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An introduction to mathematical algorithms and Artificial Intelligence

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Abstract. Artificial Intelligence (AI), as a hot topic in today's science and technology, has penetrated into every aspect of our lives. The relationship between AI and mathematics and algorithms has become increasingly close. Mathematics, as a basic science, provides the theoretical basis for AI algorithms and models and is an essential part of the development of AI. The algorithm is an element that acts as a bridge between mathematics and AI and is a series of steps taken to solve a particular problem or reach a clear result [1]. The purpose of this paper is to explore the application and impact of mathematics and algorithms in the field of AI, and to explore how AI can bring about far-reaching changes in our society and human development.

Keywords: AI, Mathematics, Algorithms, Relationships.

1. Introduction

AI is essentially the embodiment of mathematics, probability theory, statistics, and various mathematical theories. Modern electronic computers are no longer just simple arithmetic but have the ability to do logic and reasoning. The basis of arithmetic is still mathematics. Mathematics makes AI a standardized science, is the indispensable foundation for the development of AI, and plays a key role in all stages of the development of AI. The algorithm is the cornerstone of AI, the source of the vigor of machine learning, and the degree of intelligence of the machine, depending on the "algorithm." The principle of AI is summarized in one sentence as a field that closely integrates mathematics, algorithm theory, and practice.

2. Core Principles

2.1. Development of Artificial Intelligence

The abbreviation of artificial intelligence is AI [1]. Accompanied by the development and progress of computer science, machine intelligence [1] was created for the first time in 1941, 1956, in the DARTMOUTH Institute of Artificial Intelligence. Experts and scholars from various countries have been invested in the theoretical study of hundreds of experts and scholars, which in turn extends the development of a number of theoretical theories of artificial intelligence, and the development and progress of other technologies play an important role in influencing the progress of artificial intelligence.

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Artificial intelligence is a branch of computer science that refers to the research and development of simulation and extension of human intelligence theory, method, and application of technical science [1]. By studying the nature of intelligence, it explores the development of new, intelligent machines that resemble the way of human intelligence, including robots, language recognition, image recognition, natural language processing, and expert systems. With the continuous improvement of theoretical technology and the expansion of the scope of application, artificial intelligence thinks like a human being, simulates human thought and consciousness, and surpasses human intelligence, etc..

Artificial intelligence is an extremely challenging science, covering a wide range, such as machine learning, computer vision and so on [2].

2.2. Artificial Intelligence Technology

In the field of science and engineering, computer systems can simulate human intelligent behaviors and thoughts, and the level of intelligence even exceeds that of human beings in terms of learning, reasoning, problem-solving, understanding language, and perceiving the environment. Artificial intelligence theories focus on solving different problems and apply to different scopes [2]. The specifics are as follows.

2.2.1. Machine Learning. It is one of the important methods of artificial intelligence, providing computer systems with big data and algorithms, from which they learn laws and patterns and then make decisions.

2.2.2. Deep Complex Learning. Is a special learning method that excels in dealing with complex big data, for example, it can simulate the way of thinking of the human brain by learning the hierarchical structure of artificial neural networks.

2.2.3. Linguistic Interaction. Through the computer and human language communication and interaction, in terms of speech recognition and other technologies, to achieve the ability to understand and master natural language.

2.2.4. Visual Recognition. To enable computers to read and understand image videos, including image recognition, object detection, image generation, etc.

2.2.5. Expert Decision Making: Learn to master the decision-making process of expert reasoning and decision-making.

2.2.6. Intensive Learning. Reinforcing the intensity and depth of learning to obtain rewards that maximize expectations.

2.2.7. Robotics. Combines the knowledge of mechanical engineering, electrical engineering, and computer science to design, build, and manipulate robots that can perform tasks.

2.2.8. Intelligent Applications: For example, making real-time intelligent decisions in the field of quantitative investment, case investigation, etc.

2.2.9. Genetics and Evolution. Discover and explore issues such as heredity based on the laws of nature and the theory of evolution of superiority and inferiority.

2.2.10. Recommended Applications. Tailor relevant, intelligent products and services, etc., according to the personalized needs of different users.

The above is the introduction and description of the relevant aspects of artificial intelligence. With the progress of science and technology and the continuous expansion of the scope of application,

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artificial intelligence will be in all aspects of society for the development of human society to provide more and more comprehensive and more intelligent innovation and services.

2.3. Artificial Intelligence Technology Application Process

In order to ensure that the system can effectively solve the problem, the process of applying artificial intelligence in practice is as follows:

2.3.1. Problem Formulation and Goal Setting. Identify the problem to be solved and clarify the desired results and goals.

2.3.2. Data collection and Preparation. Organizing and preprocessing: discover errors, missing values or outliers in the data, and perform conversion and preprocessing to ensure the quality of sample data.

Data labeling and feature extraction: extract key features to the data to improve the machine recognition mode.

2.3.3. Selection and Design. Selection of suitable samples: Match suitable machines or deep learning models according to the problem requirements and data characteristics.

Design: Combine the neural network hierarchy, algorithm selection and other key factors to design a suitable model structure and architecture.

2.3.4. Training and Evaluation. Data segmentation: Segment into training set, validation set and test set.

Training: train the model through the training set to master the patterns and laws in the data.

Evaluation: Evaluate the model performance through the validation set or test set, test the performance on new data, and optimize and adjust.

2.3.5. Optimization and Adjustment. Adjustment of model parameters: Adjust the parameter structure of the model according to the evaluation results to continuously improve the performance and generalization ability.

Feature engineering: further optimizing feature extraction or selection to improve the model effect.

2.3.6. Deployment and Application. Deployment: Deploy the trained model to the actual application environment to process new data and make predictive decisions.

Continuous monitoring and maintenance: monitor the model performance, regularly update and maintain the model to adapt to new data and environmental changes.

2.3.7. Result Analysis and Interpretation. Result analysis: analyze the output of the model to understand how well the model solves the problem.

Interpretation of decisions: explain the decision-making process of the model to improve its interpretability.

In the practical application of artificial intelligence, the practical use of mathematical theories and algorithms is indispensable everywhere, and although there may be changes or overlaps in them, or even repeated iterations, the mathematics and algorithms are constantly improved and optimized for artificial intelligence.

2.4. Mathematics

Widely used in many fields as a pan-fundamental theoretical discipline, it is also the scientific and theoretical basis of artificial intelligence and has assumed an important and irreplaceable role in the development of artificial intelligence. Mathematical knowledge is an essential cornerstone in the understanding and application of artificial intelligence, including linear algebra, probability theory and statistics, calculus, and optimization algorithms [2].

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2.4.1. Linear Algebra that Formalizes Objects of Study. Modern mathematics uses linear algebra as the main method of analysis, and linear algebra is the core foundation of artificial intelligence. From quantum mechanics to image processing are inseparable from the use of vectors and matrices. Behind vectors and matrices, it provides an abstract view of the world: everything can be abstracted into combinations of certain features and viewed statically and dynamically under a framework of predefined rules. The essence of the concept of abstraction, in terms of its interpretation, lies in the abstraction of concrete things into mathematical objects describing their static and dynamic properties; vectors are essentially stationary points in n-dimensional linear space; linear transformations describe changes in the vectors, or in the coordinate system that serves as a frame of reference, and are represented by matrices; and the eigenvalues and eigenvectors of the matrices describe the speed and direction of the changes. Linear algebra is a fundamental toolset.

2.4.2. Probability Theory Describing Statistical Laws. In addition to linear algebra, probability theory is a necessary mathematical foundation in AI research. In today's world of exploding data and exponentially increasing computational power, probability theory has played a central role in machine learning. Like linear algebra, probability theory represents a way of looking at the world, with its focus on ubiquitous possibilities.

2.4.3. Advanced Mathematical Statistics in a Small Way. Mathematical statistics are equally indispensable in the study of artificial intelligence. The underlying statistical theory helps to explain the algorithms of machine learning and the results of data mining, and the value of the data can only be realized if a reasonable interpretation is made. Mathematical statistics study random phenomena based on data obtained from observation or experiments and make reasonable estimates and judgments on the objective laws of the research object.

Although mathematical statistics to probability theory as the theoretical basis, but there is an essential difference in methodology between the two. The role of probability theory is based on the premise that the distribution of random variables is known, according to the known distribution, to analyze the characteristics and laws of random variables; mathematical statistics is the object of study is an unknown distribution of random variables, the research method is to carry out independent repeated observations of random variables, according to the results of the observations obtained on the original distribution to make inferences.

In a loose but intuitive phrase, mathematical statistics can be seen as reverse probability theory. The task of mathematical statistics is to infer the nature of the aggregate from an observable sample in reverse; the tool for inference is the statistic, which is a function of the sample and is a random variable; parameter estimation estimates the unknown parameters of the aggregate distribution by means of randomly drawn samples, including point and interval estimation; and hypothesis testing accepts or rejects a certain judgment about the aggregate by means of randomly drawn samples.

2.4.4. Optimization Theory for Finding Optimal Answers. Essentially, the goal of artificial intelligence is to make optimal decisions in complex environments and multi-body interactions. Therefore, optimization theory is also a necessary basic knowledge of artificial intelligence. The problem of optimization theory is to determine whether the maximum or minimum value of a given objective function exists and to find the maximum or minimum value of the objective function. If the given objective function is regarded as a mountain range, the optimization process is to determine the location of the peak and find the path to reach the peak.

2.4.5. Information Theory of Quantitative Uncertainty. Scientific research in recent years has continuously confirmed that uncertainty is the essential property of the objective world. The world of uncertainty can only be described using probabilistic models, which led to the birth of information theory. Information theory uses the concept of "information entropy" to explain the amount of information from

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a single source and the amount and efficiency of information transfer in communication and builds a bridge between the uncertainty of the world and the measurability of information.

Information theory deals with uncertainty in the objective world; conditional entropy and information gain are important parameters in classification problems; KL scatter is used to describe the difference between two different probability distributions; and the principle of maximum entropy is a common criterion for summarizing classification problems.

2.4.6. Logical Forms for Realizing Abstract Reasoning. In the infancy of AI, the founders, including future Turing Award winners such as John McCarthy, Herbert Simon, Marvin Minsky, and others, had a vision of having "programs with the ability to think abstractly explain how synthetic matter can have a human mind." In layman's terms, the ideal AI would have the ability to learn, reason, and generalize in an abstract sense and would be more general than algorithms for solving concrete problems such as chess or Go.

2.4.7. Support Vector Machines for Classification in Feature Space. Support vector machine is one of the main classification methods in artificial intelligence, and its mathematical basis is the kernel function. The theory of computability is an important theoretical foundation and tool for artificial intelligence, and in order to answer the question of whether or not there is an undecidable problem, mathematical logicians have come up with definitions of algorithms (formalizing general mathematical reasoning as a logical deduction). To be computable means to find an algorithm for solving a problem. Outside of incomputability, a problem is said to be unworkable if the computational time required to solve it grows exponentially with the size of the instance, and the study of this problem gives rise to computational complexity. Computability and computational complexity provide a mathematical foundation for AI to determine the likelihood of problem solutions.

2.4.8. Solving Uncertainty Problems Bayesian Statistics and Decision Making. The birth of the discipline of artificial intelligence was characterized by the emergence of conditional probability and Bayes's theorem based on probability theory, which laid the foundation of modern methods for uncertain reasoning in most AI systems.

Bayesian networks, which originated from conditional probability, are a graphical network model that describes uncertain causal relationships between variables and are currently a mathematical tool used in artificial intelligence, typically for a variety of reasoning. The transfer algorithm provides an effective algorithm for Bayesian nets, laying a mathematical foundation for its entry into the practical field. Later, object-oriented ideas were introduced into Bayesian nets to solve the problem of modeling large complex systems. The introduction of temporal quantities into Bayesian nets resulted in the formation of dynamic Bayesian nets, which provide tools for modeling and reasoning over time. The compatibility of Bayesian network nodes with discrete and continuous numerical variables forms a hybrid Bayesian network, which has a greater advantage in mining and reasoning about massive data. The applications of Bayes in the field of artificial intelligence mainly include fault diagnosis, system reliability analysis, air traffic management, vehicle type classification and so on [3].

2.5. Algorithms

In fact, algorithms have been used throughout the history of human development. 1+1=2 is the simplest algorithm. It is only the rise of deep learning that makes the algorithm have special importance. In essence, an algorithm is "a simple set of instructions constructed to accomplish a task"[4], and "a series of steps taken to solve a particular problem or achieve a definite result"[5]. For artificial intelligence and machine learning, an algorithm is "a limited, deterministic, efficient method of solving a problem that is suitable for implementation in a computer program" [6], and is "a coding process that transforms input data into a desired output based on a specified computation" [7]. Simply put, an algorithm is a logical operation where we import data into the algorithm, and the algorithm outputs the result [8]. The specific

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steps to solve a problem are the "algorithms" needed to write a program, which is the finite number of steps required for a computer to complete a task.

AI algorithms are different from artificial intelligence, but are the most central component of worker intelligence [9]. An AI system is a human-designed software or possibly hardware system that, given a complex goal, senses its environment through data acquisition, interprets the structured or unstructured data it acquires, performs intellectual reasoning, processes the information it obtains from the data, and decides on the best course of action for the given goal in order to ultimately take action, either physically or digitally.

3. Applications

Mathematics makes artificial intelligence a standardized science, which is the indispensable foundation for the development of artificial intelligence and plays a key role in all stages of its development. Its main application process is the abstract mathematical theory applied to the computer and then solves the problems that exist in reality. mathematical methods in the application of artificial intelligence scenarios continue to improve and develop so as to promote "intelligent, scientific" artificial intelligence. [10]

Kant said, "A discipline reaches perfection only when it successfully applies mathematics." Mathematics plays an important role in the field of artificial intelligence, which provides the theoretical foundation and methodology for the development of artificial intelligence and promotes the innovation and application of artificial intelligence technology. Statistics and probability theory, linear algebra and matrix theory, optimization theory and other branches of mathematics provide strong theoretical support and algorithmic tools for AI. At the same time, mathematics has also promoted the rapid development and wide application of artificial intelligence technologies such as machine learning, deep learning, and natural language processing. The close relationship between mathematics and artificial intelligence will further promote the development of intelligent science and technology and create a more intelligent world for human beings [11].

Artificial intelligence can simulate human thought, cognition, and process, which includes the basic knowledge of mathematics and the relevant theory of algorithms. Artificial intelligence and mathematics are complementary; mathematics is a solid foundation for the development and innovation of artificial intelligence science, and artificial intelligence is the flower blossomed by the theory of mathematical science. Therefore, mathematics is the essential connotation of artificial intelligence, while artificial intelligence is the external appearance of mathematical theory.

Algorithms demonstrate the innovative ability and potential of AI technology in different fields and applications. Algorithms determine the intelligence, humanization and generalization ability of models, and also affect the speed of innovation and the scope of application. Algorithms are the core element of AI competition and the driving force for progress.

Mathematics is basic and fundamental in algorithms, and due to the development of intelligent algorithms, the display of mathematics in algorithms has never been higher. Algorithms are considered "programs for solving a given type of mathematical problem" [12], and Kevin Slavin of the MIT Media Lab suggests "rethinking the role of contemporary mathematics-not just financial mathematics. But also general math. It transforms what we extract and derive from the world into something that actually shapes the world around us and the world within us. It's specialized algorithms, basically, the math that computers use to decide things. They acquire a sensitivity to truth because they repeat themselves over and over again, they ossify and calcify, they become reality." [13] The further association of mathematics with both "truth" and "reality" prompts us to focus on the profound changes in the relationship between formalized logic, represented by mathematics, and concrete experience, and to think about the experience of mathematics in intelligent algorithms. With the development of information civilization, the importance of the experience dimension may decrease, while the importance of mathematics and the entire formal logic dimension may increase. In intelligent algorithms, the role of mathematics and to the entire formalized logical dimension is highlighted [14].

In conclusion, it is known that mathematics and algorithms play an important and irreplaceable role in the process of artificial intelligence journey.

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4. Conclusion

Artificial intelligence is one of the most promising and influential sciences and technologies today, which has penetrated various industries and fields and brought great changes and value to society [15]. Mathematics and algorithms play a crucial role in the development of artificial intelligence, making great contributions to society and bringing convenience to human life. In the era of information explosion, we should make full use of artificial intelligence, constantly learn to explore its operating logic, and combine theory and practice so that artificial intelligence can better serve society and contribute to human development.

References

[1] Ma Lingyun, 2018, Artificial Intelligence Still Needs Cold Thinking, Don't Make the Boom Turn into a Cold Wave, Chinese Engineering Journal
[2] Zhang Miao, Summary of ten years of development of artificial intelligence (2011-2020)
[3] Minglin Zhu. A Brief Discussion on the Development Prospects of Artificial Intelligence [J]. Journal of Yuncheng College, 2003(03)
[4] 2018.4, UK Artificial Intelligence Council "Al in the UK: Ready, Willing and Able" (Plans, Capabilities and Aspirations for the Development of Artificial Intelligence in the UK)
[5] Juan Huang, Jun Wang. Optimization research of mathematical modeling based on different artificial intelligence algorithms [J]. Automation and Instrumentation,2018(06)
[6] Huang Xiaojun. Application of mathematical algorithms in artificial intelligence[J]. Electronic Technology,2021,50(03)
[7] Wu Lei. Analysis of mathematical algorithms to promote the development of computer artificial intelligence [J].2020(20)
[8] Xu Zongben. 10 major mathematical and scientific foundation problems of artificial intelligence[J]. Science in China: Information Science,2021,51(12)
[9] Lei Duan, Changjie Tang, etc. Michael Sipsey: Introduction to the Theory of Computing, translated, Mechanical Industry Press, 2015, p. 114
[10] Zhihu, What is the connection between math and artificial intelligence, http://www.zhihu.com/question/341091555
[11] Pedro Dominguez (translated by Huang Wanping.), The Ultimate Algorithm: How Machine Learning and Artificial Intelligence Are Reshaping the World, CITIC Press, 2017, pp. 4, 9
[12] EU Ethics Guide lines for Trust worthy AI 2019.4.8
[13] Kevin Slavin, "How Algorithms Shape Our World", 2014-03-14
[14] Tienen Wang, "Big Data and the Epistemology of Creation", Journal of Shanghai University (Philosophy and Social Science Edition) 2021, No. 1
[15] Min Li. An overview of mathematical theoretical foundations of artificial intelligence [J]. Internet of Things Technology, 2017.7