this study. This dual strategy, which takes into account both objective and subjective components of image quality enhancement, ensures a well-rounded review. According to our research, correlation denoising consistently proves to be the most practical and effective method across a range of noise types and image categories, although modal maxima denoising and threshold denoising show promising outcomes in some situations. The thorough statistical analysis of our findings supports this conclusion, making it a convincing option for real-world picture denoising applications. In conclusion, this research broadens the range of image denoising techniques, which benefits both the field of image processing and the mathematical community. This study provides a holistic view of image denoising by incorporating wavelet analysis and other sophisticated algorithms, ultimately giving practitioners a deeper understanding of the techniques available for improving image quality in the presence of noise.

Keywords: Image denoising, Modal maxima denoising, Correlation denoising, Threshold denoising.

1. Introduction

Image denoising and image classification are essential fields of research in image processing. The presence of factors that interfere in the image and reduce clarity and comprehension is known as image noise. Many people discuss different algorithms for image denoising which include local smoothing, frequency domain filters and wavelet analysis¹. While frequency domain filters aim at reducing noise and reconstructing main geometrical configurations, they fail to preserve fine structures and textures.

The use of wavelet analysis in image denoising is an effective mathematical method for filtering noise from images. Evaluating the effects of different algorithms for image denoising is crucial, and it can be achieved through objective algorithmic evaluation and subjective human opinions. This process helps to increase the efficiency of these methods and obtain better results in image processing.

Different algorithms for image denoising exist, such as modal maxima denoising, correlation denoising and threshold denoising method, each with its unique strengths and weaknesses. It is also imperative to consider computation time when selecting an algorithm as it can affect the overall

performance of the algorithm. In this paper, I will analyze the advantages and disadvantages of each algorithm through machine evaluation and manual evaluation.

2. Literature Review

2.1. Introduction

2.1.1. Research problem restatement

Transmission image is going to be convenient and fast today. Many people transfer pictures in their daily lives. Which results in the image quality decrease and some image noise happening in the transmission. My research problem is to use wavelet analysis and other algorithms for the process of image denoising in the mathematical field. In order to improve the image quality and delete some image noises which generate in the process. My research goal is to command the process of image denoising and use wavelet analysis to do the image denoising work individually. Also, I would like to study other algorithm and apply it to the process of image denoising. Making a comparison between wavelet analysis and other algorithms can find the advantages and disadvantages of wavelet analysis.

2.1.2. Overview of the literature

Some literature discusses different algorithms for image denoising and image classification [1]. Some literature discusses the application of wavelet analysis in different image algorithms [2]. This shows that wavelet analysis plays an important role in the field of image processing. At the beginning of the literature review, I would like to share some definitions of my program. The noise in the image refers to Image noise refers to unnecessary or redundant interference information in image data. The existence of noise seriously affects the quality of remote-sensing images, which must be corrected before image enhancement and classification processing. All kinds of factors in the image that prevent people from receiving its information can be called image noise [1].

Image denoising is the process of reducing noise in digital images. In the process of digitalization and transmission, the digital image, in reality, is often affected by the interference of imaging equipment and the noise of the external environment, which is called noise image or noise image [1].

2.2. Research on image denoising methods with different algorithms

2.2.1. Introduction

There are many methods for image denoising, and the local smoothing method is one of them. and the frequency domain filters aim at noise reduction and at a reconstruction of the main geometrical configurations but not at the preservation of the fine structure, details, and texture [1].

Also, there's a lot of research on things like image processing by using a wavelet algorithm. They all have their advantages and limits. In a method of crack identification based on wavelet analysis, the basic vibration modes of cracked cantilever beams are analyzed by continuous wavelet transform, and the location and size of cracks are estimated [4]. The results show that the location of small cracks cannot be detected only by using the wavelet decomposition or curvature mode, but can be accurately identified according to the FD singularity of the detailed signal [4].

2.2.2. Conventional filter

According to the statistical distribution of image features and the different probability distribution of noise in the image, denoising algorithms are mainly divided into spatial domain denoising and frequency domain denoising [4].

2.2.3. Spatial domain image denoising

Spatial domain image denoising algorithms are relatively simple, most of them are directly connected to the gray value matrix of noisy image pixels for matrix transformation and data operation. Spatial

domain filters can be divided into linear and nonlinear types. The median filter belongs to linear type filter and the median filter belongs to nonlinear type filter [2].

The mean filter is a linear smoothing filter, which uses a small window with adjustable shape and size to move on the image, and calculates the gray value of each pixel through the gray value of the window. If the pixel is noise, it is smoothed into the mean value of the neighborhood, so as to achieve the purpose of denoising [8].

Median filtering is similar to mean filtering, which also realizes denoising by filtering high-frequency components. Median filtering is to replace the gray value of a pixel according to the median value of the gray value sequence of all pixels in a certain large and small neighborhood, that is, to move a small window on the image, and replace the gray value of the center position of the small window with the median value of the gray value of all pixels in the window [8].

2.2.4. Frequency domain image denoising

Frequency domain denoising is to apply Fourier transform, cosine transform or wavelet transform to the noisy image, transform the spatial domain signal into the frequency domain signal, and then do some processing to the conversion coefficient, and then reverse transform, in order to achieve the purpose of removing noise. Generally, the noise of the image is in the high frequency part, and the noise information of the high frequency component is filtered by the low pass filter, so as to get the denoised image information [2].

The recent debate on this subject is focused on the traditional image-denoising method and whether the new image-denoising method is better. Some people hold that traditional denoising has the advantage that it has a good denoising effect from the perspective of subjective vision. While others hold that although the data of subjective evaluation is not good, considering that the subtle differences in the denoising results cannot be well identified based on subjective visual evaluation, it is necessary to combine the objective evaluation method. After using PSNR and SSIM evaluation indexes to evaluate the denoising effect, the new algorithm has the highest value [3].

2.3. Application of wavelet analysis in different image algorithms

Wavelet analysis plays an important role in image denoising and image fusion [2]. It also supports many other subjects. We define the small wavelet analysis as a small wave of light. The so-called "small" means that it has attenuation; The term "wave" refers to its volatility, a pattern of oscillations with positive and negative amplitudes. Compared with the Fourier transform, the wavelet transform is a localized analysis of time (space) and frequency. It gradually refines the signal (function) by stretching and translating operation, and finally achieves time subdivision at high frequency and frequency subdivision at low frequency. It can automatically adapt to the requirements of time and frequency signal analysis, so as to focus on any details of the signal. The difficult problem of the Fourier transform is solved [6].

2.4. Conclusion

2.4.1. Summary and synthesis of major ideas from literature

The research and improvement of image denoising have been very mature and diversified, but some traditional image-denoising methods will cause the loss of image details [2]. At present, there are some new image-denoising methods that can restore the original image details and other aspects, but this kind of algorithm engineering is large. I hope to improve the wavelet analysis algorithm and get a more effective image-denoising model.

Maybe it is difficult for me to improve the algorithm and build a new model. I can build a new evaluation model after I have mastered the wavelet analysis image denoising code, which will be easier for me because I am still not familiar with the principle behind wavelet analysis. What I master more is the practical application of wavelet analysis in daily life. Therefore, I can make innovations in evaluation and construct an evaluation system based on people's subjective views. Instead of using the traditional

numerical evaluation model. I also need to balance the impact of various evaluation factors on the overall evaluation.

2.4.2. Research gap

In the field of mathematics, there are few specialized types of research on the practical application of wavelet analysis in image denoising, most of them are based on the given examples for image denoising. My goal is to denoise the image after the transmission of the image so that the actual method of image denoising can be applied in daily life. In addition, in the papers I found, there is very little research on image quality after image denoising, and most of them introduce a specific evaluation coefficient into the code. In this way, it is difficult for the audience to distinguish the effect of image denoising from the most basic visual aspects.

In my opinion, the image denoised model needs to be evaluated by human subjective evaluation. Instead of using some evaluation coefficient. I think I need to establish various evaluation methods for image denoising according to the image denoised. In this way, I can complete the innovation of this research project. At the same time, people's understanding of image denoising is deepened. A more scientific evaluation model reflecting the subjective evaluation of the image-denoising model is obtained.

2.4.3. Research questions

The method of using digital data and making a questionnaire to ask how bright the image is after the image denoising process. The following summary and gap which I mentioned caused my research questions:

How to use wavelet analysis in image denoising in the mathematical field?

How to evaluate the effects of different algorithms which we use in the process of image denoising?

3. Methodology

3.1. Data collection

3.1.1. Experiments

I found "MATLAB Wavelet Analysis and Applications: 30 Case Studies", a book in the MATLAB-based development examples series. This book broadly explains several wavelet-based signal filtering algorithms, "Since signals and noise have different characteristic representations at different scales, based on this principle, Mallat, Xu and Donoho have each proposed wavelet-based signal filtering algorithms [10]." The book focuses on three wavelet-based non-Bayesian algorithms: modal maximal reconstruction filtering, wavelet-domain threshold filtering, and null-domain correlation filtering - scale-space correlation filtering.

Regarding the choice of pictures, I chose to take 4 daytime pictures and 6 night pictures respectively on campus, the reason why I chose 6 night pictures is because there is more noise in the night pictures, so it is relatively better for investigating the denoising effect of different image denoising algorithms, and it is more likely to show the difference of different denoising methods.



Figure 1. Original Image



Figure 2. Image after modal maximal denoising process

3.1.2. Principle

The modal maxima denoising method is based on the wavelet transform of the observed value function $f(t)$, the singularities of the true value $x(t)$ and the noise $\varepsilon(t)$ have different trends of change with different scales, so as to make a distinction to remove the singularities brought by the noise, and then reconstruct the signal from the singularities corresponding to the true value to achieve the purpose of denoising.

1. The noisy signal is subjected to a wavelet transform of scale $S = 2^j, j=1,2,\dots,J$, and find the modal maxima of the transform coefficients at each scale.

2. Starting from the largest scale (e.g., $J=4$), determine a threshold T , remove the extreme points on the scale whose mode maxima are smaller than T , and keep the others to obtain a new set of mode maxima on the largest scale.

3. Make a neighborhood of each extreme value point retained on the scale function $j=J$, such as $N(t_i, \varepsilon_j)$, find the propagation points (extreme value points) corresponding to the extreme value points in the neighborhood $N(t_i, \varepsilon_j)$ on the $j-1$ scale, retain these extreme value points and remove the other extreme value points to obtain a new set of extreme value points on the $j-1$ scale.

4. Set $j=j-1$, and repeat step 3 until $j=2$.

5. The wavelet coefficients of the saved extreme points at $j=2$ is used to reconstruct the signal using appropriate methods [10].

In actual operation, this algorithm will take a very long time and computing power to calculate, so after the program update, I reduced the number of update iterations in this method in order to get results more effectively.

The threshold denoising method is to first perform wavelet decomposition on the noise-containing signal, threshold the wavelet coefficients, i.e., process the wavelet coefficients that are greater (or less) than a certain threshold, and then reconstruct the original signal using the processed structure [10].

There is a relevant threshold denoising command in MATLAB, so we can apply it directly.

The correlation denoising principle is to compare the normalized correlation coefficients of each layer at each position and determine from the magnitude of the correlation whether it is a signal or a noise-controlled point.

Specific steps:

1. Given the number of decomposition layers M , select the wavelet function and calculate the wavelet coefficients $W_{j,k}$ for each layer of the noisy signal.

2. For comparing $\widetilde{W}_{j,k}$ and $W_{j,k}$, determine:

If $\widetilde{W}_{j,k} \geq W_{j,k}$, it is considered that the large correlation corresponds to the characteristics of the signal, take $\widetilde{W}_{j,k} = W_{j,k}$, and set $W_{j,k} = 0$.

If $\widetilde{W}_{j,k} < W_{j,k}$, the wavelet coefficients at point k are considered to be controlled by the noise, keep $W_{j,k}$, set $\widetilde{W}_{j,k} = 0$.

3. Recalculate $\widetilde{W}_{j,k}$ at each scale.

4. Repeat the above procedure.

5. Each point in the taken $\widetilde{W}_{j,k}$ corresponds to the point controlled by the signal, while all points in $W_{j,k}$ correspond to the noise [10].

3.1.3. Procedure

In this book, the authors provide a detailed theoretical explanation and a detailed case study of these three methods. After understanding the principles, I copied several codes of the relevant methods to MATLAB in the textbook and ran the codes after adding the time conditions. After fixing several bugs in the code, I got the sample images before and after denoising, and got the raw data for machine evaluation.

3.1.4. Questionnaire survey

The requirement of this research is participants is not color blind. Therefore, they can successfully identify the difference between the original image and three different after denoised image. Besides, there is no other requirements on age, gender, education background or territory. The questionnaire was posted on Questionnaire Star and received 211 responses with 183 valid responses.

During this survey, Participants were asked to choose the de-noised image they thought was of the highest quality after comparing it with the original image, that is, comparing the similarity between the de-noised image and the original image.

Although the images themselves were not so clear after being compressed on the site, there was no way for participants to definitively tell the difference between the images, some of the judgments about the quality of the images were good choices for participants.

3.2. Data analysis

3.2.1. Data visualization & Summary statistics

In the evaluation of the algorithmic model, I started by calculating the mean and standard deviation of the different values of each image under different algorithms. The different values here refer to the PSNR, MSE, NC values and the time required by each algorithm. What these values mean and what they represent in the data will be described in the next Result section. Here I just analyze the overall sample trend and exclude potential outliers. I obtained their means and standard deviations, as well as the maximum and minimum values for each group of values through Microsoft Excel, a software program. Also, they were made into graphs.

Moreover, not only the theoretical aspects of the performance of the noise of nighttime images and daytime images are different, but the performance of the two is also very different in the actual results, so I have categorized and discussed the two, and made graphs corresponding to the denoised values of daytime images and the denoised values of nighttime images, respectively.

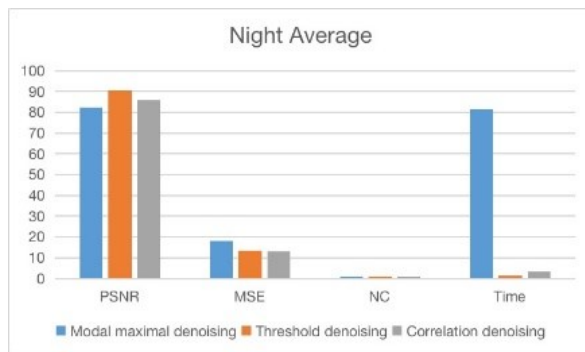


Figure 3. The average of different values under different algorithms of night image

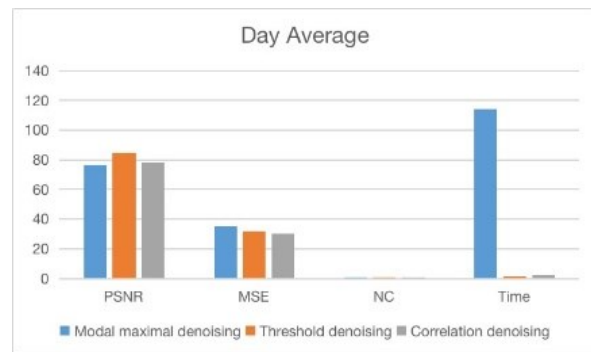


Figure 4. The average of different values under different algorithms of day image

The two images from the top are the images of the mean of different values under different algorithms for nighttime images and the images of the mean of different values under different algorithms for daytime images, respectively. We can see that the difference between the PSNR and NC values of the daytime image with different algorithms and the nighttime image is not very big. In terms of time and MSE values, there is still a difference between the denoised values of the nighttime images and the denoised values of the daytime images.

In the comparison chart, the differences between NC values of different results are not noticeable because the magnitude of the NC values is small, typically ranging from 0 to 1, while the magnitudes of other values are much larger, ranging from 0 to 100. As a result, the differences in NC values are almost invisible in the shared chart. In terms of actual results, the NC values of each algorithm are more in line with expectations.

According to the algorithm, the threshold algorithm is considered superior in terms of PSNR, MSE, NC and the time required by the algorithm. Also, the denoising effect is relatively similar in daytime and nighttime, which is a more desirable algorithm.

Besides, the time required for Modal maximal denoising algorithm is the most significant. Not only the denoising time of 114.4 seconds and 81.7 seconds for daytime and nighttime images respectively, the longest denoising time is actually 190.9 seconds and even the shortest denoising time is 54.7 seconds. Compared with the longest denoising time of 0.8356 seconds for threshold denoising and the longest denoising time of 5.0549 seconds for correlation denoising, the denoising time used by the modulo-maximum denoising method is far greater than them.

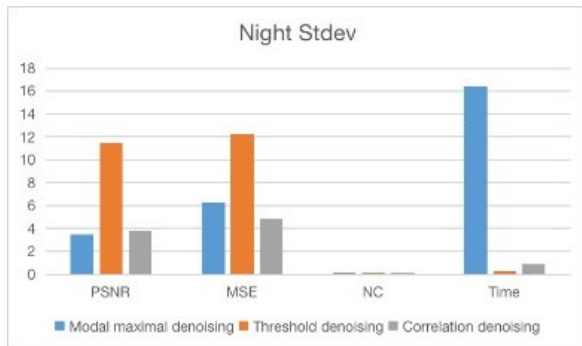


Figure 5. The standard deviation of different values under different algorithms of night image

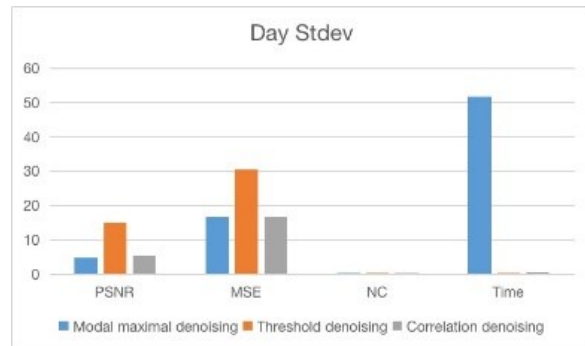


Figure 6. The standard deviation of different values under different algorithms of day image

From the standard deviation of daytime and nighttime images, the standard deviation of denoising time is the largest for the modal maxima method, which is due to the longest time of denoising time for the modal maxima method. And there is another value with a relatively large standard deviation that is different for the threshold algorithm, and it is the value with the largest standard deviation for PSNR, NC, and MSE in both daytime and nighttime denoising value statistics. This may indicate that although the average value of the thresholding algorithm is more in line with expectations, its denoising effect does vary widely for different images.

In the evaluation of the questionnaire, I counted the number of images in the valid sample that were closest to the original image for each group. The sample results were likewise divided into nighttime pictures and daytime pictures recorded in Excel and charted.

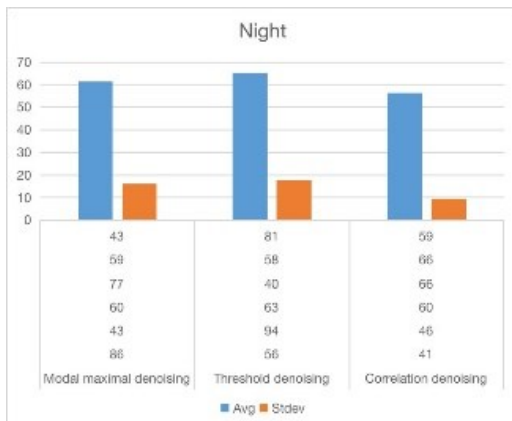


Figure 7. Mean and standard deviation of sample preferences under different algorithms in night pictures

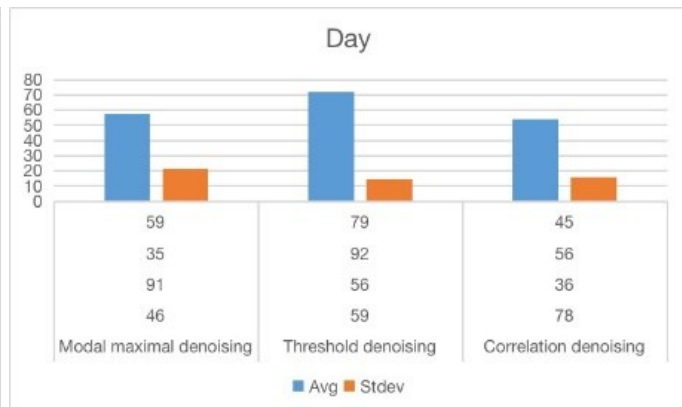


Figure 8. Mean and standard deviation of sample preferences under different algorithms in day pictures

From the above two charts, it can be seen that there is almost no difference in the selection of the best denoised image for the nighttime images, but the average value of the threshold algorithm denoised image is selected more than the other two in the selection of the best denoised image for the daytime images. This result is more in line with the previous values calculated by the machine based on the algorithm and also the best denoising of the threshold algorithm images.

At the same time, we can see that the standard deviation of each algorithm is relatively large in the results tallied by the questionnaire, with a standard deviation of about 15 for both nighttime and daytime images. This indicates that the samples are not evenly distributed and the results of the questionnaire survey may have credibility problems, but the larger number of samples solves this problem and the results can be said to be still more credible.

4. Result

4.1. Different algorithm evaluation value

Evaluating how closely the denoised image approximates the original image can be measured by the peak signal-to-noise ratio. Peak signal-to-noise ratio (PSNR) is an engineering term that refers to the ratio between the maximum possible power of a signal and the destructive noise power that affects its performance fidelity [11]. A higher value of peak signal-to-noise ratio (PSNR) indicates a better approximation.

Normalized correlation (NC) is a measure of similarity between two signals, where the signals are normalized to have zero mean and unit variance. It is commonly used in template matching and feature detection [12]. Meanwhile, the closer the NC value is to 1, the more similar the two images are.

MSE (Mean Squared Error) is a commonly used metric for evaluating the difference between predicted and actual values in regression analysis. It calculates the average of the squared differences between the predicted and actual the smaller the value (Probabilitycourse.com). MSE (mean square error), the smaller the difference between the two pictures, that is, the more similar the pictures are. MSE measures the difference between two images by calculating the average of the sum of squares of the difference between the corresponding pixel points in the two images. When MSE is 0, the two images are identical [10].

4.2. Modal maxima method

According to the evaluation model and the data in the questionnaire, the modulo-maximum algorithm has no outstanding points in terms of evaluation value, it has about the same effect in daytime and nighttime denoising, and probably the more significant point is that its MSE value is higher than the other two denoising methods. This indicates that it is not as good as the other two methods in terms of MSE algorithm evaluation because the higher MSE means the worse the similarity with the original image.

The most outstanding point of the modulo-maximum algorithm is in its denoising time, that is, its program running time, which is almost tens of times longer than the other two methods, and this is on the basis of only 5 iterations. If we want to increase its accuracy, we need to increase the number of iterations of its program, which means that increasing the accuracy will lead to increasing the computing time of the program. So, it is clearly not suitable for everyday use because of the large amount of running time it requires.

4.3. Thresholding method

According to the evaluation model, the threshold denoising algorithm maintains good results for a variety of different evaluation values. Its average PSNR value is the highest value among the three algorithms for both daytime and nighttime images. Also, its average MSE, NC values and its denoising time are both approximate to the correlation denoising method. The above data can show that it has the most significant denoising effect.

From the results analyzed by the questionnaire, threshold denoising performs relatively the best during the daytime. On average, 20 samples will choose the image denoised by the threshold denoising method in comparing the image similarity. Threshold denoising also seems to be the most suitable method in the popular evaluation.

However, at the same time, the standard deviation of the different values of the threshold denoising algorithm is also the largest in comparison in terms of the standard deviation of the data. The standard deviation is almost more than twice that of the other two algorithms. Although the average effect of the threshold denoising algorithm is the most significant, the distribution of the image quality produced by the threshold denoising algorithm is also the most uneven. This situation is also reflected in the results of the questionnaire. Therefore, if the threshold denoising method is used in an image denoising algorithm in everyday life, there is a high probability that the results will be extremely different from the original image.

4.4. Correlation denoising method

According to the evaluation model, the correlation denoising algorithm maintains significant results for a variety of different evaluation values, similar to the threshold denoising method. But the different point is that it has smaller standard deviations for each of them, so it shows that it is a more stable and average denoising algorithm. It is relatively more suitable for use in daily life image denoising algorithms, and although it is not as popular as the threshold denoising algorithm in questionnaire results, it is at least passable as an image denoising algorithm.

5. Discussion

5.1. Image denoising effect

Correlation denoising is better for nighttime image denoising because it has less error than the original image and is closer to the original image, compared to the other two algorithms.

The PSNR values, MSE values and NC values after correlation denoising with the denoised images are close to the threshold denoising method regardless of daytime and nighttime, possessing better denoising effects. Also, the computation time of both is relatively small compared to the modal maxima algorithm.

But on the other hand, the standard deviation of correlation denoising is smaller, which indicates that the image quality distribution after correlation denoising is more concentrated. And the threshold denoising method with large standard deviation indicates that the distribution is more dispersed. It also indicates that the image quality after denoising is uneven. Although the threshold denoising method is more effective in questionnaire surveys, using correlation denoising in daily image denoising algorithms will give safer and more stable quality results.

5.2. Subjective evaluation under questionnaire survey

I do not think that the results in the questionnaire can effectively be one of the bases for the evaluation of the algorithmic model because of the inaccuracy and instability of its results.

First of all, if the results of the questionnaire in the daytime picture are true, then the threshold denoising effect in the nighttime should also have the result of the most sample selection as the result of the daytime picture, according to the analysis of the results in the algorithmic model. However, the fact is different from the hypothesis, and the probability of being selected for each image after denoising in different algorithms at night is the same. Therefore, the results in the questionnaire are not indicative of the denoising effect of each algorithm.

Second, the standard deviation of the results in the questionnaire reflects the instability of the questionnaire, which is around 15 for almost all samples, which indicates the selection of each option under each image and its unevenness. It also shows that the results are relatively similar after denoising for each image, so no more significant differences can be obtained.

Therefore, the questionnaire survey it results of inaccuracy and instability and not suitable to be a factor in the evaluation algorithm. Or rather, the questionnaire form needs to be improved to overcome these two defects in order to become a reasonable subjective factor evaluation.

5.3. Limitations

There are several main limitations of this inquiry project in the form of method selection, sample selection, repeated experiments and questionnaires.

Regarding method selection, I think the more likely place for flaws is the threshold denoising method. This is because in the threshold denoising method, the threshold value can have different choices, which causes the threshold denoising method to show no better results than the other two methods. I can use different thresholds to denoise the image on further research afterward.

Regarding the sample selection, I selected the samples from my own sources and ignored the noise-containing quality of the samples while retaining the originality. With today's technological advances, the noise in images has been significantly reduced, so it is likely that the target noise of the noisy images I selected already contains very little noise. So, the results of the questionnaire are unreliable also because they are more similar to the original images.

Regarding the repetition of the experiment, the denoising effect of the modal maxima denoising method is related to the number of iterations. In order to better explore the denoising effect of the modal maxima denoising method, I should repeat the experiment and increase the number of iterations of different modal maxima denoising methods to get different denoising results.

Regarding the form of the questionnaire, the platform, Questionnaire Star, does not respond well to the image differences. So, after that, in order to get better results after the questionnaire, the questionnaire can be collected by offline way. Or some details of the images can be selected to compare with the original image to get a better picture about the most similar to the original image.

6. Conclusion

6.1. Summary

Correlation denoising is better for nighttime image denoising because it has less error than the original image and is closer to the original image, compared to the other two algorithms.

The PSMR values, MSE values and NC values after correlation denoising with the denoised images are close to the threshold denoising method regardless of daytime and nighttime, possessing better denoising effects.

But on the other hand, the standard deviation of correlation denoising is smaller, which indicates that the image quality distribution after correlation denoising is more concentrated. And the threshold denoising method with large standard deviation indicates that the distribution is more dispersed. It also indicates that the image quality after denoising is uneven. Although the threshold denoising method is more effective in questionnaire surveys, using correlation denoising in daily image denoising algorithms will give safer and more stable quality results.

6.2. Reflection

Consider the specific characteristics of the input images and choose denoising methods accordingly. While correlation denoising offers significant benefits for night images, alternative techniques like threshold denoising may be more suitable for denoising daytime images.

Because the effects of threshold denoising and correlation denoising are relatively similar, and the best results of threshold denoising were reported in the questionnaire survey. Although the effect of questionnaire survey is not reliable, the result of threshold denoising is more reliable due to the high mean value.

6.3. Future Application

The experiment will be re-run in the future after improving the questionnaire method. After applying the new denoising algorithm, a software is generated based on MATLAB by image denoising method. So that people can also use this software for image denoising in their daily life in order to reduce the impact of image noise in people's daily life.

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