# Probabilistic prediction models of pounding between adjacent buildings during seismic

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Abstract. In recent years, many researchers have studied the probability of collisions between adjacent buildings under earthquake action and proposed some prediction methods, but their accuracy still needs to be improved. This study adopts a machine learning method based on logistic regression models to propose a probability prediction model for adjacent building collisions during earthquake processes. The main content of this study includes: 1. A logistic regression model was used to analysis the factors affecting the seismic impact of adjacent buildings, and an impact prediction model was established; 2. Verify the effectiveness of the logistic regression model through comparative probability analysis.

**Keywords:** Prediction model, Pounding structure, Seismic.

#### 1. Introduction

When an earthquake occurs, adjacent buildings may collide, which could lead to structural damage or even collapse [1], posing a serious threat to human safety. Therefore, studying the probability of pounding between adjacent buildings under the earthquakes can help address some structural engineering problems in the future [2]. In recent years, many researchers have conducted in-depth research on this issue and proposed some predictive methods, but their accuracy still needs to be improved. This study employs a machine learning approach, based on a logistic regression model, to propose a probability prediction model for pounding between adjacent buildings during seismic. The advantage of the logistic regression model lies in its relatively high accuracy in prediction. The main contents of this study include: 1. Using the logistic regression model to analyse the factors affecting the pounding of adjacent buildings earthquakes and pounding a prediction model [3,4]; 2. Verifying the effectiveness of the logistic regression model through comparative probability analysis. Through this research, it is hoped that the performance of the probability prediction model can be improved, the risk assessment ability of building structures under the action of earthquakes can be enhanced, and the threat of earthquakes to human safety can be reduced.

# 2. Selection of buildings parameters

To better understand the construction and prediction process of the logistic regression model, first select some building parameters. These parameters affect the behaviour of buildings in earthquakes and the probability of pounding, so they are crucial to the performance of the model.

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# 2.1. Methods for selecting building parameters

When selecting building parameters, we need to comprehensively consider the following aspects:

- (1) Data availability: This study has selected some common parameters to ensure the universality of the model. All the data used in this article are simulated data, and the building parameters used by other researchers during experiments are re-processed. This is a significant shortcoming of this study, and related research should select more complete initial experimental data for investigation.
- (2) Correlation of parameters: The selected building parameters should have a certain correlation with the probability of pounding between adjacent buildings during seismic. In this article, the height, natural frequency, distance, earthquake magnitude, and maximum displacement of different floors of two buildings during earthquakes are selected as parameters for modelling analysis and prediction.
- (3) Significance of parameters: The selected building parameters should be significant, that is, they should have a significant impact on the probability of pounding between adjacent buildings during seismic [5]. During the data selection, some insignificant data are deleted during the data screening stage to ensure better model performance during analysis.

# 2.2. Selection and acquisition of building parameters

Building parameters are important indicators for measuring the characteristics of a building and its possible behaviour in an earthquake. Reasonable selection of building parameters can help better understand the probability of collision between adjacent buildings under the action of an earthquake. In this study, the following key building parameters are mainly concerned with:

- (1) Building height(H): The height of a building will affect its force distribution and acceleration response during an earthquake. Generally, buildings with higher heights will be subject to greater forces during an earthquake, thus having a higher collision probability. This study involves two buildings with different heights: Building A is 8(F) stories high, with each floor being  $H_1$ =2.85m; Building B is 4(F) stories high, with each floor being  $H_2$ = 2.85m.
- (2) Building natural frequency(T): The natural frequency of a building will affect the probability of collision between buildings during an earthquake. If the natural frequencies of two buildings are the same, there is a high likelihood that they will not collide during an earthquake and will not be destroyed due to collision, which is inconsistent with the original intention of this study. Therefore, this study uses different building natural frequencies, with Building A's natural frequency T being  $T_1$ = 0.821Hz and Building B's natural frequency T being  $T_2$ =0.544Hz.
- (3) Distance between buildings(L): The distance between buildings directly affects the possibility of collision between adjacent buildings under the action of an earthquake. Buildings that are closer together have a higher probability of collision during an earthquake. The distance between adjacent buildings used in this study is L=0.4m.
- (4) Earthquake intensity(E): Different intensities of earthquakes have different effects on buildings. The higher the intensity of an earthquake, the greater the impact on the shaking degree and the probability of collision between adjacent buildings. When selecting the earthquake intensity in this study, the data of longitudinal wave earthquakes are removed because the behaviour and destruction principles of buildings during longitudinal and transverse wave earthquakes are different. The earthquakes with smaller intensities that cannot cause significant shaking or collision in buildings are removed, and the remaining standard earthquakes are numbered 1-10.
- (5) Maximum displacement(X): During an earthquake, different floors of a building will have horizontal displacements. If the horizontal displacement exceeds a certain value, the building will collapse. Therefore, this study regulates the maximum displacement of different floors of buildings under different earthquake intensities to eliminate the interference factor of building collapse due to horizontal displacement exceeding the maximum value when no collision occurs. The horizontal displacement in the positive x-axis direction is defined as a positive value, and the horizontal displacement in the negative x-axis direction is defined as a negative value [6,7].

Based on the above principles, the height and distance between buildings are selected as key building parameters in this study Next, we will use logistic regression models to model these parameters to predict the probability of collision between adjacent buildings during a seismic.

#### 3. Model development

In the context of an earthquake, predicting the probability of collision between adjacent buildings is of great significance for urban earthquake disaster reduction and seismic engineering design. This chapter will introduce the application of logistic regression models in predicting the probability of collision between adjacent buildings during a seismic. Logistic regression is a commonly used regression analysis method, particularly suitable for dealing with binary classification problems.

#### 3.1. Basic principles of logistic regression models

Logistic regression models are regression models based on logistic functions that analyse the logical relationship between the dependent variable and the independent variables, used for binary classification problems. Its basic principle is to establish a mapping relationship between the input variables and the output variables using the logistic function (sigmoid function). The goal of the logistic regression model is to minimize the error between the input variables and the output variables [8,9].

The basic form of the logistic regression model is as follows:

$$P(Y|X) = \sigma \; (\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_n X_n) \label{eq:posterior}$$

Where P(Y|X) X are deterministic regressor,  $\sigma$  represents the logistic function,  $\beta_0$ ,  $\beta_1$ ,  $\beta_2$ ... $\beta_n$  are the regression coefficients, and the H, T, L, E, X independent variables, corresponding to  $X_n$  in the formula.

The key to logistic regression models is to determine the appropriate independent variables and regression coefficients. The selection of independent variables can be determined based on factors such as professional knowledge, data availability, and significance. The determination of the regression coefficients can use optimization methods such as least squares and gradient descent to solve.

In the prediction of collision probability between adjacent buildings under the action of an earthquake, we can use building parameters under the action of an earthquake (such as building height, building natural frequency, distance between buildings, earthquake magnitude, maximum displacement, etc.) as independent variables, and collision probability as the dependent variable. Logistic regression models can establish a logical relationship between the dependent and independent variables, thereby predicting the collision probability between adjacent buildings under the action of an earthquake.

#### 3.2. Data preparation

In this chapter, we will use the dataset selected in Chapter 2. First, we need to pre-process the data, including missing value treatment, outlier treatment, and data normalization.

	precision	recall	f1-score	support
0	0.31	1.00	0.47	8
1	1.00	0.18	0.31	22
accuracy			0.40	30
macro avg	0.65	0.59	0.39	30
weighted avg	0.82	0.40	0.35	30

**Table 1.** Table with precision, recall, and F1-score and support of the test set.

Secondly, the dataset is divided into training and testing sets for performance evaluation after training the model. In order to improve the performance of the model, this study set up 200 sets of data for modeling.

#### 3.3. Model Training

Develop using logistic regression models. During the training process, model performance is optimized by adjusting model parameters such as learning rate and iteration times. In addition, methods such as cross validation will be used to evaluate the generalization ability of the model.

#### 3.4. Model Evaluation

After the model training is completed, evaluate the model using a test set.

**Table 2.** Table with Building parameters.

	E	F	$\mathbf{X_1}$	$\mathbf{X_2}$	$H_1$	$H_2$	$T_1$	$T_2$	Y
0	1	8	0.088943	NaN	2.85	NaN	0.821	0.544	0
1	1	4	0.058378	0.080624	2.85	2.85	0.821	0.544	0
2	1	3	0.042810	0.074749	2.85	2.85	0.821	0.544	0
3	1	2	0.037225	0.053944	2.85	2.85	0.821	0.544	0
4	1	1	0.020946	0.027551	2.85	2.85	0.821	0.544	0
				•••					
195	10	8	-0.260473	NaN	2.85	NaN	0.821	0.544	1
196	10	4	-0.177382	-0.090363	2.85	2.85	0.821	0.544	1
197	10	3	-0.138577	-0.076865	2.85	2.85	0.821	0.544	1
198	10	2	-0.094610	-0.054562	2.85	2.85	0.821	0.544	1
199	10	1	-0.047851	-0.027762	2.85	2.85	0.821	0.544	1

200 rows×9 columns

Evaluation metrics include accuracy, precision, recall, and F1-score. The optimal model is chosen for further analysis by comparing the performance of the model under different parameters and features.

In this chapter, we developed a model using logistic regression to predict the probability of collision between adjacent buildings under earthquake influence. By analysing and processing relevant data, the model performance was improved. The research in this chapter provides a relatively universal model for subsequent studies to analyse the collision probability of buildings with different heights and other influencing factors under different frequency earthquakes.

## 4. Probability of Pounding between Adjacent Buildings During Seismic

In Chapter 3, we developed a model using logistic regression to predict the probability of collision between adjacent buildings under earthquake influence. This chapter will further study the probability of collision between adjacent buildings under different heights, natural frequencies, and earthquake frequencies under earthquake action. The influence of factors such as building height and earthquake frequency on the collision probability of adjacent buildings will be analysed through the developed model in Chapter 3.

#### 4.1. Prediction Model

Using the logistic regression model developed in Chapter 3, the probability of collision between adjacent buildings under different heights, natural frequencies, and earthquake frequencies is predicted. The impact of factors such as building height and earthquake frequency on the collision probability of buildings is analysed.

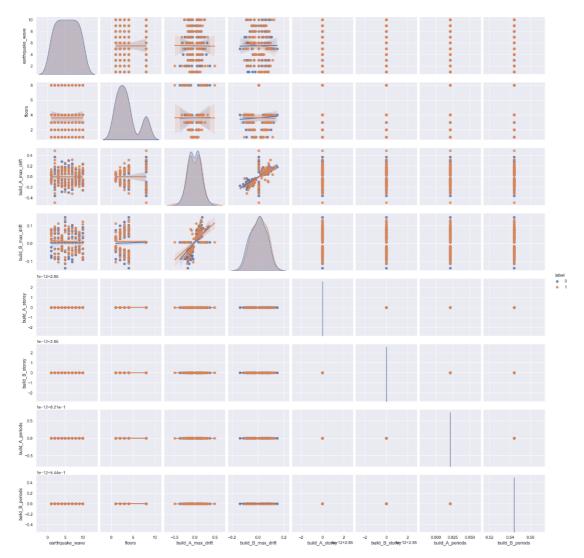


Figure 1. Figure Probability distribution diagram of building collisions influenced by different factors.

Firstly, we analyse the collision situation of different building heights under the same earthquake frequency. As shown in Figure 1, the probability of collision between adjacent buildings A and B under earthquake action increases as the building height increases. This is because the weight and inertia of the building increase with the increase of building height, resulting in greater displacement during the earthquake, thereby increasing the probability of collision between adjacent buildings A and B.

Secondly, we analyse the collision situation of different earthquake intensities and building heights. As shown in Figure 1, the probability of collision between buildings of different heights increases with the increase of earthquake intensity, including some cases without collision. This is because as the earthquake intensity increases, the ground motion becomes more severe. In addition, buildings with higher heights have a higher probability of collision themselves, thereby increasing the probability of collision between adjacent buildings under earthquake action.

Thirdly, the maximum displacement of adjacent building floors is simulated to predict the probability of collision between adjacent buildings A and B under earthquake action. The results show that the increase in the maximum displacement of adjacent building floors increases the probability of collision between adjacent buildings A and B under earthquake action. This is because the increase in the maximum displacement of the floor expands the movement range of the building. If the two adjacent

buildings are moving towards each other during the earthquake, the probability of collision between them will increase.

#### 4.2. Result discussion

According to the research in Chapter 4, the following conclusions can be drawn:

- 1. Under earthquake action, the different heights of buildings A and B have a significant impact on the probability of collision. Generally, buildings with higher height are subjected to greater forces during earthquakes, and the probability of collision also increases accordingly.
- 2. Different earthquake frequencies have a certain impact on the collision probability of buildings A and B. When the earthquake frequency is lower, the shaking duration is longer, which may cause the force applied to Building A to be greater, thereby increasing the probability of collision between Buildings A and B under earthquake action.
- 3. Factors such as the distance between adjacent buildings, natural frequencies, and other factors also affect the collision probability. For example, buildings with different natural frequencies collide and suffer damage during an earthquake due to their different movement situations.

The research in this chapter helps us better understand the probability prediction model of pounding between adjacent buildings during seismic, and understands the probability of collision between adjacent buildings under different building heights and earthquake frequencies. This provides some reference conclusions for seismic protection measures and future research on building structures;

#### 5. Conclusion

#### 5.1. Summary of Research Results

This study developed a probability prediction model of pounding between adjacent buildings under earthquake action based on logistic regression and analysed the frequency of pounding between buildings of different heights under different frequency earthquakes. The main research findings are as follows:

- 1. Selected some building dataset with universal research significance and pre-processed the data through programming, laying a good foundation for model development.
- 2. Developed a prediction model of pounding probability between adjacent buildings during seismic based on logistic regression, and verified the universality and accuracy of the model through analysis and discussion.
- 3. Analysed the probability of pounding between buildings of different heights under different frequency earthquakes and found that building height and earthquake frequency have a significant impact on the pounding probability of adjacent buildings during seismic [10].
- 4. We used practical cases to analyse the value of probability prediction models for pounding between adjacent buildings during seismic in practical applications, providing theoretical support for earthquake prevention and urban planning.;

## 5.2. Research significance and limitations

This study has important value in understanding the probability of collisions between adjacent buildings under earthquake action, and provides a theoretical basis for earthquake prevention and urban planning. However, this study still has the following limitations:

- 1. The building dataset used in this study may have certain limitations. In the future, it is possible to consider collecting more types of building data to improve the generalization ability of the model.
- 2. In this study, relatively few factors were considered, such as the distance between buildings, structural type, and soil type on the ground. Further research can explore the impact of these factors on collision probability.

3. The case analysis of this study is mainly based on logistic regression models and does not involve other prediction methods. In the future, the performance of different prediction methods can be compared to choose a more suitable model.

#### 5.3. Future research directions

In response to the limitations of this study, future research can be expanded in the following directions:

- 1. Collect more types of building data, optimize feature engineering, and improve the generalization ability of the model.
- 2. Consider more influencing factors, such as the distance between buildings, structural type, and soil type on the ground, to more accurately predict the probability of collisions between adjacent buildings.
- 3. Research other prediction methods, such as support vector machines, decision trees, etc., and compare the performance of different models to choose a more suitable prediction method.
- 4. Apply the research results to practical earthquake prevention and urban planning work, providing scientific basis for earthquake risk assessment and urban safety planning.

In summary, this study has achieved certain results in predicting the probability of collisions between adjacent buildings under earthquake action, providing useful insights for subsequent research. We look forward to more research in the future that can make breakthroughs on this basis and make greater contributions to earthquake prevention and urban planning.

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# Appendix

**Table 3.** Table with database

E	F	$X_1$	$X_2$	$H_1$	$H_2$	$T_1$	$T_2$	Y
1	8	0.088943		2.85		0.821	0.544	FALSE
1	4	0.058378	0.080624	2.85	2.85			FALSE
1	3	0.04281	0.074749	2.85	2.85			FALSE
1	2	0.037225	0.053944	2.85	2.85			FALSE
1	1	0.020946	0.027551	2.85	2.85			FALSE
1	8	-0.09934		2.85				FALSE
1	4	-0.07011	-0.07884	2.85	2.85			FALSE
1	3	-0.06056	-0.06862	2.85	2.85			FALSE
1	2	-0.05174	-0.04269	2.85	2.85			FALSE
1	1	-0.03081	-0.02124	2.85	2.85			FALSE
1	8	0.136897		2.85				TRUE
1	4	0.099905	0.065923	2.85	2.85			TRUE
1	3	0.082442	0.057563	2.85	2.85			TRUE
1	2	0.05943	0.043416	2.85	2.85			TRUE
1	1	0.031042	0.022983	2.85	2.85			TRUE
1	8	-0.13203		2.85				TRUE
1	4	-0.09905	0.080624	2.85	2.85			TRUE
1	3	-0.08087	0.074749	2.85	2.85			TRUE
1	2	-0.05784	0.053944	2.85	2.85			TRUE
1	1	-0.03067	0.027551	2.85	2.85			TRUE
2	8	0.373167		2.85				FALSE
2	4	0.185826	0.086766	2.85	2.85			FALSE
2	3	0.140779	0.076771	2.85	2.85			FALSE
2	2	0.101288	0.056978	2.85	2.85			FALSE
2	1	0.054996	0.030528	2.85	2.85			FALSE
2	8	-0.36654		2.85				FALSE
2	4	-0.24692	-0.08697	2.85	2.85			FALSE
2	3	-0.19937	-0.07624	2.85	2.85			FALSE
2	2	-0.14189	-0.0566	2.85	2.85			FALSE
2	1	-0.0735	-0.03004	2.85	2.85			FALSE
2	8	0.492867		2.85				TRUE
2	4	0.31787	0.096228	2.85	2.85			TRUE
2	3	0.25177	0.082601	2.85	2.85			TRUE
2	2	0.175687	0.064256	2.85	2.85			TRUE
2	1	0.09038	0.035056	2.85	2.85			TRUE
2	8	-0.49279		2.85				TRUE
2	4	-0.31685	-0.08552	2.85	2.85			TRUE
2	3	-0.25043	-0.07577	2.85	2.85			TRUE
2	2	-0.174	-0.05795	2.85	2.85			TRUE
2	1	-0.0893	-0.03111	2.85	2.85			TRUE
3	8	0.174215		2.85				FALSE
3	4	0.119268	0.086766	2.85	2.85			FALSE

Table 3. (continued)

$\overline{E}$	F	<i>X</i> <sub>1</sub>	$X_2$	H <sub>1</sub>	H <sub>2</sub>	<i>T</i> <sub>1</sub>	T <sub>2</sub>	Y
3	3	0.094873	0.076771	2.85	2.85			FALSE
3	2	0.0677	0.056978	2.85	2.85			FALSE
3	1	0.035574	0.030528	2.85	2.85			FALSE
3	8	-0.16898		2.85				FALSE
3	4	-0.12097	-0.07624	2.85	2.85			FALSE
3	3	-0.09312	-0.0566	2.85	2.85			FALSE
3	2	-0.06571	-0.03004	2.85	2.85			FALSE
3	1	-0.03466	-0.08781	2.85	2.85			FALSE
3	8	0.182294		2.85				TRUE
3	4	0.101298	0.082601	2.85	2.85			TRUE
3	3	0.084083	0.064256	2.85	2.85			TRUE
3	2	0.065189	0.035056	2.85	2.85			TRUE
3	1	0.036186	0.11274	2.85	2.85			TRUE
3	8	-0.16898		2.85				TRUE
3	4	-0.12097	-0.07577	2.85	2.85			TRUE
3	3	-0.09312	-0.05795	2.85	2.85			TRUE
3	2	-0.06571	-0.03111	2.85	2.85			TRUE
3	1	-0.03466	-0.10525	2.85	2.85			TRUE
4	8	0.28274		2.85				FALSE
4	4	0.177216	0.039721	2.85	2.85			FALSE
4	3	0.150933	0.034303	2.85	2.85			FALSE
4	2	0.11188	0.027008	2.85	2.85			FALSE
4	1	0.060641	0.018514	2.85	2.85			FALSE
4	8	-0.28512		2.85				FALSE
4	4	-0.18002	-0.04007	2.85	2.85			FALSE
4	3	-0.15614	-0.03498	2.85	2.85			FALSE
4	2	-0.1163	-0.02752	2.85	2.85			FALSE
4	1	-0.06181	-0.01542	2.85	2.85			FALSE
4	8	0.19452		2.85				TRUE
4	4	0.156962	0.046347	2.85	2.85			TRUE
4	3	0.147653	0.040311	2.85	2.85			TRUE
4	2	0.106346	0.03209	2.85	2.85			TRUE
4	1	0.05043	0.021263	2.85	2.85			TRUE
4	8	-0.19359		2.85				TRUE
4	4	-0.16393	-0.03825	2.85	2.85			TRUE
4	3	-0.15664	-0.03286	2.85	2.85			TRUE
4	2	-0.10434	-0.02433	2.85	2.85			TRUE
4	1	-0.04606	-0.01484	2.85	2.85			TRUE
5	8	0.24373		2.85				FALSE
5	4	0.183645	0.14851	2.85	2.85			FALSE
5	3	0.148292	0.12243	2.85	2.85			FALSE
5	2	0.104276	0.10122	2.85	2.85			FALSE
5	1	0.053558	0.07459	2.85	2.85			FALSE

Table 3. (continued)

$\overline{E}$	F	<i>X</i> <sub>1</sub>	$X_2$	<i>H</i> <sub>1</sub>	<i>H</i> <sub>2</sub>	$T_1$	$T_2$	Y
5	8	-0.23904		2.85				FALSE
5	4	-0.17173	-0.1381	2.85	2.85			FALSE
5	3	-0.14031	-0.11423	2.85	2.85			FALSE
5	2	-0.1001	-0.09281	2.85	2.85			FALSE
5	1	-0.05246	-0.04601	2.85	2.85			FALSE
5	8	0.260237		2.85				TRUE
5	4	0.157506	0.133688	2.85	2.85			TRUE
5	3	0.13175	0.102527	2.85	2.85			TRUE
5	2	0.097998	0.081083	2.85	2.85			TRUE
5	1	0.0558	0.063913	2.85	2.85			TRUE
5	8	-0.28537		2.85				TRUE
5	4	-0.18412	-0.11253	2.85	2.85			TRUE
5	3	-0.15206	-0.09128	2.85	2.85			TRUE
5	2	-0.11189	-0.0775	2.85	2.85			TRUE
5	1	-0.06014	-0.03887	2.85	2.85			TRUE
6	8	0.307112		2.85				FALSE
6	4	0.211585	0.083387	2.85	2.85			FALSE
6	3	0.165902	0.073122	2.85	2.85			FALSE
6	2	0.114027	0.054152	2.85	2.85			FALSE
6	1	0.058116	0.028863	2.85	2.85			FALSE
6	8	-0.3031		2.85				FALSE
6	4	-0.20923	-0.08637	2.85	2.85			FALSE
6	3	-0.16301	-0.07568	2.85	2.85			FALSE
6	2	-0.11124	-0.05569	2.85	2.85			FALSE
6	1	-0.05651	-0.02938	2.85	2.85			FALSE
6	8	0.191288		2.85				TRUE
6	4	0.141717	0.096065	2.85	2.85			TRUE
6	3	0.120024	0.081951	2.85	2.85			TRUE
6	2	0.091992	0.062686	2.85	2.85			TRUE
6	1	0.050918	0.034597	2.85	2.85			TRUE
6	8	-0.20972		2.85				TRUE
6	4	-0.17149	-0.05963	2.85	2.85			TRUE
6	3	-0.14647	-0.05196	2.85	2.85			TRUE
6	2	-0.10526	-0.03771	2.85	2.85			TRUE
6	1	-0.05428	-0.0198	2.85	2.85			TRUE
7	8	0.241849		2.85				FALSE
7	4	0.170142	0.104331	2.85	2.85			FALSE
7	3	0.133821	0.092141	2.85	2.85			FALSE
7	2	0.092478	0.069312	2.85	2.85			FALSE
7	1	0.047685	0.037511	2.85	2.85			FALSE
7	8	-0.24681		2.85				FALSE
7	4	-0.16974	-0.10353	2.85	2.85			FALSE
7	3	-0.13518	-0.09	2.85	2.85			FALSE

Table 3. (continued)

$\overline{E}$	$\overline{F}$	<i>X</i> <sub>1</sub>	X <sub>2</sub>	$H_1$	H <sub>2</sub>	$T_1$	$T_2$	Y
	2	-0.09593	-0.06714	2.85		1 1	12	FALSE
7					2.85			
7 7	1 8	-0.05074	-0.03725	2.85 2.85	2.85			FALSE TRUE
7		0.320201	0.006464		2.05			
	4	0.20818	0.086464	2.85	2.85			TRUE
7	3	0.168522	0.075025	2.85	2.85			TRUE TRUE
7	2	0.120525	0.055624	2.85	2.85			
7	1	0.063775	0.028897	2.85	2.85			TRUE
7	8	-0.32175	0.06492	2.85	2.05			TRUE
7	4	-0.20736	-0.06482	2.85	2.85			TRUE
7	3	-0.16826	-0.05492	2.85	2.85			TRUE
7	2	-0.12291	-0.03929	2.85	2.85			TRUE
7	1	-0.06535	-0.02288	2.85	2.85			TRUE
8	8	0.139687	0.050064	2.85	2.05			FALSE
8	4	0.07561	0.072264	2.85	2.85			FALSE
8	3	0.063304	0.065741	2.85	2.85			FALSE
8	2	0.0533	0.054915	2.85	2.85			FALSE
8	1	0.032539	0.029606	2.85	2.85			FALSE
8	8	-0.13762		2.85				FALSE
8	4	-0.1031	-0.06384	2.85	2.85			FALSE
8	3	-0.08722	-0.04578	2.85	2.85			FALSE
8	2	-0.0709	-0.03411	2.85	2.85			FALSE
8	1	-0.0417	-0.01856	2.85	2.85			FALSE
8	8	0.165655		2.85				TRUE
8	4	0.088252	0.088063	2.85	2.85			TRUE
8	3	0.066754	0.081822	2.85	2.85			TRUE
8	2	0.045028	0.065033	2.85	2.85			TRUE
8	1	0.023449	0.035518	2.85	2.85			TRUE
8	8	-0.17468		2.85				TRUE
8	4	-0.11331	-0.0532	2.85	2.85			TRUE
8	3	-0.09344	-0.04522	2.85	2.85			TRUE
8	2	-0.06712	-0.03703	2.85	2.85			TRUE
8	1	-0.03619	-0.02205	2.85	2.85			TRUE
9	8	0.135284		2.85				FALSE
9	4	0.077535	0.054572	2.85	2.85			FALSE
9	3	0.066847	0.047757	2.85	2.85			FALSE
9	2	0.050004	0.040152	2.85	2.85			FALSE
9	1	0.027388	0.022969	2.85	2.85			FALSE
9	8	-0.14455		2.85				FALSE
9	4	-0.10233	-0.04321	2.85	2.85			FALSE
9	3	-0.08548	-0.03933	2.85	2.85			FALSE
9	2	-0.05979	-0.0318	2.85	2.85			FALSE
9	1	-0.03109	-0.01528	2.85	2.85			FALSE
9	8	0.190812		2.85				TRUE

Table 3. (continued)

$\overline{E}$	F	<i>X</i> <sub>1</sub>	$X_2$	<i>H</i> <sub>1</sub>	$H_2$	<i>T</i> <sub>1</sub>	<b>T</b> <sub>2</sub>	Y
9	4	0.136529	0.065649	2.85	2.85			TRUE
9	3	0.111004	0.05866	2.85	2.85			TRUE
9	2	0.080082	0.045916	2.85	2.85			TRUE
9	1	0.042819	0.026384	2.85	2.85			TRUE
9	8	-0.19017		2.85				TRUE
9	4	-0.13293	-0.051	2.85	2.85			TRUE
9	3	-0.10979	-0.04517	2.85	2.85			TRUE
9	2	-0.07877	-0.0351	2.85	2.85			TRUE
9	1	-0.04094	-0.01849	2.85	2.85			TRUE
10	8	0.235008		2.85				FALSE
10	4	0.120311	0.091505	2.85	2.85			FALSE
10	3	0.094087	0.079513	2.85	2.85			FALSE
10	2	0.078408	0.058534	2.85	2.85			FALSE
10	1	0.050123	0.031332	2.85	2.85			FALSE
10	8	-0.25494		2.85				FALSE
10	4	-0.16589	-0.09092	2.85	2.85			FALSE
10	3	-0.15138	-0.07942	2.85	2.85			FALSE
10	2	-0.11797	-0.0578	2.85	2.85			FALSE
10	1	-0.06549	-0.03024	2.85	2.85			FALSE
10	8	0.241905		2.85				TRUE
10	4	0.167936	0.123842	2.85	2.85			TRUE
10	3	0.132152	0.108342	2.85	2.85			TRUE
10	2	0.091272	0.08027	2.85	2.85			TRUE
10	1	0.046661	0.042561	2.85	2.85			TRUE
10	8	-0.26047		2.85				TRUE
10	4	-0.17738	-0.09036	2.85	2.85			TRUE
10	3	-0.13858	-0.07687	2.85	2.85			TRUE
10	2	-0.09461	-0.05456	2.85	2.85			TRUE
_10	1	-0.04785	-0.02776	2.85	2.85			TRUE