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On the Dissemination of Negative Public Opinions on a Certain Stage

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Abstract. In the previous study we have already known that SIR model can be used to deal with the public opinion problem. On the basis of this, in this paper we proposed a new model which can describe the dissemination of the public opinion, especially the negative ones more precisely and efficiently. We separate people into 4 parts to describe their different state when received the information and use method of dynamic system to find out the final state of the system. From this study, we give a explanation about the phenomenon that public opinions always disappear.

Keywords: SIR, Public opinion

1. Introduction

The method of dealing with the negative public opinion always plays an important role nowadays. Since everyone who know the information has the possibility to be a transmitter, it's hard to predict the tendency of the public opinion timely.

Many researchers have applied SIR model and its variants to the dissemination of public opinion and they gave some meaningful conclusion. In 2021, Jiangjun Yuan et al. applied SIR model to the public opinion polarization [1]. In the same year, Tinggui Chen et al. considered the individual heterogeneity between different people [2]. This paper will propose a new and more precise model of the public opinion dissemination which is negative for people in a certain social networking platform to make the analysis more accurate.

The paper is organized as follow: Chapter II will give a brief and necessary introduction of the SIR model and its variants. In Chapter III, the new model is proposed and dynamic system differential equations are given. In Chapter IV, together with the calculation of the Disease-free Equilibrium point (DFE) and the basic reproductive number, we give the analysis of the result. Chapter V concludes.

2. A brief introduction of SIR model

SIR model, with its variant, is a compartmental model in epidemiology proposed in the early 20th century. In the origin model, people are divided into 3 parts:

S are the number of **s**usceptible individuals,

I are the number of infectious individuals, and

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R are the number of removed (and immune) or deceased people who are no longer have the possibility to be infected.

These variables represent the number of the relative people in a particular time and thus are vary over time. Because of this, the notation $\mathrm{S(t)}$, $\mathrm{I(t)}$ and $\mathrm{R(t)}$ are often used to indicate that they're actually functions of t (time). By assuming the transmission rate γ between I and R and the cross infectious rate β, we can draw a schematic diagram as follow:

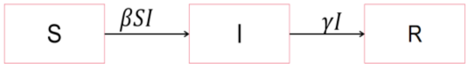

Figure 1. The original model of SIR

The relative differential equations are in (2.1):

$$\begin{cases} \frac{dS}{dt} = -\beta SI \\ \frac{dI}{dt} = \beta SI - \gamma I \\ \frac{dR}{dt} = \gamma I \end{cases} \tag{2.1}$$

This model gives reasonably prediction for the infectious diseases which are transmitted from human to human, and where recovery confers lasting resistance.

Out of the consideration for the complexity of a real infectious disease, people proposed some variants of the SIR model, such as SIS (for disease like influenza which nobody can immune forever), SCIR (C for the carriers who are infected but don't have the ability to infect others temporarily) and so on. All these models have one thing in common: they divided the people into different parts and give the dynamic system differential equations by considering the transmission of people from one part to another. Based on this, this paper gives a different SIR model to describe the different people during the dissemination of the public opinion.

3. The Construction of the Model

This model concentrates on people in a particular social networking platform and the public opinion caused by some negative events. The word 'negative' means the comments are more likely to be criticism and aggressive.

In this model, when an incident happened, people are separated into 4 parts:

U indicates unknown people, who are able to be the transmitter of the information but don't know anything right now.

L indicates people with lower will to transmit this information, such as people don't like to record their life by social media.

H indicates people with higher will to transmit this information, such as people who are deep-users of a social networking software.

T indicates people who are tired of this kind of information and decided to ignore it temporarily.

Here are some assumptions for this model:

1. The influence of other events which may be related to this information is omitted. That is to say, this model only concentrates on a certain event or a certain information.

2. Every time there are people who joined in or quit the platform. People who joined in the platform is always assumed to be an outsider, and it is possible for people in every compartment to exit this platform.

3. Unless claimed, every parameter should be considered as a positive constant.

All the parameters with their meaning are shown in Table 1:

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Table 1. Parameters

PARAMETERS	INTERPRETATION
Λ	Increase number of users
μ_0	Natural exiting rate
μ_H	Banned rate
ρ	Ratio constant
m	Process rate from L to H
n	Process rate from H to L
t_L	Basic process rate from T to L
t_H	Basic process rate from T to H
l_T	Basic process rate from L to T
h_T	Basic process rate from L to H
$\alpha(t)$	Control decaying function
$\gamma(t)$	Control decaying function
β_L	Cross dissemination rate caused by the contact between L and U
β_H	Cross dissemination rate caused by the contact between H and U

More detailed description is as follow:

μ_H gives the banned rate of the users because of their aggressive comments against the event. People with higher will to transmit the information are more likely to be banned. Thus, we use the assumption that this rate can only appear in part H.

The parameter ρ gives the ratio of people who changed their style from U to L. The total people who will go out of the style U in a unit time are assumed to be $K = \beta_L LU + \beta_H H$ and thus the people come in L from U are just ρK .

The functions $\alpha(t)$ and $\gamma(t)$ indicates the decaying exchanging progress between T and H, L. As time increasing, more and more people feel tired about the same event and thus they are more likely to be in T. These functions are just continuous functions who has the limit 0 when t goes to infinity.

Based on discussion above and the figure 2, the model is formulated as follow:

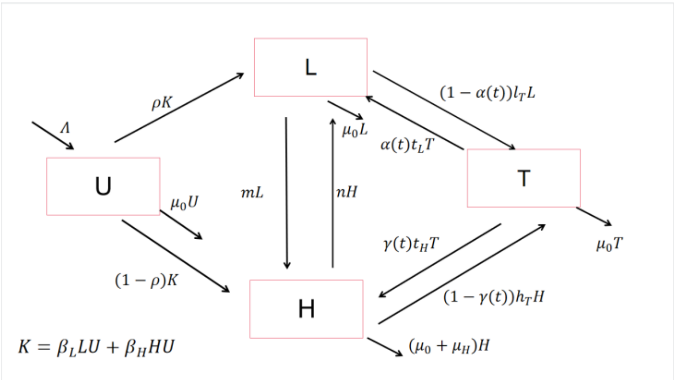

Figure 2. Flowchart of the dissemination for the ODE model

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$$\begin{cases} \dfrac{dU}{dt} = \Lambda - K - \mu_0 U \\ \dfrac{dL}{dt} = \rho K + nH + \alpha(t) t_L T - \big(1-\alpha(t)\big) l_T L - mL - \mu_0 L \\ \dfrac{dH}{dt} = (1-\rho)K + mL + \gamma(t) t_H T - nH - \big(1-\gamma(t)\big) h_T H - (\mu_0 + \mu_H) H \\ \dfrac{dT}{dt} = \big(1-\alpha(t)\big) l_T L + \big(1-\gamma(t)\big) h_T H - \mu_0 T \end{cases} \tag{3.1}$$

Where $K = \beta_L LU + \beta_H HU$ and $\alpha(t), \gamma(t)$ are notated as α, γ.

4. Calculations and Analysis

4.1. Calculation progress

The method here we use is inspired by Van den et al. [3].

First of all, by letting all of the differential equations in (3.1) equals to 0 and $L = T = H = 0$ as well, we can find the DFE is just

$$x_0 = (L_0, H_0, T_0, U_0) = (0,0,0,\Lambda/\mu_0)$$

Then, we claim that all parts except S are negative state because if there is even one person in L, H or T, they have the possibility to transmit the information. This isn't accord with out will.

Consider the flows go into the negative state from positive ones, let

$$\mathcal{F} = \begin{pmatrix} \rho K \\ (1-\rho)K \\ 0 \\ 0 \end{pmatrix}$$

And

$$\mathcal{V} = \begin{pmatrix} (1-\alpha) l_T L + mL + \mu_0 L - nH - \alpha t_L T \\ nH + (1-\gamma) h_T H + (\mu_0 + \mu_H) H - mL - \gamma t_H T \\ \mu_0 T - \big(1-\alpha(t)\big) l_T L - \big(1-\gamma(t)\big) h_T H \\ K + \mu_0 U - \Lambda \end{pmatrix}$$

Such that $\mathcal{F} - \mathcal{V}$ is just the right side of the differential equations after the order is rearranged as (L, H, T, U).

Use lemma 1 in [3], by consider $x = (L, H, T, U)$ as the variable, we find the derivative of $\mathcal{F}$ and $\mathcal{V}$ at the DFE are just

$$D\mathcal{F}(x_0) = \begin{pmatrix} F & 0 \\ 0 & 0 \end{pmatrix}$$

Where F is a 3×3 non-negative matrix and

$$D\mathcal{V}(x_0) = \begin{pmatrix} V & 0 \\ a_1 & a_2 \end{pmatrix}$$

Where V is also 3×3 non-singular matrix while $a_2 > 0$.

By definition,

$$F = U_0 \begin{pmatrix} \rho\beta_L & \rho\beta_H & 0 \\ (1-\rho)\beta_L & (1-\rho)\beta_H & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

And

$$V = \begin{pmatrix} m + (1-\alpha) l_T + \mu_0 & -n & -\alpha t_L \\ -m & (1-\gamma) h_T + n + \mu_0 + \mu_H & -\gamma t_H \\ -(1-\alpha) l_T & -(1-\gamma) h_T & \mu_0 \end{pmatrix}$$

Here we introduce an important lemma:

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Lemma 4.1: Suppose $N = (n_{ij})_{3\times 3}$ is any matrix and N is a 3×3 matrix has the form

$$M = \begin{pmatrix} km_1 & km_2 & 0 \\ (1-k)m_1 & (1-k)m_2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Where k is a constant. Then

$$MN = \begin{pmatrix} A & B & e_1 \\ C & D & e_2 \\ 0 & 0 & 0 \end{pmatrix}$$

And the spectral radius of MN is just

$$\rho(MN) = \max\{0, A + D\}$$

Proof 4.1: Calculate the product MN, we got

$$MN = \begin{pmatrix} A & B & e_1 \\ C & D & e_2 \\ 0 & 0 & 0 \end{pmatrix}$$

Where

$$\begin{aligned} A &= k(m_1 n_{11} + m_2 n_{21}) \\ B &= k(m_1 n_{12} + m_2 n_{22}) \\ C &= (1-k)(m_1 n_{11} + m_2 n_{21}) \\ D &= (1-k)(m_1 n_{12} + m_2 n_{22}) \end{aligned} \quad (4.2)$$

If we consider the eigenvalues of the matrix $K = MN$, then we have

$$\lambda I - K = \begin{pmatrix} \lambda - A & B & e_1 \\ C & \lambda - D & e_2 \\ 0 & 0 & \lambda \end{pmatrix}$$

Then the eigenvalues are just the roots of the polynomial

$$p(\lambda) = \det(\lambda I - K)$$

Expand this determinant from the last raw, we find

$$p(\lambda) = \lambda[(\lambda - A)(\lambda - D) - BC]$$

From (3.2) we observe that $AD = BC$, and thus

$$p(\lambda) = \lambda^2(\lambda - (A + D))$$

This polynomial has 2 roots and thus the spectral radius equals to the larger one, i.e.

$$\rho(MN) = \max 0, A + D$$

And we've done the proof.

By theorem 2 in [3], the basic reproductive number

$$\mathcal{R}_0 = \rho(FV^{-1})$$

Simple algebra gives

$$\rho(FV^{-1}) = \frac{U_0}{\det(V)} \rho(F_0 V^*) \quad (4.3)$$

Where V^* is the adjugate matrix of V, and

$$F_0 = \frac{F}{U_0} = \begin{pmatrix} \rho\beta_L & \rho\beta_H & 0 \\ (1-\rho)\beta_L & (1-\rho)\beta_H & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Suppose

$$V^* = \begin{pmatrix} v_{11} & v_{12} & v_{13} \\ v_{21} & v_{22} & v_{23} \\ v_{31} & v_{32} & v_{33} \end{pmatrix}$$

Simple calculation of V^* gives

$$\begin{aligned} v_{11} &= h_T(1-\gamma)(\mu_0 - t_H\gamma) + \mu_0(n + \mu_0 + \mu_H) \\ v_{12} &= (1-\gamma)\alpha h_T t_L + n\mu_0 \\ v_{21} &= (1-\alpha)\gamma l_T t_H + m\mu_0 \\ v_{22} &= l_T(1-\alpha)(\mu_0 - t_L\alpha) + \mu_0(m + \mu_0) \end{aligned} \quad (4.4)$$

Notice that if we take $M = F_0$ and $N = V^*$ in lemma 4.1, combined with (4.3), we find that

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$$\mathcal{R}_0(t) = \frac{U_0}{\det(V)} \max\{0, A + D\} \tag{4.5}$$

And

$$U_0 = \Lambda/\mu_0$$
$$A = \rho(\beta_L v_{11} + \beta_H v_{21})$$
$$D = (1-\rho)(\beta_L v_{12} + \beta_H v_{22})$$

Where $v_{ij}(1 \le i.j \le 2)$ is defined in (4.4) and V is defined above.

Take $t \to +\infty$ gives

$$\lim_{t\to+\infty} \mathcal{R}_0(t) = c$$

With $\mathcal{R}_0(t)$ is a continuous function, we find that the DFE is essentially stable when $c < 1$ and unstable when $c > 1$.

4.2. Analysis Progress

Use the substitution $U' = U - \Lambda/\mu_0$, the stability of the DFE in system (3.1) changes to the stability of 0 for the system below:

$$\begin{cases} \dfrac{dU'}{dt} = -K' - \mu_0 U' \\ \dfrac{dL}{dt} = \rho K' + nH + \alpha(t)t_L T - \big(1-\alpha(t)\big)l_T L - mL - \mu_0 L \\ \dfrac{dH}{dt} = (1-\rho)K' + mL + \gamma(t)t_H T - nH - \big(1-\gamma(t)\big)h_T H - (\mu_0 + \mu_H)H \\ \dfrac{dT}{dt} = \big(1-\alpha(t)\big)l_T L + \big(1-\gamma(t)\big)h_T H - \mu_0 T \end{cases} \tag{4.6}$$

Let $V(x) = U'$ for $x = (L, H, T, U')$. Firstly $V > 0$ for all $x \ne 0$,and then we have

$$\dot{V}(t) = \frac{\partial V}{\partial U'}\frac{dU'}{dt}$$
$$= -K' - \mu_0 U'$$

Which is obviously less than 0 since

$$K' = \beta_L L U' + \beta_H H U' + \frac{\Lambda}{\mu_0}(\beta_L L + \beta_H H) > 0$$

Thus, $V(x)$ is indeed the Lyapunov function. And from Lyapunov Theorem, the zero solution $x = (0,0,0,0)$ is locally asymptotically stable.

If we additionally assume that

(A) $\alpha(t)$ and $\beta(t)$ as two small constants.

Let $L = \{x: \dot{V}(x) = 0\}$, then it's obvious that

$$L = \{U' = 0\}$$

Thus, by La Salle's Invariance Principle we know that

$$\lim_{t\to+\infty} U'(t) = 0$$

Let M be the maximal invariant set in L , if either $L \ne 0$ or $H \ne 0$ will vary U ,thus they must be 0. By the same discussion for $\frac{dS}{dt}$ we know that $T = 0$.

This is to say, $M = \{x = 0\}$ and thus by the La Salle's Invariance Principle, the zero solution is global asymptotically stable. Back to differential equations (3.1), we find the DFE is global asymptotically stable when (A) holds.

5. Concludes

Negative public opinion always causes lot of trouble and thus it's necessary to control the scale of it. I realized that from the analysis above, as long as (A) holds, every initial state will go to DFE when the time goes to infinity. This gives an explanation about why all the scale public opinion go smaller and smaller. This means all of the public opinion will be controlled naturally as time goes by.

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If (A6) isn't assumed, the conclusion is that if the initial state isn't far away from the DFE, then even if $\alpha(t)$ and $\beta(t)$ are functions, the system state will also go to the DFE. When we notice that in the initial state, we always have

$$L = H = T = 0$$

The only thing we need to concern is whether U is far away from $\frac{\Lambda}{\mu_0}$.

Also, if we got some values of the parameters in Table 1, then we can use the basic reproductive number to find that the DFE is stable or not.

Finally, I should mention that the individual heterogeneity is not considered in this model. It might affect the dissemination rate in the dynamic system as some researchers suggested. Also, there's still some trouble when it is needed to estimate the velocity of convergence of the solution. These questions are important and need further exploration.

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