Proceedings of the 2nd International Conference on Functional Materials and Civil Engineering
DOI: 10.54254/2755-2721/66/20240957

Study on the truss damage measurement based on Bayesian updating

Ziru Zhang

College of Civil Engineering, Taiyuan University of Technology, Taiyuan, China

zhangziru2458@link.tyut.edu.cn

Abstract. Bayesian updating can integrate new observations in real time. This online updating property makes the Bayesian approach particularly suitable for dynamic monitoring of structural damage. In this paper, the node displacements are obtained by the finite element software OpenSees, assuming that the bars are damaged and the stiffness is reduced. Rejection Sampling method is used to determine the stiffness drop interval. During the sampling process, different load sizes, load locations, errors and types are selected to realize the updating of the stiffness of the top chord bar, bottom chord bar, vertical web bar, and diagonal web bar. A Pratt-type truss is used as an example, combined with displacement data. The proposed stiffness damage measurement method is demonstrated by updating the stiffness of four types of bars separately. The results shows that damage in structures can be accurately identified by Bayesian updating methods. This method not only pinpoints the location of the damage, but also quantifies the extent of the damage.

Keywords: Bayesian updating, truss, Rejection Sampling, Stiffness damage measurement.

1. Introduction

In the use of civil engineering buildings and structures, fatigue of materials and damage to members cannot be avoided. For the damage identification of trusses, scholars have proposed methods based on deep neural networks, decision tree algorithms, statistical steady-state strain eigenfunctions [1-3]. Bayesian updating is an emerging method in civil engineering in recent years. It has been extensively employed in geotechnical applications, including geotechnical parameter updating, landslide time prediction, and slope reliability assessment. Bayesian updating methods also have applications in the structural context. Liu et al. utilized the physical parameter estimation method to estimate the residual strength of bridges [4]. Zhang et al. developed a probabilistic model of wave power response of deepwater bridge abutment based on Bayesian updating [5]. Chen proposed a reliability prediction methodology for reinforced concrete bridge members [6]. The unique feature of Bayesian updating is its ability to integrate new observations in real time. The feature makes the Bayesian approach particularly suitable for dynamic monitoring of structural damage. In practical applications, with the continuous operation and use of engineering structures such as Bridges and buildings, their health conditions will gradually change. Engineering structure performance may decline due to factors such as structural aging, disaster damage or overload operation. Damage in structures can be accurately identified through regular or continuous monitoring combined with Bayesian updating methods. This

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approach not only accurately locates the damage, but also quantifies the extent of the damage, thus providing engineers with comprehensive information about the state of structural health.

2. Research Methods

2.1. Bayesian updating

Combining the prior distribution with the likelihood function derived from the data measurements and applying probabilistic techniques to produce the posterior distribution constitutes the fundamental method of Bayesian updating [7].

$$f_X(X|Y) = \frac{1}{c} L(X) \cdot f_X(x) \quad (1)$$

$f_X(x)$ is the probability density function (PDF) of the prior distribution, which is derived from empirical judgments. L(X) is the likelihood function. $f_X(X|Y)$ is the PDF of the posterior distribution. In general, equation (1) is difficult to find, so a large number of methods for solving this equation have been developed. The best known is the Markov Chain Monte Carlo method (MCMC). It does this by constructing a Markov chain with a smooth distribution as the posterior distribution. When the Markov chain reaches the limiting distribution, the resulting sample is the posterior distribution. In some cases, traditional Markov chain Monte Carlo methods are inefficient. Several improved algorithms have been developed, i.e., adaptive Metropolis-Hastings methods, Transitional Markov chain Monte Carlo method, and hybrid Monte Carlo methods [8-10]. In addition, there are some methods used as an alternative to the Markov chain Monte Carlo method. Straub et al. proposed a structural reliability-based Bayesian model updating method with the method of Rejection Sampling [11].

2.2. Likelihood function

There are many uncertainties in the updating of the finite element model. The PDF of the observed data reflects how well the updated structural model explains the observed data x. It is called the likelihood function and is expressed as $L(X)$. The impact of the observed data on the posterior probability's PDF is reflected in the likelihood function.

When new observed data is generated, it is usually based on the parameters of the mechanical model. Therefore, the likelihood function has to reflect this relationship in order to linking the observed data and the parameters through the function h(x). The observation error is defined as $\varepsilon_i = y_i - h_i(x)$. Its PDF is f_{ε_i} [7].

$$L_i(x) = f_{\varepsilon_i}[y_i - h_i(x)] \quad (3)$$

$$\mathrm{L(x)} = \prod_{i=1}^{m} \mathrm{L_i(x)} \quad (4)$$

In most cases, the likelihood function conforms to a normal distribution.

$$\mathrm{L(x)} \propto \prod_{i=1}^{m} \mathrm{L_i(x)} = \prod_{i=1}^{m} \exp\left\{-\frac{1}{2}\frac{[\mathrm{h_i(x)} - \mathrm{y_i}]^2}{\sigma_\varepsilon^2}\right\} \quad (5)$$

In lower dimensions, the posterior function can be solved by mathematical derivation. In general, it is difficult to write the full form of the posterior function and derive it by mathematical derivation. Therefore, it is important to find a sampling method that works for any distribution. One effective method is MCMC and the other one is Rejection Sampling. The traditional Monte Carlo method is inefficient, so the rejection sampling method is used.

2.3. Rejection Sampling

The basic principle of Rejection Sampling method is assuming the PDF is f(y), and finding an arbitrary distribution g(y) and a constant c that can be sampled directly. For all y, it should be satisfied: $\frac{f(y)}{g(y)} \leq c$.
Sampling is then carried out as follows:

(1) Taking a sample y_i from g(y);

(2) Sampling a random number u_i from a uniform distribution of U (0,1);

(3) If $u_i \leq \frac{f(y)}{cg(y)}$, accepting the sample y_i, otherwise, rejecting the sample.

3. Case analysis and discussion

3.1. Case description

In this paper, a Pratt-type truss (figure 1) is used as an example, assuming that the bars are damaged and the stiffness decreases. Nodal displacements are obtained by loading the loads with the finite element software OpenSees. The stiffness decrease interval is calculated using Bayesian updating. During the sampling process, attempts are made to optimize the loading method and measurement nodes. The efficiency of obtaining valid information is further improved so that the posterior distribution PDF can be found quickly and accurately to realize the updating of the stiffness of the top chord bar, bottom chord bar, vertical web bar, and diagonal web bar.

Using the above method, the damaged Pratt-type truss model is updated. The model suffer damage to bars 10 and 18 whose stiffness is reduced. Empirical judgment indicates that the prior distribution of bars 10 and 18 follows a uniform distribution U(0.76,1.54) and U(0.68,1.26), respectively. The stiffness of other bars is shown in table 1. It is necessary to analyze the structure, design the loading and measurement scheme. A normal distribution with a 0.5 standard deviation is followed by the measurements. The posterior distributions of the stiffness of the two damaged bars are obtained by performing Bayesian updating on the basis of the data.

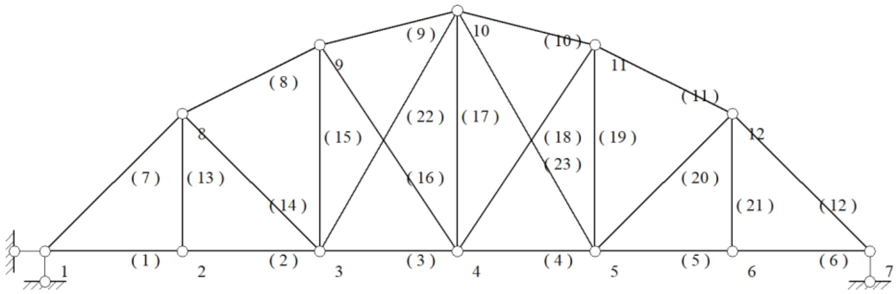

Figure 1. Case Truss Schematic (1-12 is the node number, (1)-(23) is the bar number).

Table 1. Bar stiffness

Bar (Node)	1-2	2-3	3-4	4-5	5-6	6-7	1-8
EA/10^5 kN·m^2	1.4	1.4	1.4	1.4	1.4	1.4	1.54
Bar(Node)	8-9	9-10	11-12	3-10	12-7	2-8	3-8
EA/105 kN·m^2	1.428	1.428	1.428	1.134	1.54	1.26	1.19
Bar (Node)	3-9	4-9	4-10	5-11	5-12	6-12	5-10
EA/10^5 kN·m^2	1.19	1.19	1.33	1.19	1.19	1.26	1.134

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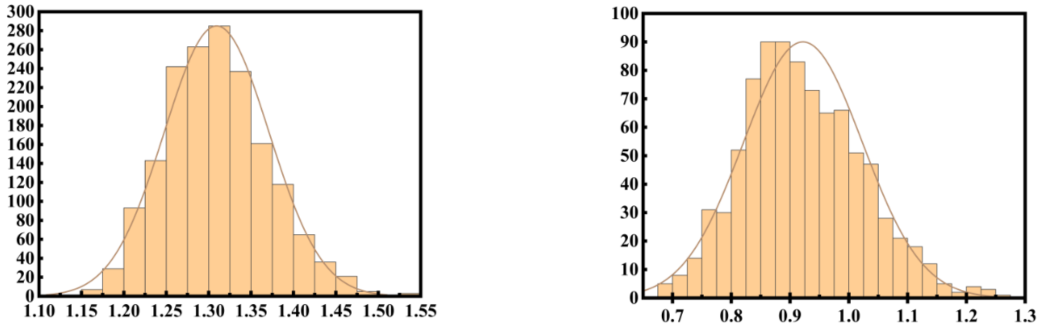
3.2. Impact of the loading scheme on the effectiveness of the update

Identifying more efficient loading scheme can lead to more efficient update and more accurate results. Therefore, different loading schemes are used in this study to study their effects on updating effectiveness.

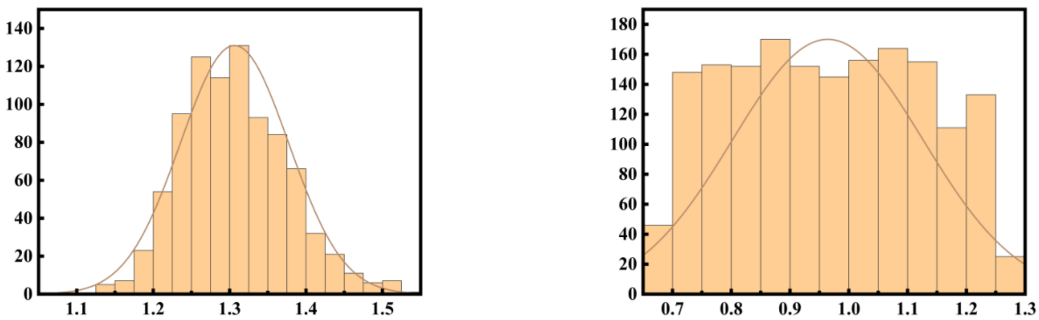
3.2.1. Load magnitude. In order to determine the effect of load magnitude on the Bayesian updating results, the model nodes 11 and 12 are loaded together with 25kN, 50kN, and 100kN and update. The acceptance rates of the samples in the 10,000 samples are 0.3982, 0.1667, and 0.0439 for the three loading states. The larger the acceptance rate, the more general the updating effect of the sample, so it could be concluded that the larger the load, the better the updating effect.

3.2.2. Load position. A load of 100 kN is loaded and updated at points 10, 11, and 12. The acceptance rates for the samples at the three load states are 0.1708, 0.0874, and 0.3092, respectively. Therefore, it can be judged that node 11's updating effect is better. However, when importing the data into Origin and analyzing it, it is found that while the node 10's updating effect update is mediocre for the bar 10 compared to the node 11, it is better for the bar 18. The loading results are shown in figures 2 and 3.



(a) Bar 10: Average-1.306, Variance:0.0709 (b) Bar 18: Average-0.921, Variance-0.101257

Figure 2. Node 10 loading results



(a) Bar 10: Average-1.3093, Variance-0.0617 (b) Bar 18: Average-0.9641, Variance-0.1618

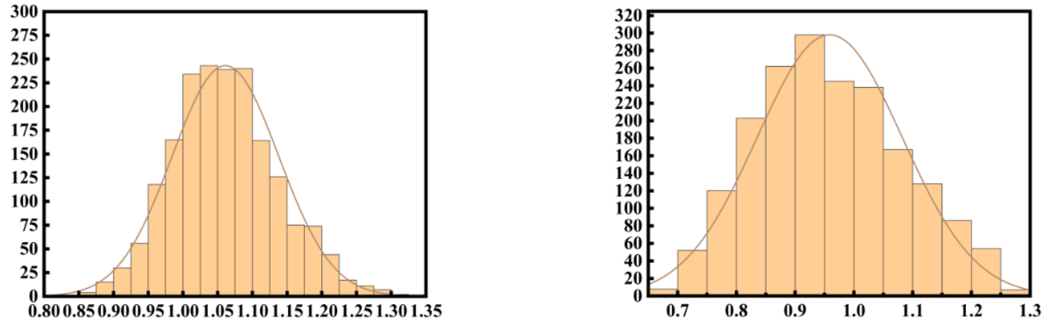
Figure 3. Node 11 loading results

3.3. Impact of the measurement program on the effectiveness of the update

After determining the impact of the loading scheme on the update effect, in order to make the update effect of bar 10 and bar 18 better, the data of bar 10 is updated with node 11, and the data of bar 18 is updated with node 10. In order to be able to load the loads at two different locations separately, the following Bayesian update is performed using a quadratic sampling method.

DOI: 10.54254/2755-2721/66/20240957

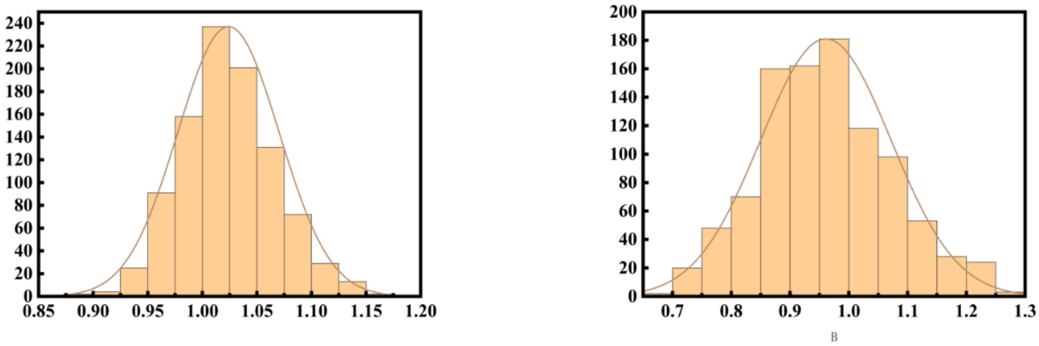
3.3.1. Selection of errors. The absolute error of the update is 0.5 mm and the relative error is 0.05%. With a relative error of 0.05%, the updated result is shown in figure 4.



(a) Bar 10: Average-1.061192, Variance-0.0769 (b) Bar18: Average-0.9587, Variance-0.1236

Figure 4. Relative error of 0.05%

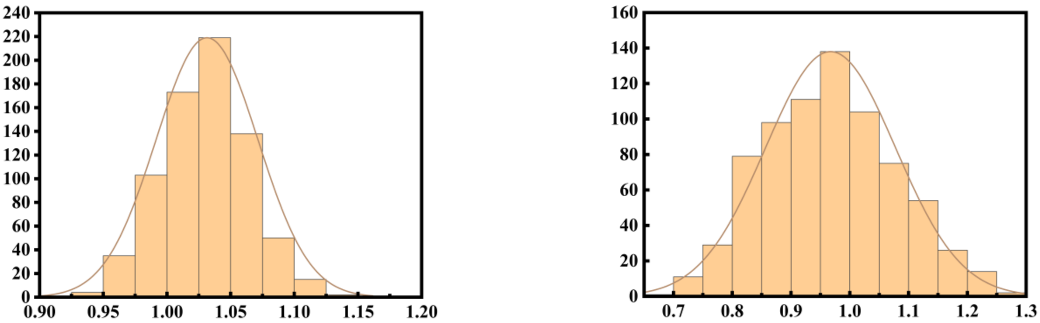
The results for an absolute error of 0.0005m are shown in figure 5. By comparison, it can be concluded that the update using absolute error is better than relative error.



(a) Bar 10: Average1.0239, Variance-0.0465 (b) Bar 18: Average-0.9608, Variance-0.111

Figure 5. Absolute error of 0.0005

3.3.2. Selection of measurement points. Sampling with the absolute error is better for updating, but for further improvements in accuracy, the degree to which the likelihood function explains the structure needs to be increased. From displacement measurements at two measurement points, there are now three measuring points instead of only two. The result of the update is shown in figure 6.



(a) Bar 10: Average1.0314Variance0.04001 (b) Bar 18: Average0.9671Variance0.110649

Figure 6. Updated results for three measurement points

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Table 2. Displacement of each node when loading 100kN at node 11

Direction	x (Direction 1)	y (Direction 2)
Node 2	0.002389	-0.020727
Node 3	0.004709	-0.033025
Node 4	0.007967	-0.042126
Node 5	0.013829	-0.044182
Node 6	0.018604	-0.030348
Node 8	0.01438	-0.020709
Node 9	0.016531	-0.03367
Node 10	0.013978	-0.040073
Node 11	0.004737	-0.049132
Node 12	0.005426	-0.030205

Judging by the results of the update, it has worked well for the bar 10. There is room for refinement of the distribution of the bar 18 posterior function. The use of lower absolute errors and more measurement points, while improving measurement accuracy, reduces the speed of sampling and increases the time and arithmetic power needed for updates.

The secondary sampling method was adopted. First, 100kN is loaded at node 11 and the stiffness of rod 10 is updated according to the displacement at nodes 10, 11 and 12. Then 100kN is loaded at node 10 and the stiffness of rod 18 is updated according to the displacement of nodes 4, 10 and 11. The displacement of each node is shown in table 2 and table 3.

Table 3. Displacement of each node when loading 100kN at node 10

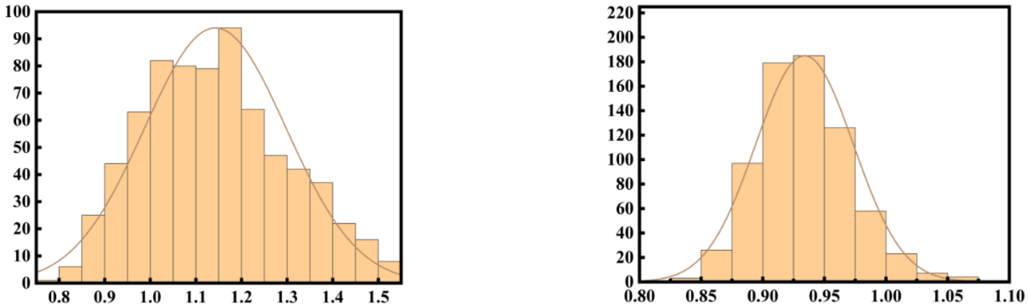
Direction	x (Direction 1)	y (Direction 2)
Node 2	0.003443	-0.026521
Node 3	0.006986	-0.040343
Node 4	0.012884	-0.045619
Node 5	0.01821	-0.040814
Node 6	0.021784	-0.026731
Node 8	0.016893	-0.026068
Node 9	0.017421	-0.039778
Node 10	0.013584	-0.048279
Node 11	0.008192	-0.039767
Node 12	0.008221	-0.026473

3.4. Updated approach to diagonal web bar 18

In order to improve the update effect of 18 rods, the original loading scheme can be replaced. Node 4 is loaded with 100kN upward load, and node 11 is loaded with 100kN downward load. After 15,000 samples are taken, the absolute error is accepted within 0.03mm. The displacement of the four nodes is shown in table 4. The update results are shown in figure 7.

Table 4. Load node 4 with an upward 100kN load and node 11 with a downward 100kN load

Direction	x (Direction 1)	y (Direction 2)
Node 4	-0.0038889	0.0103398
Node 5	-0.003365	-0.0027806
Node 10	0.0008012	0.0057415
Node 11	-0.0008556	-0.0067643



(a) Bar 10: Average-1.1425, Variance-0.154188 (b) Bar 18: Average-0.933, Variance-0.03622

Figure 7. Updated results for the diagonal web bar 18

A significant improvement in renewal can be found for bar 18, but this method is generally effective for bar 10. In practice, different methods can be used to update the two bars.

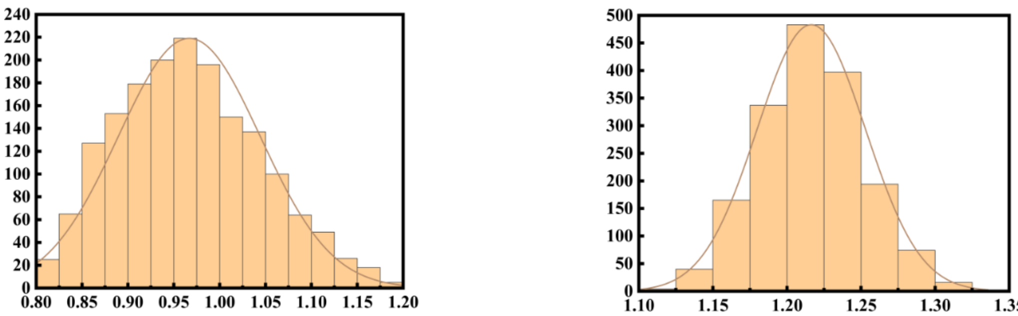
3.5. Updated approach to bottom chord bar 4, vertical web bar 19.

In the same truss model, it is assumed that damage occurs to bars 4 and 19. The preceding distribution of bars 4 and 19 both follow a uniform distribution U(0.85,1.4) and U(0.81,1.26), respectively.

Loading a downward 100kN load at node 5 for 15,000 samples, the absolute error of the accepted result is within 0.03mm. The displacements of nodes 4, 5, 10, and 11 are measured to perform Bayesian updates, as shown in table 5. The result of the update is shown in figure 8.

Table 5. Displacement of each node when loading 100kN at node 5

Direction	x (Direction 1)	y (Direction 2)
Node 4	0.0083137	-0.03967
Node 5	0.013838	-0.0481193
Node 10	0.0134987	-0.0392187
Node 11	0.0071897	-0.0419291



(a)Bar19: Average-0.966829, Variance-0.077213 (b)Bar4: Average-1.216625, Variance-0.0355

Figure 8. Updated results for the bars 4 and 19

The results show that while the renewal of the vertical web bar 19 has potential for improvement, the renewal of the lower chord bar 4 is better. However, after trying some different loading schemes, it is found to be difficult to improve the updating effect on the vertical web bar 19.

4. Conclusion

In this paper, the finite element software OpenSees is utilized to investigate the application of Bayesian updating to truss stiffness damage measurements. Using the rejection sampling method and assuming a damaged Pratt-type truss, the updating of the stiffness of the top chord bar, bottom chord bar, vertical web bar, and diagonal web bar is realized by choosing different load sizes, load locations, errors, and types of loads. The following main conclusions are obtained:

(1) The effectiveness of the updating depends critically on the quantity of measurement points. The more points measured, the more accurate the updated results, but this also requires more samples and slows down the rate of updating.

(2) The size and location of the loads also play a key role in the updating. In general, the higher the load the better the renewal effect. The location of the loads is important in order to achieve the goal of improving the updating effect without increasing the number of measurement points. For the top chord bar, bottom chord bar, vertical web bar, a more accurate update can be obtained by loading at one end of the bar. In the case of the diagonal web bar, if a set of loads in opposite directions is applied at both ends of the bar, a more accurate update can be obtained without using too many measurement points.

(3) The arithmetic is updated in the ideal state. In the actual project, there are more factors to consider. The stiffness update of the vertical web bar does not achieve the same results as the other bars, and its update program still has room for improvement.

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