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Leader-follower consensus for nonlinear multi-agent systems under directed topology

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Abstract. This paper investigates the consensus problem for multi-agent systems (MASs) under directed topology. The primary objective is to design the distributed control protocol such that all agents can converge to the state of leader. The distributed control protocol is designed, and we derive sufficient conditions by using Lyapunov stability theory to achieve consensus. Theoretical analysis and numerical simulations are provided to verify the effectiveness of the proposed control protocol.

Keywords: Multi-agent systems, Consensus, Stability , Distributed control.

1. Introduction

In the past decades, cooperative control has gradually become a research focus in the scientific community due to the wide range of applications of multi-agent systems(MASs) in many fields, such as biology, physics, and artificial intelligence [1]. Research of MASs mainly includes consensus, formation, and controllability problem etc [1]. The task of consensus lies in designing a control input protocol that enables all agents to converge to the same state in the end. However, in many real-world scenarios, the convergence of controllers does not achieve a certain expected effect, for example, multiple unmanned aerial vehicles need to fly to a specified speed for real-time continuous control of the environment and so on. MASs controllability is an emerging area of research after the study of multi-agent systems coherence. For a network of agents, external inputs are applied to the leader such that the followers reach any expected final state from any initial state.

In terms of consensus and controllability, research on MASs is mainly divided into leaderless consensus and leader-follower consensus [1-2]. And the key to analyze the topic is to design an input control [3]. Lu etc. present two non-smooth leader -following formation protocols for non-identical Lipschitz nonlinear MASs [3]. Hui Q. proposed a nonlinear consensus algorithm for first-order systems, expressed as [4]:

$$u_i(t) = \sum_{j=1, j\neq i}^{N} \Phi_{ij}(x_i(t), x_j(t))$$

Stability conditions of the system under this nonlinear consensus algorithm were obtained by giving Lyapunov method[8], and analysis was conducted on the nonlinear consensus under switching topologies. Lin et al. study the consistency problem for a continuous-time nonlinear system and give conclusion that the system achieves consensus if and only if the directed switching topological network of the system have a sufficiently large connectivity range and strength [5]. Furthermore, the vector field of each individual in the system must fall within a minimal sector made up of the individual itself and its dependent individuals. Meanwhile the consensus problem usually involves the asymptotic stability of the differential equation, which needs to be analyzed by means of the Lyapunov function. For continuous time nonlinear system consensus algorithm, Moreaul designed Lyapunov function for continuous time nonlinear system consensus algorithm [6]. As an extension of classic Lyapunov function method, non-monotonically decreasing Lyapunov function method (NMDLF method), (Aeyels & Peuteman, Citation1999) is applicable to complex time-varying dynamics, especially for fast time-varying systems [7-8].

Motivated by the above analysis, the purpose of this paper is to analyze that the consensus of MASs converges to the expected state under the condition that the designed control input protocol and the corresponding parameters are satisfied. By giving a lemma, we transform the consensus problem for MASs into an asymptotic stability problem for error systems. By investigating the error system, and analyzing the asymptotic stability of the error systems, thus prove the consensus of the MASs converges to the expected state.

Through theoretical analysis and numerical simulation, we verify the effectiveness of the control input protocol and show the process of consensus for the MASs. The research in this paper not only provides a solution to the consensus problem of MASs, but also provides theoretical support and practical guidance for the design and realization of distributed control systems.

2. Preliminaries and problem formulation

2.1. Graph theory

Firstly, some notations will be given about the structure of an agent as well as definitions. $G = (V, E, \vartheta)$ refers to the graph of N-agents, where $V = \{v_1, v_2, \ldots, v_N\}$ denotes the vertex set of graph G. $(v_i, v_j) \in E$ if and only if the j-th agent can receive the information of the i-th agent. What's more, v_i is also the neighbourhood of v_j so let $N_i = \{v_j \in V | (v_j, v_i) \in E\}$. Next $A = [a_{ij}] \in \mathbb{R}^{N\times N}$ is a weighted adjacency matrix, where $a_{ii} = 0$, and $a_{ij} > 0$ if $(v_i, v_j) \in E, a_{ij} = 0$ otherwise. The Laplacian matrix of G is that $L = [l_{ij}] \in \mathbb{R}^{N\times N}$, where $l_{ii} = \sum_{i=1,k\neq i}^{N} a_{ik}$, and $l_{ij} = -a_{ij}, i \neq j$. A directed path from node v_{i_1} to node v_{i_l} is equivalent to the existence of a sequence of ordered edges $\{(v_{i_1}, v_{i_2}), (v_{i_2}, v_{i_3}), \ldots, (v_{i_{l-1}}, v_{i_l})\}$ in the directed graph G. If there exists a node called the root, which has no parent node, such that the node has directed paths to all other nodes in the graph, then the directed graph G contains a directed spanning tree. We define $B = diag\{b_1, b_2, \ldots, b_N\}$ as the attenuation coefficient matrix associated with G, where $b_i > 0$ if the leader is a neighbour of i-th agent and otherwise $b_i = 0$. It is assumed that the leader is self-active or moving independently. That is the followers could receive information from the leader while the leader needs no information from any follower.

2.2. Problem formulation

Consider the following nonlinear MASs with follower i can be described by :

$$\dot{x}_i = f(x_i) + u_i i = 1,2, \ldots, N \tag{1}$$

And the dynamics for leader is described by:

$$\dot{x}_0 = f(x_0) \tag{2}$$

Where $x = (x_1, x_2, \ldots, x_N) \in \mathbb{R}^{n\times N}$ denotes the state variables of the N-agents, $i = 1,2, \ldots, N, x_i(t) \in \mathbb{R}^n$ is the state vector of the i-th agent and $f(x) \in \mathbb{R}^n$ represents the nonlinear function, $u \in \mathbb{R}^n$ is the control input protocol to be designed, $x_0 = (x_{0_1}, x_{0_2}, \ldots, x_{0_N})^T$ denotes the state of the expected state vector, $i = 1,2, \ldots, N, x_0(t) \in \mathbb{R}^n$ is the state of the leader which is also the expected state.

In order to obtain the main result, the following assumptions are needed:

The topological structure G for MASs includes a directed spanning tree with the leader being the root.

There is a positive constant μ such that for any $x_1, x_2 \in \mathbb{R}^n$, there hold

$$(x_1 - x_2)^T(f(x_1) - f(x_2)) \leq \mu(x_1 - x_2)^T(x_1 - x_2)$$

Remark 1: Assumption 2 takes the practicality into consideration, and there are many practical systems can meet the requirement of Assumption 2 such as chaotic systems like Chen system, the Lorenz system, and the unified chaotic system have been verified to satisfy this assumption.

When discussing the speed of change of an agent's state variables, we categorize the factors affecting the speed of change into internal and external factors, and an object's state variables such as displacement, velocity are often affected by the constraints of the fields inside the space in which it is located and the effects of the external environment such as the interactions of other agents on itself, and thus we build the above continuous time model of the i-th agent to portray the agent's state variables. But generally, the behaviour and state of these N agents are inconsistent, due to the needs of people these N agents need to make behavior that meets the expectations, the corresponding mathematical differential equation model shown in (2) denotes the state expected to be reached by the agents.

In order to gradually reach the expected state, each agent receives information from its respective neighbours and passes information to its neighbours to update the state of the agent in the current moment. The gap between the i-th agent and the other agents and the difference between each agent and the corresponding expected state should be focused on and portrayed, so in the light of the idea, we will give the following design of the control input function which is defined as consensus protocol:

$$u_i = -\rho \sum_{j=1}^{N} a_{ij}(x_i - x_j) - b_i(x_i - x_0) \qquad \forall i = 1,2,\ldots,N$$

Where ρ is a positive constant. a_{ij} is the element of weighted adjacency matrix and b_i is the element of B. So, on the basis of consensus protocol, we can get the concrete model as following:

$$\dot{x}_i = f(x_i) - \rho \sum_{j=1}^{N} a_{ij}(x_i - x_j) - b_i(x_i - x_0) \qquad i = 1,2,\ldots,N$$

The coherent control problem is mathematically defined as follows: assume that the MASs contains N agents, where the state of the i-th agent is denoted by $x_i(t), i = 1,2,\ldots,N$, if when $t \to \infty$ we have $\lim_{t\to\infty} ||x_i(t) - x_0(t)|| = 0 i = 1,2,\ldots,N$ then the MASs is said to have reached an expected state of consensus.

2.3. Stability analysis

Firstly, an overall error function is defined as $\eta(t) = (\eta_1(t), \eta_2(t), \ldots, \eta_N(t))^T$ to represent the difference from the expected state x_0 at each moment t.

Where $\eta_i(t) = x_i(t) - x_0(t)$ indicates the difference between i-th agent and the expected state. $\frac{d\eta_i(t)}{dt} = f(x_i) - f(x_0) + u_i$ $i = 1,2,\ldots,N$ is the ordinary differential equation for the i-th component of the error function.

To investigate the consensus problem of the MASs, in other words, to certify $\lim_{t\to\infty} ||x_i(t) - x_0(t)|| = 0$ $i = 1,2,\ldots,N$. We need to give a lemma to establish the asymptotic stability of differential equations error functions and the consensus of MASs is the same problem.

Lemma1: For the error function, if $L + B$ is nonsingular, one has $||x_0(t) - x(t)|| \leq \frac{||\eta(t)||}{\sigma_{\min}(L+B)}$

With $\sigma_{\min}$ being the minimum singular value.

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So, if the solution of $\frac{d\eta_i(t)}{dt} = f(x_i) - f(x_0) + u_i \quad i = 1,2,\dots,N$ is asymptotic stable, then we can get $\lim_{t\to\infty} ||\eta_i(t)|| = 0$ and $\lim_{t\to\infty} ||x_i(t) - x_0(t)|| = 0 \quad i = 1,2,\dots,N$ by the Lemma1. It can be concluded that the consensus problem of MASs can be transformed into the asymptotic stability problem of its error systems.

3. Main results

In this section, the sufficient condition for the asymptotic stability of error systems will be given. For the leader is globally reachable, at least one follower is connected to the leader, so $B \neq 0$.

Lemma2: For any $x \neq 0$, the eigenvalue for L the Laplacian matrix of G, the smallest eigenvalue is always 0, which corresponds to the eigenvector being an all-one vector. The eigenvalue λ_2 is the algebraic connectivity degree, which reflects the connectivity of the graph.

$$\lambda_2 \leq \frac{x^T L x}{x^T x} \leq \lambda_{max}$$

Where the λ_{max} is the maximum eigenvalue and λ_2 is the second smallest eigenvalue.

Theorem 3.1. Suppose that the assumptions hold, the consensus of the system (1) (2) is achieved under the following condition:

$$\rho > max_{i=1}^{N} \left\{ 0, \frac{2(\mu - \min_i \{b_i\})}{\lambda_2} \right\}$$

Proof. The Lyapunov function is designed as

$$V(t) = \frac{1}{2} \sum_{i=1}^{N} \eta_i^T \eta_i$$

Then, $\forall t_0 > 0, V(t_0) > 0$

$$\dot{V}(t) = \sum_{i=1}^{N} {\eta_i}^T \dot{\eta_i} = \sum_{i=1}^{N} {\eta_i}^T \left(f(x_i) - f(x_0) - \rho \sum_{j=1}^{N} a_{ij}(x_i - x_j) - b_i(x_i - x_0)\right)$$

Since for any $x_1, x_2 \in \mathbb{R}^n$, $(x_1 - x_2)^T (f(x_1) - f(x_2)) \leq \mu (x_1 - x_2)^T (x_1 - x_2)$

$$\begin{aligned} &\sum_{i=1}^{N} {\eta_i}^T \left(f(x_i) - f(x_0) - \rho \sum_{j=1}^{N} a_{ij}(x_i - x_j) - b_i(x_i - x_0)\right) \\ &\leq \sum_{i=1}^{N} {\eta_i}^T \left(\mu\eta_i - \rho \sum_{j=1}^{N} a_{ij}(x_i - x_j) - b_i\eta_i\right) \\ &= \sum_{i=1}^{N} (\mu - b_i){\eta_i}^T \eta_i - \rho \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij}{\eta_i}^T (x_i - x_j) \\ &= \sum_{i=1}^{N} (\mu - b_i){\eta_i}^T \eta_i - \frac{\rho}{2} \sum_{i=1}^{N} {\eta_i}^T L \eta_i \end{aligned}$$

From Lemma2, one has

$$\sum_{i=1}^{N} (\mu - b_i){\eta_i}^T \eta_i - \frac{\rho}{2} \sum_{i=1}^{N} {\eta_i}^T L \eta_i \leq \sum_{i=1}^{N} (\mu - b_i - \frac{\rho}{2}\lambda_2){\eta_i}^T \eta_i < 0$$

Thus, we can conclude that under the condition i and assumptions, the error systems are asymptotically stable, which is $\lim_{t\to\infty} ||\eta_i(t)|| = \lim_{t\to\infty} ||x_i(t) - x_0(t)|| = 0 i = 1,2,\dots,N$. Then, the consensus for the MASs is realized.

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4. Numerical simulation

In this section, an illustrative example will be presented to verify the effectiveness of our conclusion. The simulations are performed in a two-dimensional space consisting of the X-direction and Y-direction with five agents.

Give the nonlinear dynamical function of the system as follows

$$\dot{x}_i = 0.005x_i^2 - 15\sum_{j=1}^{5} a_{ij}(x_i - x_j) - b_i(x_i - x_0)\; i = 1,2,\ldots,5$$

The initial states of leader is:

$$x_0 = \begin{pmatrix}1\\1\end{pmatrix}$$

And the initial states of followers are:

$$x_1 = \begin{pmatrix}1.2\\1.1\end{pmatrix}, x_2 = \begin{pmatrix}1.1\\1.2\end{pmatrix}, x_3 = \begin{pmatrix}1.1\\0.8\end{pmatrix}, x_4 = \begin{pmatrix}0.8\\1.1\end{pmatrix}, x_5 = \begin{pmatrix}0.8\\0.8\end{pmatrix}$$

Where $\rho = 15$, the adjacency matrix and attenuation coefficient matrix is:

$$A = \begin{pmatrix}0&1&0&0&0\\1&0&1&0&0\\0&1&0&1&0\\0&0&1&0&1\\0&0&0&1&0\end{pmatrix}, B = \begin{pmatrix}0.12&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\end{pmatrix}, L = \begin{pmatrix}1&-1&0&0&0\\-1&2&-1&0&0\\0&-1&2&-1&0\\0&0&-1&2&-1\\0&0&0&-1&1\end{pmatrix},$$

$$\lambda_2 = 0.382, \mu = 2$$

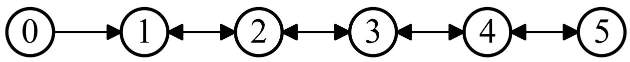

Figure 1. Topology graph.

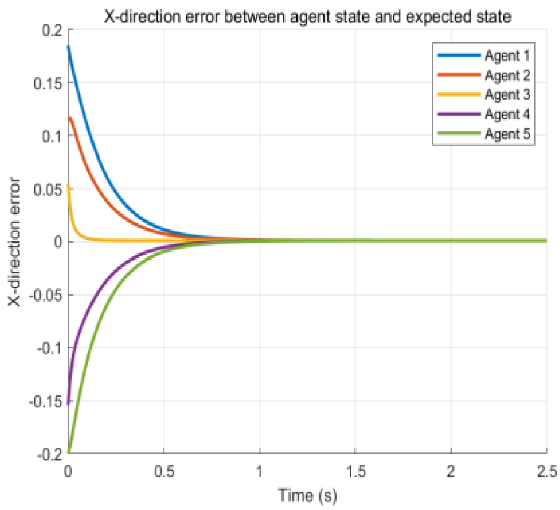

Figure 2. Error in the X-direction.

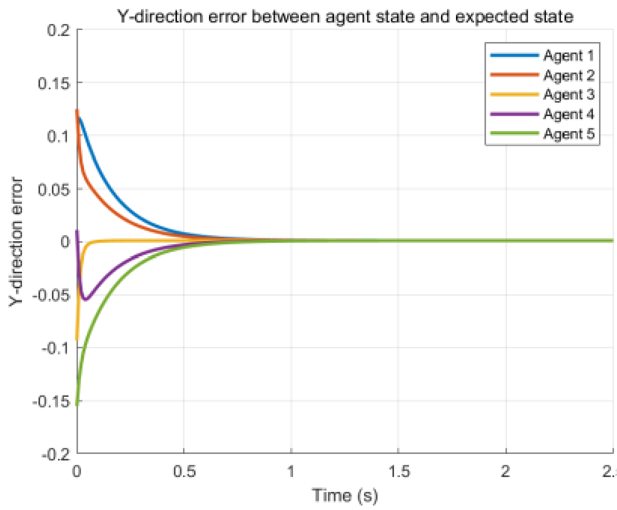

Figure 3. Error in the Y-direction.

Figure 2 and figure 3 show that the error values of each agent with respect to the expected state in the X-direction and Y-direction gradually converge to 0 over time, that is, it means that the five agents eventually converge to the expected state consistently.

5. Conclusion

The expected consensus problem for nonlinear MASs is investigated in the paper. The topology of the MASs is directed and the consensus can be realized. On the basis of the proposed distributed control protocol and the Lyapunov stability theory, a sufficient condition is derived to reach the consensus for MASs. Finally, the effectiveness of the distributed control protocol is verified by numerical simulation.

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