Proceedings of the 4th International Conference on Computing and Data Science (CONF-CDS 2022)
DOI: 10.54254/2755-2721/2/20220557

Research on traffic congestion situation in urban central area based on SIR model

Keyu Chen[1]

[1]Department of math, Xi'an Jiaotong-liverpool University, Suzhou, 215123, China

Ckycc5632@163.com

Abstract: Urban traffic congestion is similar to the spread of infectious diseases. Based on this point of view, based on the classical infectious disease model SIR, a traffic congestion situation evolution model is established. The effectiveness of the model is verified by numerically solving the equilibrium point R0 and combining with specific examples. The model effectively depicts the process and characteristics of the dynamic evolution of traffic conditions in the central area of the city, which can help the urban traffic management department to make a real-time response to traffic congestion and provide reference for traffic management.

Keywords: traffic congestion, SIR model, Central area; Kunming.

1. Introduction

With the rapid development of cities, the increasing daily travel needs of urban residents and the rapid growth of private cars, the problem of urban traffic congestion has become increasingly prominent. Taking Kunming as an example, at the end of 2020, the number of motor vehicles in Kunming reached 2.97 million, with an annual growth rate of 5.2%. The number of motor vehicles owned by one thousand people was 309, ranking the top in the country. Urban traffic congestion occurred frequently, which had a great impact on people's work and life. In order to facilitate the traffic management department to analyze the road traffic congestion, grasp the characteristics and situation of road traffic operation in real time, evaluate the degree of traffic congestion, and timely dredge the traffic congestion is an important basic work to solve the traffic congestion.

Urban traffic congestion starts from a certain road section. If it cannot be evacuated in time, it will lead to the spread of congestion in the whole network, resulting in a large area of traffic congestion. The diffusion mechanism of traffic congestion in the road network is similar to the spread of infectious diseases. Therefore, SIR model provides a new perspective for studying the spread of urban traffic congestion. Sun, H. J. and Wu, J. J. [1] established SEIR model of complex small world network to describe the diffusion process of urban traffic congestion, and simulated the correlation between congestion recovery rate and congestion infection rate through simulation. Wu, J. J. et al [2] proposed that the infection rate and recovery rate affect the spread of traffic congestion through simulation research. Meead, S. et al [3] used two characteristic indexes of system dynamics, propagation rate and dissipation rate, to describe the process of traffic congestion diffusion. Verified by the traffic data of six different cities, they believe that Sir model can better predict urban traffic congestion. Getachew K, B. [4] used the model to describe the propagation and dissipation process of traffic congestion in urban road network, and simulated the phenomena related to traffic congestion diffusion in traffic network dynamics. Teja, I. et al [5] established a propagation model of information received by vehicles during driving to predict the number of vehicles receiving road traffic congestion information at each time on the road.

Zeng, Z. L. et al [6] used SIR model to simulate the congestion propagation process of urban rail transit system, and introduced rail transit passenger Congestion Propagation Rate and passenger congestion evacuation rate to study the propagation law and dissipation law of passenger congestion. It is considered that the passenger flow of rail stations, train departure interval, initially congested stations and station capacity affect the congestion propagation rate. Shi, Z. et al [7] used SIR model to simulate the propagation process of rail transit passenger congestion, found that the propagation rate affected the passenger congestion rate, and put forward policies to control passenger congestion from the two aspects of supply control and demand control. Gurin, D. et al [8] used the improved SIR model to simulate the train delay propagation process in the railway network, and studied and considered the delay propagation between trains with different priority levels. Zhao, Y. et al [9] established the SIR model of multi station fault delay propagation in urban rail transit operation, predicted the spatial impact range of station delay, and provided the basis for the safe operation of rail transit.

Based on the above research, using the data of road traffic conditions in the central urban area of Kunming, this paper establishes a differential equation model for the evolution of urban 24-hour traffic operation situation. The model can not only describe the change process of a road traffic congestion with time from a micro perspective, but also intuitively reflect the situation of urban road network traffic congestion from a macro level, The main contents of the research include: (1) using the model to analyze the urban 24-hour traffic operation situation; (2) The traffic operation characteristics of different functional areas of the city are compared and analyzed.

2. Data sources

Use Python software to crawl real-time road condition data from Gaode map, realize data visualization through ArcGIS software, and analyze the spatio-temporal changes of road traffic conditions in the central area of Kunming throughout the day.

3. Model assumptions

Hypothesis 1: the infectious nature of road congestion can be studied by using the basic SIR model
Hypothesis 2: There is an influence between the two regions, and the equilibrium point can be calculated by using the multistrain model.

4. Model construction

The mathematical description of the operation state in the process of traffic congestion propagation is simplified to the study of the proportion of congestion mileage, and the model is abstracted and improved. The evolution mechanism of traffic operation conditions is consistent with the principle of SIR model to a great extent. The following assumptions are made for the model. The study divides the urban road network into unblocked roads, congested roads and restored unblocked roads. The traffic state on each road will change with the passage of time, the unblocked roads will become congested, and the congested roads will recover. In the model, the diffusion of traffic congestion can be described by a general measure similar to the basic regeneration number, which is called R0 in the infectious disease model. This number represents the speed of congestion spreading in the city.

5. The establishment of model

1） Urban impact areas are mainly divided into two categories: cultural and educational areas (shown in orange area) and office areas (shown in red area).

2） Applying the basic SIR model, the road will change from unobstructed to congested, and also from congested to unobstructed. The change rates of the two are different, and there is no possibility of

DOI: 10.54254/2755-2721/2/20220557

corresponding "removal" (death).

3) The selected time is Wednesday, March 16, 2022, as a sample of daily working days.

4) The calculation time interval is 1H, and a total of 24h image feedback is obtained (according to the actual situation of traffic flow, the available data is mainly 6:00-22:00).

5) Congestion transmission rateλ_1、λ_2、μ_1、μ_2 is the average congestion change rate during peak hours of the day (7:00-9:00 and 16:00-18:00), and the obtained value is rounded to two decimal places.

6) The data used in the model are all the data climbed by Gaode map, which is true and reliable.

7) The total length remains at a fixed level for the period under consideration.

8) Corresponding to the model, the calculation amount of the model is the length of the road, in meters.

9) The rate of change that deviates from the middle value and is too large or too small shall be rounded off.

In this part, we use Sir infectious disease model to check the changes of road congestion in Kunming (Yunnan, China) during the peak period, and calculate and predict them.

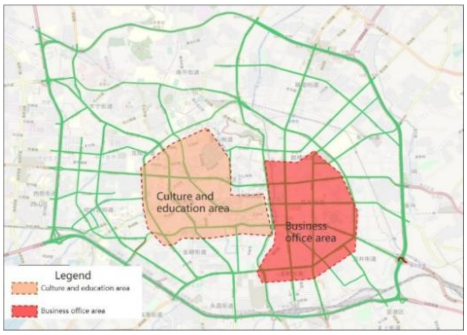
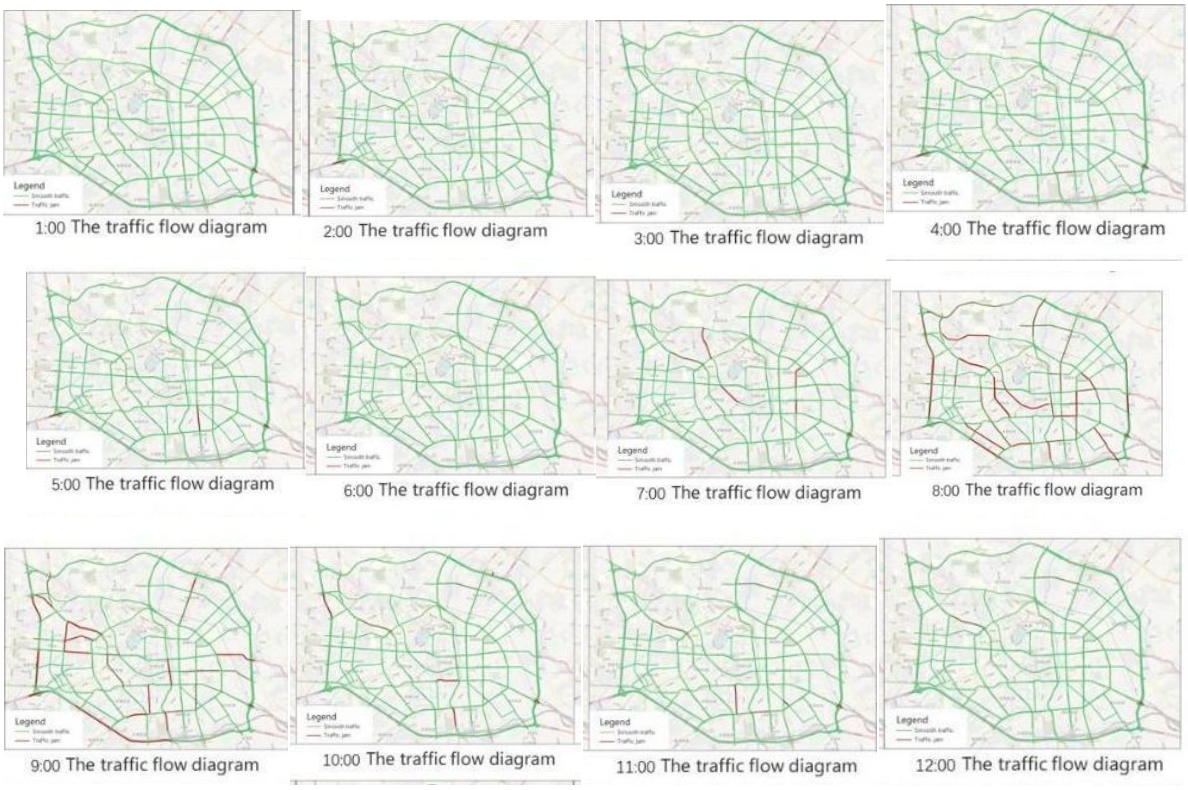


Figure 1. The two areas with different urban functions.



DOI: 10.54254/2755-2721/2/20220557

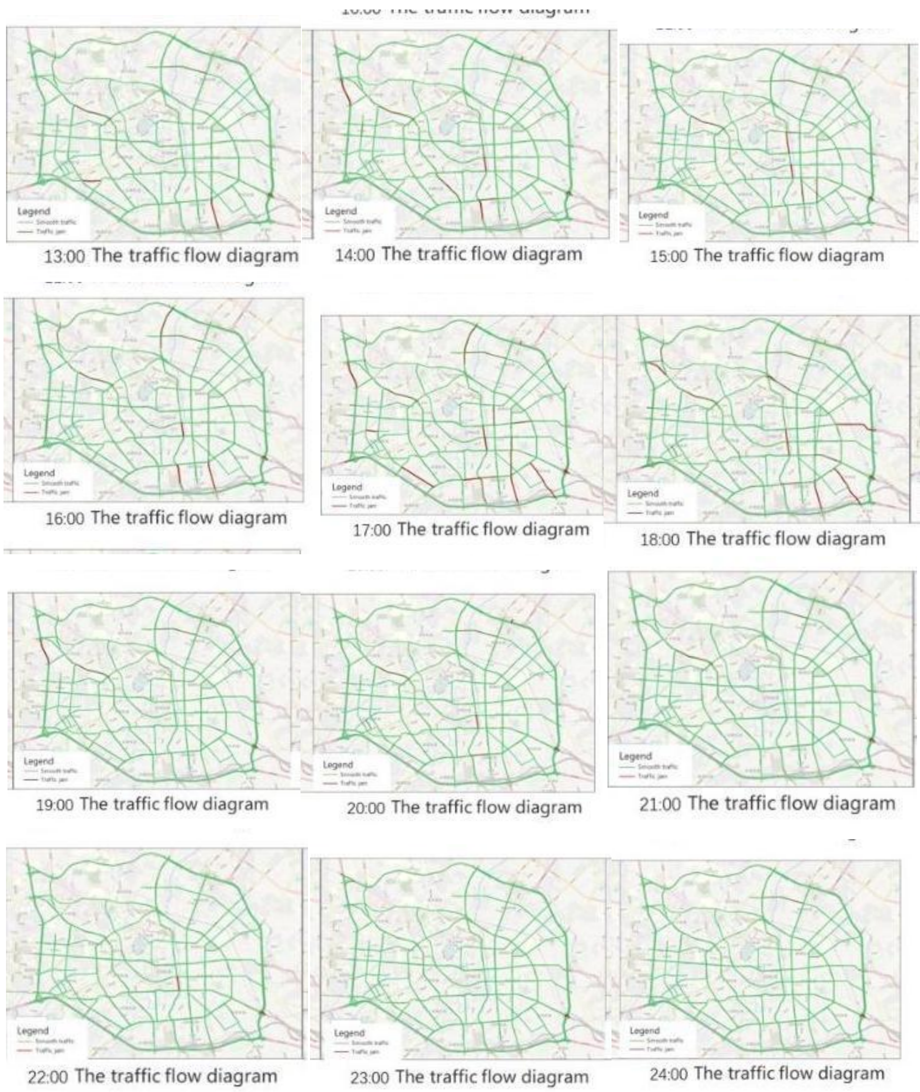


Figure 2. Round-the-clock traffic situation map of Kunming central district.

Assumption 1: the two major influence areas of the city do not interfere with each other
For cultural and educational districts:

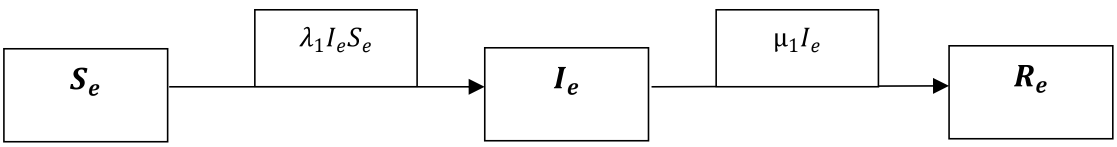


Figure 3 rogression diagram for the SIR model of (1)-(3)

λ_1: Congestion infection rate in cultural and educational districts
μ_1: Congestion recovery rate of cultural and educational districts
S_e: Proportion of the length of infectious road sections in the culture and education division to the length of Education Division
I_e: The proportion of the length of infected (congested) road section in the length of Education Division
R_e: Proportion of the length of the restored road section in the cultural and educational division to the length of the educational division

The following relation is obtained:

$$\begin{cases} \frac{dS_e}{dt} = -\lambda_1 I_e S_e & \#(1) \\ \frac{dR_e}{dt} = \mu_1 I_e \#(2) \\ \frac{dI_e}{dt} = \lambda_1 I_e S_e - \mu_1 I_e \#(3) \end{cases}$$

$$\frac{d}{dt}\left(S_e(t) + I_e(t) + R_e(t)\right) = 0\#(4)$$

$$R_e(t) = 1 - S_e(t) - I_e(t)\#(5)$$

It can be deduced that

$$\begin{cases} \frac{dS_e}{dt} = -\lambda_1 I_e S_e \#(6) \\ \frac{dI_e}{dt} = \lambda_1 I_e S_e - \mu_1 I_e & \#(7) \end{cases}$$

$$\frac{dI_e}{dS_e} = \frac{\lambda_1 I_e S_e - \mu_1 I_e}{-\lambda_1 I_e S_e} = -1 + \frac{\mu_1}{\lambda_1 S_e}\#(8)$$

$$I_e(S_e) = -S_e + \frac{\mu_1}{\lambda_1} ln\, S_e + C\#(9)$$

When $t = t0$, $I_e(t0) = I0$, $S_e(t0) = S0$, let $\rho = \frac{\mu_1}{\lambda_1}$, namely,

$$I_e(S_e) = I0 + S0 - S_e + \rho\, ln\frac{S_e}{S0}\#(10)$$

$$I_e'(S_e) = -1 + \frac{\rho}{S_e}\begin{cases} < 0, S_e > \rho & (11) \\ = 0, S_e = \rho & (12) \\ > 0, S_e < \rho & (13) \end{cases}$$

When $S_e < \rho$, $I_e(S)$ monotonically increasing; When $S_e > \rho$, $I_e(S_e)$ monotonically decreases; When I_e=**0**,$I(0) = -\infty$, $I_e(S0) = I0 > 0$,so there is a unique $S_e^*, 0 < S_e^* < S0$ let $I_e(S_e^*) = 0$.Besides, $When\, S_e^* < S_e \leq S0$, $I_e(S_e) > 0$.

From this, we can see that the road congestion rate in the area exceeds the threshold $\rho = \frac{\mu_1}{\lambda_1}$, congestion infection will spread.

Table 1. digital chart (the average changing rate λ_1 *and* μ_1 *of* cultural and educational districts)

cultural and educational districts	7:00-9:00	16:00-18:00	Average
λ_1	0.52（0.368-0.851）	0.39（0.161-0.995）	0.48
μ_1	0.40（0.186-0.749）	0.28 (0.161-0.478)	0.34

In this case, the specific value will be brought in, $\rho = \frac{\mu_1}{\lambda_1} \approx 0.71$

It can be concluded that when the vulnerable sections in the whole cultural and educational area account for 0.71 of the whole area, it exceeds the threshold value. When $\rho = \frac{\mu_1}{\lambda_1} \approx 0.71$, traffic congestion will spread (SARS Epidemic Forecast and Trend Analysis, 2021).

Similarly, for the division of office area:

$$\boxed{S_w} \xrightarrow{\lambda_2 I_w S_w} \boxed{I_w} \xrightarrow{\mu_2 I_w} \boxed{R_w}$$

Figure 4 rogression diagram for the SIR model of (14)-(16)

λ_2: Congestion infection rate in office area

DOI: 10.54254/2755-2721/2/20220557

μ_2: Congestion recovery rate of office area
S_w: Proportion of the length of infectious road sections in the office division to the length of the office division
I_w: Proportion of the length of infected (congested) road sections in the office division to the length of the office division
R_e: Proportion of the length of the restored road section in the office partition to the length of the office partition
Get the following relation

$$\begin{cases} \frac{dS_w}{dt} =- \lambda_2 I_w S_w & (14) \\ \frac{dR_w}{dt} = \mu_2 I_w & (15) \\ \frac{dI_w}{dt} = \lambda_2 I_w S_w - \mu_2 I_w & (16) \end{cases}$$

Table 2. digital chart (the average changing rate λ_2 and μ_2 of office area).

office area	7:00-9:00	16:00-18:00	A
λ_2	0.41 (0.154-0.664)	0.44 (0.309-0.560)	0.43
μ_2	0.36 (0.127-0.514)	0.24 (0.123-0.358)	0.30

In this case, the specific value will be brought in, $\rho = \frac{\mu_2}{\lambda_2} \approx 0.70$

It can be concluded that when the vulnerable sections in the whole cultural and educational area account for 0.70of the whole area, it exceeds the threshold value. When $\rho = \frac{\mu_2}{\lambda_2} \approx 0.70$, traffic congestion will spread (SARS Epidemic Forecast and Trend Analysis, 2021).

To sum up, when there are many vulnerable sections during morning and evening peak hours and the corresponding congestion is difficult to dredge, the traffic congestion infection will spread quickly.

Assumption 2:

If there is mutual influence between the two zones (combined with the facts, the influence of the early peak Education Zone on the office area and the influence of the late peak office area on the education area), in order to study the traffic congestion infection diffusion state of the two mutually affected zones, a new model is established.

1. The model has only one susceptible compartment, but there are two traffic infection factors, each of which is modeled as a simple SIS model. The SIS model is constructed to correspond to two simultaneous interpreting rates [10]. Since only the evening peak is studied, according to people's travel judgment, infection transmission will occur from the office area to the education area.

2. The value of B in this model is 0, because usually for roads, the section distance will not be increased or reduced for a long time.

3. In the evening peak, the education area will not infect the office area in turn.

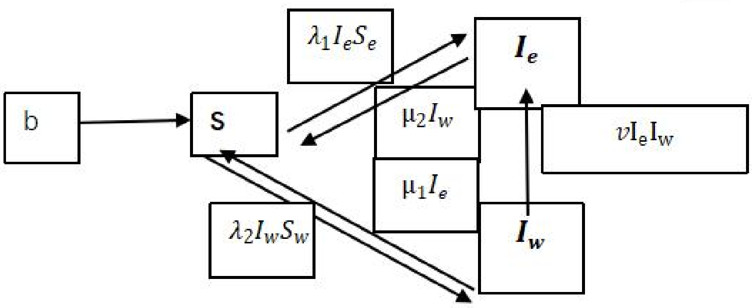


Figure 5. Progression diagram for the multistrain model of (17)-(19)

DOI: 10.54254/2755-2721/2/20220557

Get:

$$\begin{cases} \dot{I_e} = \lambda_1 I_e S_e - (b+\mu_1) I_e + v I_e I_w & (17) \\ \dot{I_w} = \lambda_2 I_w S_w - (b+\mu_2) I_w - v I_e I_w & (18) \\ \dot{S} = b - (\lambda_2 I_w S_w + \lambda_1 I_e S_e) + \mu_1 I_e + \mu_2 I_w & (19) \end{cases}$$

DFE is $x_0 = (0,0,1)^t$

$$F=\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \qquad V=\begin{pmatrix} b+\mu_1 & 0 \\ 0 & b+\mu_2 \end{pmatrix} \tag{20}$$

FV^{-1}, has the two eigenvalues

$$R_e = \frac{\lambda_1}{(b+\mu_1)} = \frac{\lambda_1}{\mu_1} \approx 1.41 \tag{21}$$

$$R_w = \frac{\lambda_2}{(b+\mu_2)} = \frac{\lambda_2}{\mu_2} \approx 1.43 \tag{22}$$

$$R_0 = \max_{i\in\{e,w\}} R_i = 1.43 \tag{23}$$

6. Analysis of model results

1） From hypothesis 1, we can see that during the peak period, the threshold values of the two zones are very similar, both around 0.7. This shows that, to some extent, although the urban nature of the two regions is different, the trend of congestion transmission during peak periods is basically the same. That is, the transmission occurs when it exceeds about 0.7.

2） From hypothesis 2, we can see that when two partition areas are placed in a compartment, there will be a mutual influence to promote diffusion, so as to obtain the basic regeneration number· R_0. due to $R_0 > 1$， the disease free equilibrium is locally asymptotically unstable, then the DFE is unstable and invasion is always possible.

Deficiencies and deficiencies of the model:

In model 1, the infection rate and recovery rate are obtained by taking the average value of the road section in the whole region and its morning and evening peak hours. The available range of these values is very small, so there is no way to infer the situation other than this.

In model 2, the multistrain model is adopted, but for diseases with birth rate and mortality b > 0, it will be more in line with the application of the model. For traffic, there will be no new roads and old roads damaged for a long time.

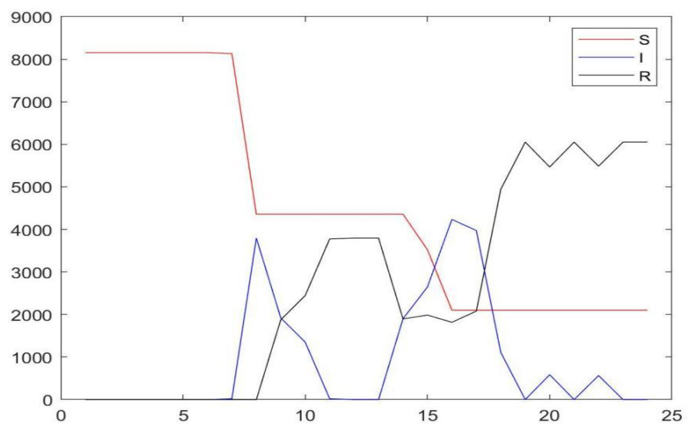


Figure 6. the changes of the Youth Road’s vulnerable sections, infected sections and recovered sections within 24h.

7. Conclusion

The figure 6 above shows the typical Youth Road (at the junction of culture and education area and office area), and measures the changes of vulnerable sections, infected sections and recovered sections within 24h. It can be seen from the figure that the peak period (7:00, 8:00, 16:00, 17:00) is the high incidence period and outbreak period of infection. Especially at 8:00 in the morning, the congested sections rise sharply. According to the actual situation, it can be inferred that at this time, it is likely to go to work after sending their children to school, and cross from one area to another at the same time, resulting in traffic congestion. After 6:00 p.m., the recovery section increased sharply. According to the actual situation, it can be inferred that at this time, most parents are likely to rush home after picking up their children. To sum up, we can see the spread trend of infection. The early peak is from the culture and education area to the office area, and the late peak is from the office area to the culture and education area.

Through the above two mathematical models and the change analysis of 24h images of typical road sections, the first conclusion can be drawn is that the road congestion is indeed infectious, and the regional thresholds of different urban properties are similar. Without considering other influencing factors, the defined value of road infection spread is basically fixed in the whole road area. Secondly, the equilibrium point of road congestion is unstable, indicating that there may be a variety of other factors, such as whether the road is a one-way street, whether it is temporarily blocked due to construction, etc. Finally, by combining with the reality of life, we can draw the same conclusion as Figure 8: the trend of traffic congestion transmission is often related to the trend of human life. To sum up, SIR model is suitable for analyzing urban traffic conditions and can effectively predict and infer the change trend of congestion infection to a certain extent.

References

[1] Sun, H. J. and Wu, J. J. (2005) Urban traffic congestion spreading in small world networks. International Journal of Modern Physics B: Condensed Matter Physics; Statistical Physics; Applied Physics, Vol. 19 Issue 28, pp. 4239-4246. 8p. 7 Graphs.

[2] Wu, J. J., Gao, Z. Y., Sun, H. J. (2004) Simulation of traffic congestion with SIR model. Modern Physics Letters B, Vol. 18 Issue 30, pp. 1537-1542. 6p.

[3] Meead, S., Mudabber, A., Homayoun, H., Amir, H. S., Gu, Z. Y., Sajjad, S., Divya J, C., Vinayak, D., Lauren, G., Travis, W. S., Marta C, G. (2019) A simple contagion process describes spreading of traffic jams in urban networks. [online]Available at <http://login.ez.xjtlu.edu.cn/login?url=https://search.ebscohost.com/login.aspx?direct=true&db=edsarx&AN=edsarx.1906.00585&site=eds-live&scope=site > (04/04/2022 16:53)

[4] Getachew K, B. (2021) 2021 55th Annual Conference on Information Sciences and Systems (CISS) Information Sciences and Systems (CISS). In: Baltimore, MD, USA. pp. 1-3.

[5] Teja, I., Kaan, O., Sandeep, M. (2012) Analytical modeling of vehicle-to-vehicle communication using spread of infection models. 2012 IEEE International Conference on Vehicular Electronics and Safety (ICVES 2012) Vehicular Electronics and Safety (ICVES). In: Istanbul, Turkey. pp. 217-222.

[6] Zeng, Z. L. and Li, T. X. (2018) Analyzing Congestion Propagation on Urban Rail Transit Oversaturated Conditions: A Framework Based on SIR Epidemic Model. Urban Rail Transit, 4(3):130-140

[7] Shi, Z., Zhang, N., Zhu, l. (2019) Understanding the Propagation and Control Strategies of Congestion in Urban Rail Transit Based on Epidemiological Dynamics Model. Information (Switzerland), Vol. 10 Issue 8, pp. 258-258, 1p.

[8] Gurin, D., Prokhorchenko, A., Kravchenko, M., Shapoval, G. (2020) Development of a method for modelling delay propagation in railway networks using epidemiological SIR models. Eastern-European Journal of Enterprise Technologies, Vol. 108 Issue 3, pp. 6-13, 8p.

[9] Zhao, Y. and Ding, X. B. (2021) The Research on Delay Propagation of Urban Rail Transit Operation under Sudden Failure. Journal of Advanced Transportation, pp. 1-10. 10p.

[10] Watmough, J. and van den Driessche, P. (2002). Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. Mathematical

Biosciences, 180, pp. 29-48.