Proceedings of the 4th International Conference on Signal Processing and Machine Learning
DOI: 10.54254/2755-2721/52/20241685

Shortest Hamiltonian circuit algorithm for complex environments

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Abstract. Simply taking into account the static path length is insufficient to compute the shortest Hamiltonian circuit in complicated situations. To incorporate the influence of many factors on the shortest Hamiltonian circuit, this paper introduces a multi-objective fuzzy comprehensive evaluation model and quantitatively assesses the fuzzy factors. To better correctly reflect each influencing factor's importance in the problem-solving process, this paper also builds a hierarchical model to decide the weights assigned to each one. Finally, this paper adopts the state compression dynamic programming algorithm to solve the dynamic shortest Hamiltonian loop. In this study, an example analysis is also performed to confirm the algorithm's efficacy. Compared with the traditional algorithm, the algorithm designed in this paper comprehensively considers the influence of several factors when calculating the shortest Hamiltonian loop, and can more comprehensively evaluate the advantages and disadvantages of the path.

Keywords: Hamiltonian loop, state compression algorithm, fuzzy decision-making, hierarchical modeling

1. Introduction

Path planning now plays a significant part in navigation software due to the ongoing growth of urban transportation and the rising demand for trip efficiency. However, a lot of navigational software merely takes the path's length into account, omitting to fully account for all influencing aspects in the complicated environment. In practice, choosing the optimum way will also be significantly influenced by the length of the journey, road capacity, and density of traffic [1]. In complex contexts, characteristics like road width and road quality have an impact on road capacity, whereas factors like road capacity and traffic flow have an impact on traffic congestion density. These variables are hazy and challenging to measure precisely. Therefore, this paper adopts the hierarchical model with a multi-objective fuzzy comprehensive evaluation model to thoroughly analyze the influence of these fuzzy elements on path planning. By grouping particular factors into several levels, the hierarchical model creates the link between each factor and hence establishes the weights of each factor. The significance of each element in path planning can be more properly reflected using this method. On the other hand, the multi-objective fuzzy comprehensive evaluation model achieves the comprehensive evaluation of fuzzy factors by quantifying and allocating different weights to them. By using this model, this paper can achieve more complete and accurate results by taking into account fuzzy elements like road capacity and traffic congestion density while planning paths. In this paper, this paper will use the state compression dynamic

DOI: 10.54254/2755-2721/52/20241685

planning algorithm to solve the dynamic shortest Hamiltonian loop. This algorithm can efficiently compute the shortest path while considering the effects of multiple factors. Through the research in this paper, this paper aims to provide a more integrated and comprehensive solution for the path planning algorithm of navigation software to meet people's needs for comprehensive consideration of travel efficiency and traffic conditions.

2. Hierarchical Model

When faced with a complex system consisting of many elements that are interrelated and mutually constrained and lacking in quantitative data, hierarchical modeling combines quantitative and qualitative analyses [2]. The experience of the decision-maker determines the relative importance of each criterion for measuring whether or not the goal can be achieved and reasonably gives weight to each criterion for each decision option [3].

2.1. Hierarchy Building

According to hierarchical analysis, the highest level is the goal level, which represents the purpose of solving the problem and usually has only one overall goal. The middle level is the criteria level, which contains the criteria needed to realize the predetermined general goal [4]. The lowest level is the solution level, which includes various measures and decision options available to achieve the objectives. As shown in Fig. 1, in our problem, there is no criteria level, the goal level is the optimal path, and the solution level consists of four factors that influence the dynamic path length, with the variation of the second factor determined by six indicators and the variation of the third factor determined by two indicators.

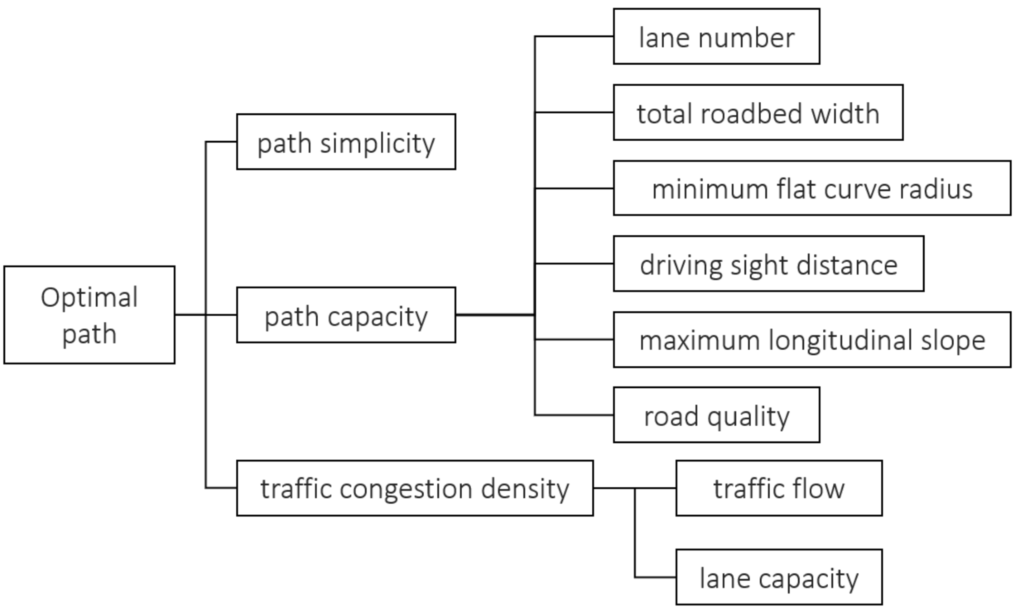

Figure 1. Hierarchy of path selection

2.2. Constructing a Judgment Matrix

There are four factors in the second level and six factors in the third level, and each factor is assigned a value based on a scale of 1-9. The scale of 1-9 is shown in Table 1. The two judgment matrices constructed based on expert opinion are as follows:

$$p2 = \begin{bmatrix} 1 & 2 & 1 \\ \frac{1}{2} & 1 & \frac{1}{2} \\ 1 & 2 & 1 \end{bmatrix} \tag{1}$$

$$p3 = \begin{bmatrix} 1 & 1 & 1 & \frac{1}{2} & 1 & \frac{1}{3} \\ 1 & 1 & 1 & \frac{1}{2} & 1 & \frac{1}{3} \\ 1 & 1 & 1 & \frac{1}{3} & 1 & 3 \\ 2 & 2 & 3 & 1 & 2 & 1 \\ 1 & 1 & 1 & \frac{1}{3} & 1 & \frac{1}{3} \\ 3 & 3 & 3 & 1 & 3 & 1 \end{bmatrix} \tag{2}$$

Table 1. Meaning of Scale

Scale	1	3	5	7	9
Degree	equal importance	slightly importance	significantly importance	strongly importance	extreme importance

2.3. Consistency Test

The maximum eigenvalue of matrix p2 is 3.0, and the corresponding RI value is 0.525 according to the RI table, so CR=CI/RI=-0.0≤0.1, which passes the one-time test.

The maximum eigenvalue of matrix p3 is 6.028, and the corresponding RI value is 1.25 according to the RI table, so CR=CI/RI=0.004≤0.1, which passes the one-time test [5].

2.4. Indicator Weights

Based on the root method to calculate the weight results, the weight coefficient vector of the second layer $A_2 = (a_{21}, a_{22}, a_{23}) = (0.4,0.2,0.4)$ and the weight coefficient vector of the third layer $A_3 = (a_{31}, a_{32}, a_{33}, a_{34}, a_{35}, a_{36}) = (0.11,0.11,0.1,0.26,0.1,0.3)$.

3. Multi-Objective Fuzzy Comprehensive Evaluation Model

Applying the principle of fuzzy relationship synthesis, it is a method of comprehensively judging the subordinate grade status of the evaluated things from multiple indicators, and the specific steps are as follows:

(1). Set up the factorization domain U of the evaluated object, $U = (u_1, u_2, \cdots, u_n)$;

(2). set up a rubric hierarchy thesis domain V, $V = (v_1, v_2, \cdots, v_n)$;

Usually, there are different levels of rubrics:

$$V = (\text{very high}, \text{high}, \text{moderately high}, \text{moderately low}, \text{low}, \text{very low}).$$

The different levels of rubrics can be chosen on a case-by-case basis [6].

(3). Make a one-factor judgment and establish a fuzzy relationship matrix R.

$$R = \begin{pmatrix} r_{11} & r_{12} & \cdots & r_{1m} \\ r_{21} & r_{22} & & r_{2m} \\ \vdots & & \ddots & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{nm} \end{pmatrix} 0 < r_{ij} < 1 \tag{3}$$

r_{ij} represents the membership relationship of factor u_i in U to level v_j in V;

(4). Determine the weight vector of evaluation factors $A_2 = (a_1, a_2, \cdots, a_n)$, A is the affiliation of each factor in U to the thing being evaluated, which depends on the focus of people when making comprehensive fuzzy judgment, i.e., assigning weights according to the importance of each factor at the time of judgment.

(5). Select the synthesis operator for evaluation and synthesize A and R to obtain B as the result of fuzzy comprehensive evaluation [7].

$$B = A^{\circ}R = (b_1, b_2, \cdots, b_n) \tag{4}$$

$$B = (a_1, a_2, \cdots, a_n)^{\circ}\begin{pmatrix} r_{11} & r_{12} & \cdots & r_{1m} \\ r_{21} & r_{22} & & r_{2m} \\ \vdots & & \ddots & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{nm} \end{pmatrix} \tag{5}$$

$^{\circ}$ is the fuzzy operator, which is used as $M(\cdot, +)$

Obtain the affiliation vector B and quantize it. In this paper, this paper use the percentage scoring method, noting $90 \le c_1 \le 100$ (excellent), $80 \le c_2 < 90$ (good), $70 \le c_2 < 80$ (medium), $60 \le c_2 < 70$ (good), $0 \le c_2 < 60$ (poor). This results in a vector of scores for the rubric $C = (c_1, c_2, c_3, c_4, c_5)$.After obtaining the score vector, the score can be calculated accordingly.

$$S = \frac{\sum_{i=1}^{n} b_i \cdot c_i}{\sum_{i=1}^{b} b_i} \tag{6}$$

Here, the c_i represents an interval, and the upper limit of each interval is used for the calculations in this paper.

The above mathematical model applies to the third level of the hierarchical model, i.e., the calculations regarding capacity can be made using the above methodology.

For the second layer in the hierarchical model, this paper uses a direct calculation to obtain the affiliation vectorW_1as the weights of the paths with the following formula:

$$W_1 = A_2{}^{\circ}(r_1, r_2, r_3) \tag{7}$$

4. Affinity Function

Path simplicity is an important factor limiting the choice of paths. The path length is relative in the network, so the simplicity of a path can be expressed as

$$r_1 = \frac{path\ length}{maximum\ path\ in\ the\ network} \tag{8}$$

At the same time, the capacity of a path is also a relative value, which can be expressed as

$$r_2 = \frac{path\ capacity}{maximum\ capacity\ in\ the\ network} \tag{9}$$

In addition, there are only two cases of traffic congestion density: the path is congested or not congested, which can be expressed by the term

$$r_3 = \begin{cases} 1 & congestion \\ 0 & no\ congestion \end{cases} \tag{10}$$

This can be expressed as follows.

The capacity of a path is jointly determined by the six factors in the third layer, and the subordination functions for each of these six factors are determined below. According to the Technical Standards for Highway Engineering published by the Ministry of Transportation and Communications in 1998 (Table 2), the affiliation functions for the number of lanes, total width of the roadbed, minimum radius of the level curve, and sight distance of the traveler are similar [8]. As shown in Table 2, taking driving sight distance as an example, where greater than or equal to 210 meters is considered excellent, less than 210 meters but greater than 160 meters is considered good, and so on. The number of lanes, the total width of the roadbed, the minimum radius of the level curve, and the subordinate degree function of the travel sight distance can be obtained as follows:

Table 2. Main technical indicators for each level of highway

Class of road	Freeway	First class	Second class	Third class	Fourth class
lane number	8	6	4	2	1
Total roadbed width (meters)	42.5	25.5	12	8.5	6.5
Minimum flat curve radius (meters)	650	400	250	125	60
Driving sight distance (meters)	210	160	110	75	40
corresponding commentary	excellent	good	medium	accept	poor
corresponding variable	w_1	w_2	w_3	w_4	w_5

$$U_{excellect} = \begin{cases} 1 & x \geq w_1 \\ \frac{x-w_2}{w_1-w_2} & w_2 \leq x < w_1 \\ 0 & 0 \leq x < w_2 \end{cases} \quad (11)$$

$$U_{good} = \begin{cases} 0 & x \geq w_1 \\ \frac{w_1-x}{w_1-w_2} & w_2 \leq x < w_1 \\ \frac{x-w_3}{w_2-w_3} & w_3 \leq x < w_2 \\ 0 & x < w_3 \end{cases} \quad (12)$$

$$U_{medium} = \begin{cases} 0 & x \geq w_2 \\ \frac{w_2-x}{w_2-w_3} & w_3 \leq x < w_2 \\ \frac{x-w_4}{w_3-w_4} & w_4 \leq x < w_3 \\ 0 & x < w_4 \end{cases} \quad (13)$$

$$U_{accept} = \begin{cases} 0 & x \geq w_3 \\ \frac{w_3-x}{w_3-w_4} & w_4 \leq x < w_3 \\ \frac{x-w_5}{w_4-w_5} & w_5 \leq x < w_4 \\ 0 & x < w_5 \end{cases} \quad (14)$$

$$U_{poor} = \begin{cases} 0 & x \geq w_4 \\ \frac{w_4-x}{w_4-w_5} & w_5 \leq x < w_4 \\ 1 & 0 \leq x < w_5 \end{cases} \quad (15)$$

For the maximum longitudinal slope of the road, according to the Technical Standards for Highway Engineering, 0-3% is excellent, 3%-4% is good, 4%-5% is medium, 5%-6% is acceptable, and 6%-9% is poor [9], so the maximum longitudinal slope corresponds to a similar affiliation function as above, as follows:

$$U_{excellent} = \begin{cases} 1 & 0 \leq x < 3 \\ 4-x & 3 \leq x < 4 \\ 0 & x \geq 4 \end{cases} \quad (16)$$

$$U_{good} = \begin{cases} 0 & x < 3 \\ x-3 & 3 \leq x < 4 \\ 5-x & 4 \leq x < 5 \\ 0 & x \geq 5 \end{cases} \quad (17)$$

$$U_{medium} = \begin{cases} 0 & x < 4 \\ x-4 & 4 \leq x < 5 \\ 6-x & 5 \leq x < 6 \\ 0 & x \geq 6 \end{cases} \quad (18)$$

DOI: 10.54254/2755-2721/52/20241685

$$U_{accept} = \begin{cases} 0 & x<5 \\ x-5 & 5\leq x<6 \\ \frac{9-x}{3} & 6 \leq x < 9 \\ 0 & x \geq 9 \end{cases} \tag{19}$$

$$U_{poor} = \begin{cases} 0 & 0 \leq x < 6 \\ \frac{x-6}{3} & 6 \leq x < 9 \\ 1 & x \geq 9 \end{cases} \tag{20}$$

In this paper, the pavement surface type is used to measure the road quality, which is categorized into four types according to the Technical Standards for Highway Engineering: high-level pavements include asphalt concrete and cement concrete, sub-high-level pavements include asphalt infilled, asphalt crushed stone and asphalt surface treatment, intermediate-level pavements include crushed gravel and semi-complete stone blocks, and low-level pavements include material-grain reinforced soils and improved soils, and the corresponding affiliation functions are shown in Table 3:

Table 3. Affiliation of pavement quality

road quality	evaluation	degree of affiliation	Affiliation to other rubrics
Advanced Pavement	excellent	1	0
Sub-advanced pavement	good	1	0
Intermediate pavement	medium	1	0
low-grade pavement	accept	1	0

The traffic congestion density K is determined by the traffic volume in the third layer Q and the lane capacity C. From $\mathrm{K} = \frac{\mathrm{Q}}{\mathrm{C}}$, it can be seen that the roadway has reached its maximum capacity when $\mathrm{K} = 1$, while the roadway becomes congested when $\mathrm{K} > 1$ [10].

5. Basic Idea of State-Compressed Dynamic Programming Algorithm

Know the starting vertex number startVertex and the set of vertex numbers that the path will pass through, vertexSet, with the first element of vertexSet being startVertex. Create a path list path to store the vertex numbers passed through. Calculate the total number of vertices passed n and the number of subsets m of the vertex set except the start vertex. Create a dp-table and a prev-table to record the shortest path lengths and path information from the current vertex through all the vertices in the corresponding subset once and only once and back to the start point. The basic procedure for solving the shortest Hamiltonian loop algorithm starting from startVertex is as follows:

1) Initialization: For each attraction i except the start attraction, initialize dp[i][0] to the weight from vertex i to the end vertex.

2) Process each subset array in turn: for each subset array i, iterate over each vertex j. If subset i does not contain attraction j, iterate over each element k in the subset. Compute the weights from vertex j to vertex k plus the value of dp[k][i - (1 << (k - 1))] and compare it to dp[j][i]. If smaller, update dp[j][i] and prev[j][i].

3) Calculate the path length: traverse each site i except the starting site, calculate the weight from vertex i to the starting vertex plus the value of dp[i][m - 1 - (1 << (i - 1))] and compare it with dp[0][m - 1]. If smaller, update dp[0][m - 1] and prev[0][m - 1].

4) Backtracking path: Backtrack from the prev table to get the sequence of vertices with the smallest path. Starting from the last column, find the previous vertex according to prev[row][col] and add it to the path list. Update the values of col and row until col is 0.

6. Example Analysis

A simple schematic of a road network is shown in Fig. 2, and the individual factors are summarized in Table 4. Now, it is required to compute the shortest Hamiltonian circuit starting at point 1 and passing through points 2, 3, 4, 5, and 6.

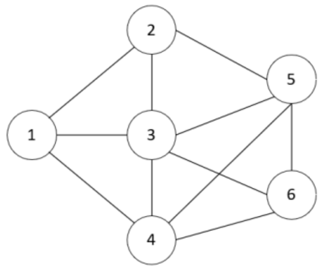

Figure 3. Schematic diagram of the road network

Table 4. Condition of each factor in the road section

way	length	lane number	Total roadbed width	Minimum flat curve radius	Driving Range	Maximum longitudinal slope	road quality	traffic flow	lane capacity
1-2	11	4	40	580	120	3	sub-high	0.9	1.1
1-3	14	6	45	280	180	5	high level	1.8	1.2
1-4	8	6	35	450	200	7	sub-high	1.3	1.8
2-3	12	8	30	200	100	8	high level	1.1	1.4
3-4	10	8	40	620	150	4	high level	0.7	0.8
2-5	13	4	38	390	210	6	sub-high	1.6	0.9
3-5	9	6	34	410	160	9	high level	1.2	1.3
4-5	6	8	32	290	180	10	sub-high	0.8	1.1
3-6	14	6	29	600	150	5	high level	1.5	0.7
4-6	8	8	25	550	100	7	sub-high	0.6	1.6
5-6	10	4	42	580	220	4	sub-high	1.4	0.5

According to the algorithm mentioned above, it can be seen that from vertex 1, the next step requires searching and comparing vertices 2, 3, and 4 [11]. The following is an example of vertex 2, calculating the combined weights of path 1-2, and for the capacity of a path, substituting the values of the corresponding factors in the above table into the affiliation function to obtain the fuzzy relationship matrix R_1.

DOI: 10.54254/2755-2721/52/20241685

$$R_1 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0.85 & 0.15 & 0 & 0 & 0 \\ 0.72 & 0.28 & 0 & 0 & 0 \\ 0 & 0.2 & 0.8 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} \tag{21}$$

Obtained from the $A_3 \circ R_1$, the affiliation vector$B_1 = (0.2655,0.3965,0.318,0,0)$, the quantization score is calculated as$S_1 = 89.464$, and since the maximum value of the capacity is 100, the fuzzy affiliation degree of the capacity is$r_2 = 0.89464$. According to the above equation, the fuzzy affiliation degree of path simplicity is $r_1 = \frac{11}{14}$, and the traffic congestion density$K_1 = \frac{0.9}{1.1} < 1$, so its corresponding fuzzy affiliation is $r_3 = 0$. For the optimal road section, the r_1,$r_3$the smaller the better, r_2 the larger the better. In this paper, the best road section is selected according to the principle of "the smaller the affiliation degree, the better," so this paper take the complementary value of r_2 [12]. The complementary value of$r_2 = 1 - 0.89464 = 0.10536$. Obtained from$A_2 \circ (r_1, r_2, r_3)$, this paper gets the combined weights of road sections 1-2$W_1 = 0.3354$.

According to the above steps, the dynamic section length of each path can be obtained after considering all the factors, and the shortest Hamiltonian loop obtained according to the state compression dynamic programming algorithm is $1 \to 3 \to 6 \to 4 \to 5 \to 2 \to 1$. The shortest Hamiltonian loop obtained according to the state compression dynamic planning algorithm is$1 \to 4 \to 6 \to 2 \to 5 \to 3 \to 1$. The method provided in this article considers the combined influence of multiple factors.

7. Conclusion

This article proposes a stratified decision-making model that confronts the challenge of inadequate quantitative data within complex systems. The model constructs a hierarchical framework delineating objectives and solutions, and employs expert appraisal alongside consistency checks to create and affirm judgment matrices, thereby determining the weight of importance for each factor. Furthermore, the study delineates a fuzzy comprehensive evaluation method, facilitating the assessment of numerous criteria, inclusive of road technical standards. This model amalgamates expertise, fuzzy logic, and weight computations, providing an innovative resolution framework for intricate decision-making scenarios, with particular pertinence to real-world issues such as transportation route selection.

References

[1] LI Zhaokui,LI Ting,ZHOU Xinshao et al. Construction of road network matching model for urban agglomeration based on fuzzy hierarchy theory and its application[J/OL]. Ea rth Science:1-11[2023-09-24].http://kns.cnki.net/kcms/detail/42.1874.P.20230305.1749.004.html.

[2] Shy Child,2019.07.16, Hierarchical Analysis Model,https://blog.csdn.net/qq_45400755/article/details/96087065

[3] Zhi Zi,2022.08.16,Mathematical Modeling - Analytic Hierarchy Process (AHP),https://blog.csdn.net/qq_45355603/article/details/126292000

[4] Zhang Yemao,Yang Xiaowu. Application of fuzzy matrix algorithm in traffic navigation system[J]. Western Transportation Science and Technology,2017(01):67-70+82.DOI:10.13282/j.cnki.wccst.2017.01.018.

[5] Xu Xiaomin. Application of the Analytic Hierarchy Process [J]. Statistics and Decision, 2008(1):156-158.

[6] Wang XiaoXiao,2019.05.07, Fuzzy Decision Analysis Method,https://blog.csdn.net/qq_29831163/article/details/89913744?ops_request_mis

[7] ZHAO Chenyang, LI Meng, MIAO Lili. Research on electric power customer risk evalu ation model based on multi-objective comprehensive evaluation[J]. Heilongjiang Electri c Power,2019,41(05):408-412.DOI:10.13625/j.cnki.hljep.2019.05.007.
[8] WU Qi-Chang,CHEN Ti-Ze,SU Yi. Optimal path analysis based on multi-objective fuzzy decision making[J]. Computer Simulation,2005(12):106-109+123.
[9] Zhang, H., Sun, D., & Ji, F. et al. (2021). Tourism route planning based on dynamic programming strategy. Computer and Digital Engineering, 49(02), 301-304+346.
[10] Ye, Z. (2022). Research on Hamiltonian circuit based on adjacency matrix and recursive algorithm. Journal of Jiamusi University (Natural Science Edition), 40(04), 164-167.
[11] Gao, Z., & Chen, Z. (2021). Path operation matrix of graph and path problems including Hamiltonian circuit. Journal of Huazhong University of Science and Technology (Natural Science Edition), 49(02), 32-36. DOI: 10.13245/j.hust.210204.
[12] Wang, Y., & Xu, C. (2018). An algorithm for determining the Hamiltonicity of a graph and solving the Hamiltonian cycle based on the adjacency matrix. Journal of Ningbo University (Natural Science & Engineering Edition), 31(02), 83-88.