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Research on probabilistic analysis of trapezoidal triangle truss

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Abstract. As a region affected by the Mongolian Siberian high-pressure system, the Northeast area of China faces extreme low temperatures and snowy weather in winter, which poses a high demand for local energy. As a relatively mature sustainable energy source, laying solar photovoltaic panels on the roofs of residential buildings relied on sufficient local sunlight resources has become a feasible measure. From the perspective of engineering safety, this paper studies the stability of a single span double slope roof steel truss with added solar photovoltaic panels under snow loads. First, the types of loads on the truss are analyzed, assuming the distribution of the load and capacity of any member in the truss structure. Then, Monte Carlo algorithm is used to simulate the failure probability, and finally the failure probability of the member as well as the overall structure is obtained. As a conclusion, this paper finds that the stability of this structure is basically not affected by the above factors, and this provides certain reference significance for balancing safety and economic benefits in practical engineering design.

Keywords: Truss, probabilistic analysis, Monte Carlo algorithm, capacity.

1. Introduction

In the northeastern region of China, affected by the temperate monsoon climate, winters are cold and dry, and the energy demand for heating has increased significantly. At the same time, the northeast latitude is high, the sun altitude Angle is large, there are many sunny days in winter, the sunshine intensity is high, and the solar energy resources are rich. The laying of rooftop solar photovoltaic panels mentioned in this project not only fills the local energy gap in winter, but also makes up for the socio-economic problems caused by the interruption of production and life in winter to a certain extent. It has high economic benefits and realizes continuous development.

However, as a temperate inland region directly affected by the Mongolian Siberian high-pressure system, the Northeast region of China experiences frequent extreme snowfall in winter, and accidents of local buildings collapsing due to snow overload often occur. From March 3rd to 4th, 2007, Liaoning Province, China experienced a rare snowstorm, with more than 300 steel structure factories of 92 enterprises in the province have suffered varying degrees of damage, including using structural systems such as portal frames or arched roofs. It is particularly pointed out that steel structures with arched roofs are more severely damaged [1, 2]. It can be seen that the impact of heavy snowfall on the stability of building structures must be considered when laying solar photovoltaic panels mentioned above.

At present, relevant fields have done some research on this issue, but there is little research on whether adding solar photovoltaic panels on the steel truss of a single span double slope roof will affect the stability of the structure during heavy snowfall. Wang et al. [2] conducted a comprehensive analysis of the portal frame light steel industrial plant in the above-mentioned accidents and found that the steel structure of light houses has advantages such as light weight, fast construction speed, and low cost. However, its own overload capacity is poor, and accidents are prone to occur when encountering rare loads. At the same time, it is summarized that the specific reasons for this accident include low structural safety, overall overload of snow load, local snow overload caused by improper building shape, and secondary disasters caused by snow clearing. Xiong and Luo [3] used ANSYS finite element software to analyze the continuous collapse performance of the roof truss of a high school art building under snow load overload. They found that the structure did not experience continuous collapse when the snow load was overloaded by 15%. When the snow load is overloaded by 30%, there is a possibility of continuous collapse. When the snow load exceeds 50%, continuous collapse will occur.

Therefore, this paper takes this as a starting point to study whether there are structural stability problems of solar photovoltaic panels installed on residential roofs in northeast China in the event of heavy snowfall. According to relevant standards, this study will select a common residential steel structure single-span double-slope roof truss in Northeast China as an example, and determine building materials, specific geometric dimensions, bearing capacity, load type and size in the design stage [4-6].

Based on relevant data, it is believed that there is discreteness in the capacity and load, and its distribution is assumed [7]. Then, Monte Carlo algorithm will be used to conduct a large number of random value sampling and the two parameters will be compared, for the propose to simulate the probability of instability of any member or the overall structure in the truss under the situation mentioned above. This study aims to demonstrate the feasibility of using Monte Carlo algorithm to simulate the stability of truss structures through this study, and to explore whether strong snowfall will affect the stability of steel single span double slope roofs with solar photovoltaic panels.

2. Engineering background

This study selects steel structure pitched roofs of common sizes in residential buildings for further research. The roof of such houses is generally made of tiles. According to relevant specifications, the weight of ceramic tiles is about 3.6~4.6 (kg/m^2). In this study, the weight is taken as 4.1 kg/m^2 [8]. For photovoltaic panels, according to related records, the size and weight of a piece of photovoltaic panel are shown in table 1.

Table 1. Parameters of photovoltaic panels.

Parameter	Corresponding value
Volume (m^3)	1.64*0.99*0.05
Weigh (N/m^2)	294

As shown in figure 1, the structure consists of 11 beams. The left end of the structure is connected to a hinge, the left end of the structure is connected to a roller, makes it a statically determinate structure. Moreover, all beams are connected with hinges so it is a truss. According to the construction standards for self-built houses in rural areas, drainage slopes are generally greater than 10%, advisable to reach a slope of 30% to 40%. Therefore, take span AE as 9 meters, take a slope of 30%. Then it can be known that AC 4.69 meters, the angle A is 17°. Additionally, the length of the house as 12 meters.

In terms of material selection, out of comprehensive considerations of material strength and economic benefits, this study chooses Q235 steel as the main building material, which is relatively low-cost and behaves well in capacity and durability. According to <GB/T 700-2006>, the properties of Q235 steel are shown in table 2. The cross- sectional diagram of the steel is shown in figure 2.

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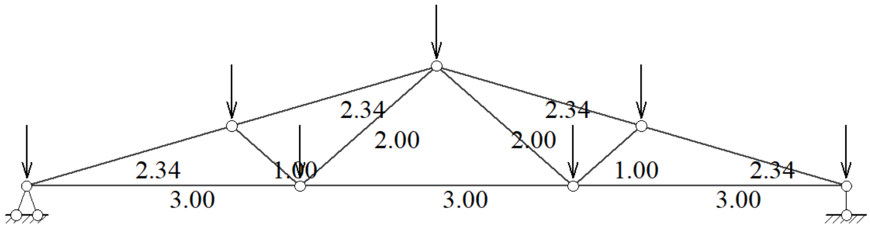

Figure 1. Side view of the structure (meter).

Table 2. Sectional dimensions and mechanical properties of Q235 steel.

Parameter	Corresponding value
Density (*10^3kg/m^3)	7.85
Section length (mm)	500
Section width (mm)	400
Wall thickness (mm)	16
Yield strength (MPa)	235
Shear strength (MPa)	135.68
Elastic Modulus (*10^5 MPa)	2.06
Density (*10^3kg/m^3)	7.85
Section length (mm)	500

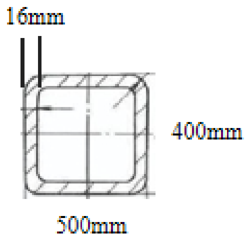

Figure 2. Cross-sectional diagram of steel.

3. Methodology

3.1. Loads design

It should be noted that, since this structure is a truss, there is no load on the middle part of the member. Therefore, all distributed loads are replaced with the concentrated ones by equal division to ensure that all loads are applied to the joints.

3.1.1. Dead loads. For the self-weight of structure, the weight of Q235 steel per meter in this project can be obtained as 2.137 kN/m.

$$q_s = [BH - (B - h) * (H - h)]\, \rho_s g \tag{1}$$

For the tiles weight of one side roof surface, it is 0.241 kN/m.

$$q_t = \rho g L/2 \tag{2}$$

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3.1.2. Live loads. (1) Weight of photovoltaic panels:

The total number of photovoltaic panels that can be installed on one side of the roof is determined as: width (w) = 4.69/0.99 = 4.74, taking 4; length (l)= 12/1.64 = 7.32, taking 7. Therefore, the weight of photovoltaic panels per meter is (defined as qP) 0.712 kN/m.

$$q_p = \frac{A\rho wl}{4AC} \tag{3}$$

(2) Snow load

According to relevant specifications, the standard value of snow load should be calculated using the following equation [5].

$$S_k = \mu_r S_0 \tag{4}$$

where S_k is snow load standard value (kN/m^2), μ_r is roof area snow distribution coefficient, and S_0 is basic snow pressure (kN/m^2).For this project, the roof type is a single-span double-slope roof, whose roof area snow distribution coefficient can be obtained as 1.0 in the standard. As for the basic snow pressure, this study chooses basic snow pressure with a return period of 50 years and it is 1.40 kN/m^2 in the standard, which is also the maximum value in the table of Heilongjiang Province [5].

$$q_n = S_k L \tag{5}$$

In total, for any member in the structure, the value of load on it is:

$$Q = l(q_s + q_t + q_p + q_n) \tag{6}$$

Then the value of all loads on every joint can be calculated after converting to concentrated loads (table. 3). By using the SM Solver, the axial force of the whole structure can be computed and the free-body diagram can be drawn, as shown in figure 3.

Table 3. Data for joints under designed loads.

Joint number	A(E)	B(D)	C	F(G)
Self-weight (kN)	3.758	3.051	9.331	9.331
Tiles	0.283	0.565	0.565	0
Photovoltaic panels	0.835	1.669	1.669	0
Snow load	9.851	19.702	19.702	0

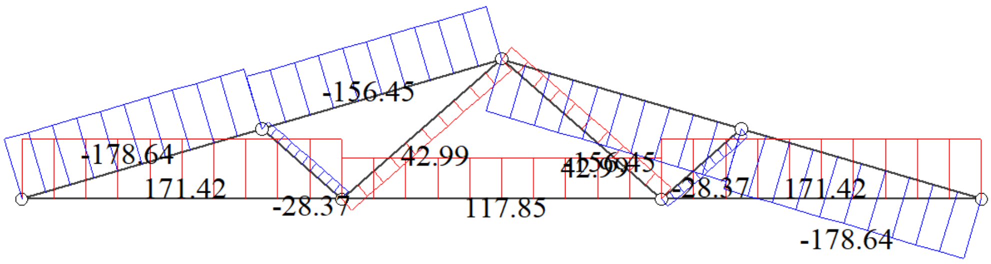

Figure 3. Free-body diagram.

3.2. Capacity verification

Before the verification, it should be noted that there is uncertainty in material properties and snow load, so that the actual value of strength should be a function, and loads on all members should be a function plus a constant instead of a constant only, which will be discussed in the following part. With the help of SM solver, it can be figured out with the axial force of all members in the truss, as shown in table 4.

The stress on a member subjected to axial force can be calculated using the following equation:

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$$\sigma_{t,max}(or\ \sigma_{c,max}) = \frac{F_N}{A} \tag{7}$$

In this equation:

$\sigma_{t,max}(\sigma_{c,max})$ — Maximum tension/compressive stress (MPa);

F_N — Member compressive stress (kN);

A — Sectional area (m^2).

It can be found that the maximum axial force on members in this structure is member (5). For hollow rectangular section: $\sigma_{c,\ max}$ = 6.431 MPa< f = 235MPa. Therefore, it can be preliminarily assumed that the structure will be safe in most of the time under the design value of load.

Table 4. Data for axial force of all members in the truss.

Member's number	Axial force (kN)
(1)	171.417613
(2)	117.84659
(3)	171.417613
(4)	-28.3673307
(5)	-178.640219
(6)	-156.447194
(7)	42.9928106
(8)	42.9928106
(9)	-156.447194
(10)	-178.640219
(11)	-28.3673307

3.3. Uncertainties

3.3.1. Material properties. The main factors affecting the mechanical properties of steel include chemical composition, metallurgical defects and rolling process, steel hardening, temperature, stress state, loading speed, and properties in the thickness direction of the steel [9]. This study divides temperature form all other factors since the temperature difference is extremely large in the project.

(1) Environment temperature

According to relevant research, when the temperature is lower than 0°C, the strength of steel will even rise slightly while its plasticity and toughness will reduce, which has obviously no significant impact on the performance of Q235. Therefore, in this project, the impact of temperature on the strength of the steel is minimal.

(2) Other factors

Clearly, all other factors will have an impact on the strength of steel, and the it is a comprehensive reflection. From the perspective of results, relevant literatures indicate that the strength of steel obeys the Lognormal distribution. All in all, it can be assumed that the strength of steel simply obeys the Lognormal distribution in this project. Since the structure is a truss in the project, here only the tensile strength will be considered, which is used in comparison with axial force.

The PDF of Lognormal distribution should be calculated using the following equation:

$$f(x) = \frac{1}{\sqrt{2\pi} * \zeta x} * \exp\left\{-\frac{1}{2} * \left[\frac{(lnx - \lambda)}{\zeta}\right]^2\right\} \tag{8}$$

In this equation:

x — Steel tensile strength sample;

λ — Mean of x after taking the logarithm;

ζ— Standard deviation of x after taking the logarithm.

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For tensile strength sample, the tensile strength range of Q235 steel can be found in relevant literature [7]. On the assumption that it follows a lognormal distribution, this study randomly generates 1000 values within this interval, use them to calculate mean and standard deviation and then get the complete expression of tensile strength distribution (Defined as t(x)):

$$t(x) = \frac{1}{\sqrt{2\pi}*0.058x} * \exp\left\{-\frac{1}{2} * \left[\frac{(lnx - 6.035)}{0.058}\right]^2\right\} \tag{9}$$

3.3.2. Loads. In this study, the structural loads include the self-weight of the structure, the weight of the roof tiles, the weight of the solar photovoltaic panels and the snow load. It is assumed that the weight of the structure, tiles and solar photovoltaic panels is relatively stable. Therefore, the uncertainty of the load is mainly reflected in the randomness of the snow load.

It can be known from the previous part that, when the roof area snow distribution coefficient (μ_r) is determined, the snow load standard value depends on the value of basic snow pressure. According to relevant standards, it indicates that the basic snow pressure should use the maximum value between Gumbel distribution or the Lognormal distribution [5].

Gumbel distribution:

The PDF of Gumbel distribution (defined as g(x)) should be calculated using the following equation:

$$g(x) = \alpha * \exp\{-\alpha(x - u) - exp[-\alpha(x - u)]\} \tag{10}$$

and

$$\alpha = C_1/\sigma \tag{11}$$

$$u = \mu - C_2/\alpha \tag{12}$$

In this equation:

x — Annual maximum snow pressure sample;

u — Positional parameter of distribution;

α — Scaling parameter of distribution;

C_1, C_2 — Coefficient chosen through relevant table;

μ, σ — Mean and standard deviation of a limited sample (whose size is n).

This study takes annual maximum snow pressure sample of 31 cities in Heilongjiang Province with a return period of 50 years, so n=31, with these data, the Gumbel distribution of snow loads can be calculated:

$$g(x) = 5.321 * \exp\{-5.321(x - 0.51) - exp[-5.321(x - 0.51)]\} \tag{13}$$

The PDF of Lognormal distribution (defined as l(x)) should be calculated using the same equation as *3.3.1. Material properties,* and for the same way, the complete expression is:

$$t(x) = \frac{1}{\sqrt{2\pi}*0.325x} * \exp\left\{-\frac{1}{2} * \left[\frac{(lnx + 0.545)}{0.325}\right]^2\right\} \tag{14}$$

To sum up, the complete distribution of snow load is:

$$f(x) = max\{g(x), l(x)\} \tag{15}$$

3.4. Assuming Random Variables and Associated Distributions

3.4.1. Capacity. With the expression in *3.3.1. Material properties*, this study performs data simulation in Matlab, then substitutes all the simulation data into the expression to get results in Excel, and draws out the PDF diagram of tensile strength distribution of Q235 steel, as shown in figure 4.

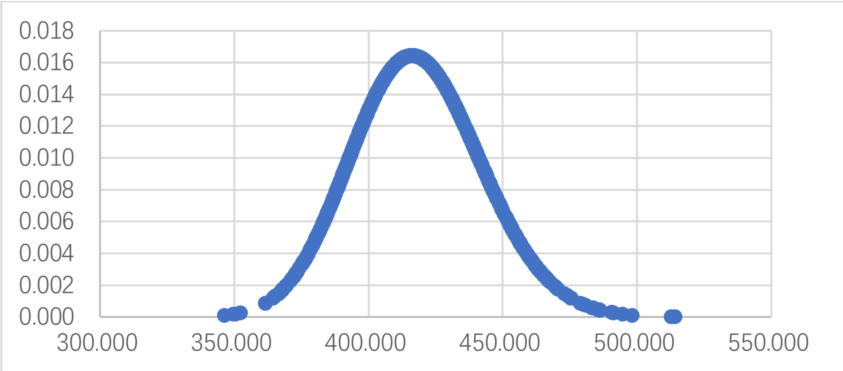

Figure 4. PDF diagram of tensile strength.

3.4.2. Demand. According to *3.3.2 Loads,* the actual value of loads on all members should be a function plus a constant, and the uncertainty of loads is only reflected in the randomness of the snow load. Therefore, in order to facilitate analysis and intuitive display, this study divides all the loads on the structure into two parts: snow load and other loads. Then with the help of SM Solver again, the two parts of loads can be added in algebraic form (which is clearly allowed for axial forces on the members), and finally the actual axial force expressions for all members in the structure can be computed, as shown in table 5.

Table 5. Actual axial force expression of all members in the truss.

Member's number	Other loads	Snow loads	Actual axial force expression
(1)	70.669	$47.966*S_0$	$47.966*S_0+70.669$
(2)	50.681	$35.974*S_0$	$35.974*S_0+50.681$
(3)	70.669	$47.966*S_0$	$47.966*S_0+70.669$
(4)	-6.000	$-7.987*S_0$	$-7.987*S_0-6.000$
(5)	-73.646	$-49.987*S_0$	$-49.987*S_0-73.646$
(6)	-68.952	$-43.739*S_0$	$-43.739*S_0-68.952$
(7)	20.625	$7.987*S_0$	$7.987*S_0+20.625$
(8)	20.625	$7.987*S_0$	$7.987*S_0+20.625$
(9)	-68.952	$-43.739*S_0$	$-43.739*S_0-68.952$
(10)	-73.646	$-49.987*S_0$	$-49.987*S_0-73.646$
(11)	-6.000	$-7.987*S_0$	$-7.987*S_0-6.000$

By using the same method as which in *3.4.1 Capacity,* the diagram of basic snow pressure distribution can be found. It can be discovered that it takes the value of the Lognormal for most x, as shown in figure 5.

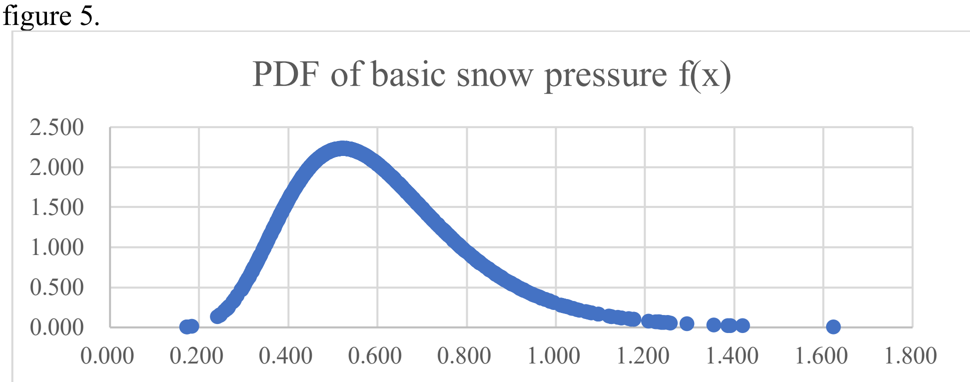

Figure 5. PDF diagram of basic snow pressure.

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3.5. Reliability Analysis and Monte Carlo Algorithm

In this part, in order to determine the failure probability of any member in the structure of this project under uncertain loads and strength, this study uses the Monte Carlo algorithm and writes code in Matlab to simulate its probability [10].

3.5.1. Determination of member's failure probability. Since the simulation process of the failure probability of each member is the same, take member (5) as shown in figure 6, and the actual axial force expression of it is (The signs represent the direction):

$$F_{N5} = -49.987S_0 - 73.646 \tag{16}$$

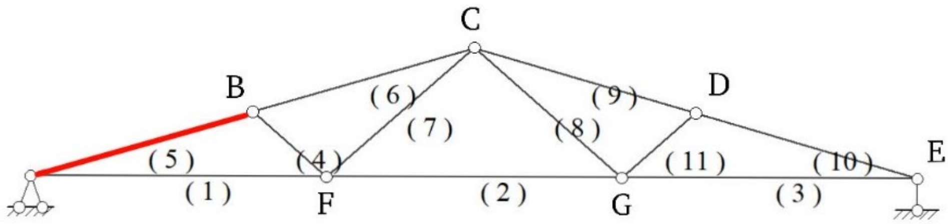

Figure 6. Diagram of member (5) in the structure.

By substituting it into the expression of Demand in the code, the failure probability of member (5) can be simulated using Monte Carlo algorithm in Matlab.

3.5.2. Determination of structure failure probability. The failure probability of the whole truss structure can be calculated in the form of a set with the connection relationship between the members in the truss. In this study, it is evident that all members are in series, and the failure of any member will lead to the collapse of the entire structure. Therefore, the failure probability of the whole truss can be expressed as

$$\mathrm{P(F)} = \sum_{i=1}^{n} p_i, \text{ while i = 1,2,3…,11} \tag{17}$$

In order to demonstrate the process of using Monte Carlo algorithm to simulate the probability of structural instability, the study still use the reduced steel tensile strength to simulate it.

4. Results and discussion

Through 100000 random simulations of Demand and Capacity according to the distribution function, and calculating the probability after comparison, result shows that the failure probability of the member is 0, which is contrary to the expected and actual situation.

In order to figure out the problem, the tensile strength is reduced as an assumption that the tensile strength of Q235 steel is too large for loads to make members in structure fail, after executing the above process again, the new result is 0.0023 as shown in figure 7, which is satisfying. This also means the structure is extremely safe under the original load and steel tensile strength, whose failure probability is too low that it is 0.

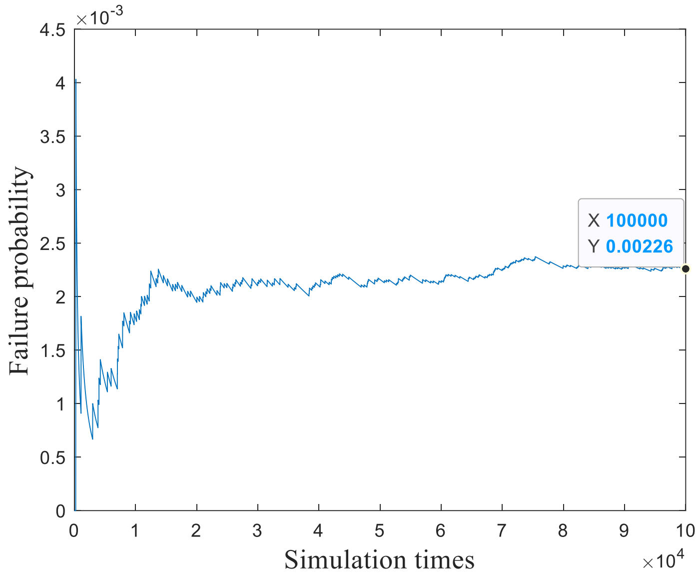

Figure 7. Diagram of failure probability of member (5).

As for the structure, the failure probability is 0.0241, which meets expectations as well. In fact, this is a very rough result because in this program, all members are the member (5) with the highest failure probability, and the actual result will be smaller than it.

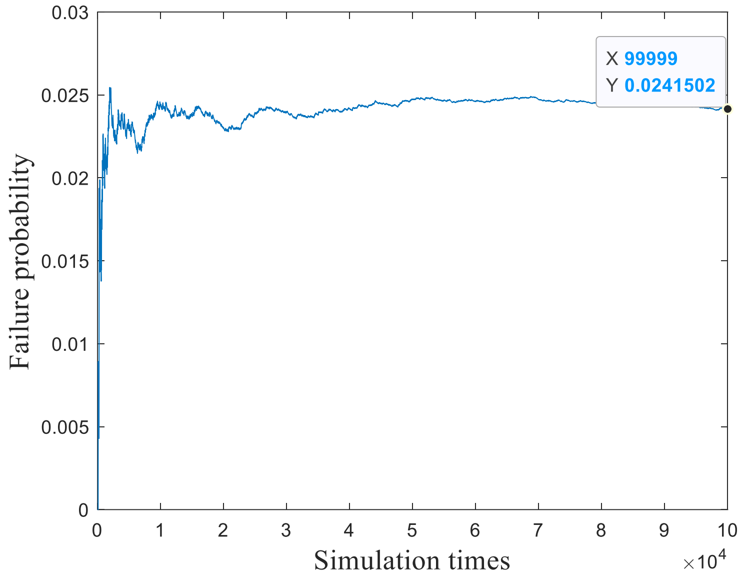

Figure 8. Diagram of failure probability of the structure.

To placed two PDFs (Demand and Capacity, the latter one is also the t(x)) in the same coordinate system in Desmos, the study substitutes the actual axial force on the member (5) into the expression of basic snow pressure(f(x)) to change the abscissa of it, and the new PDF is just the Demand. It should be noted that the Demand PDF should be divided by the coefficient of the previous transformation of x, to ensure that the integration of the Demand PDF in the interval is equal to 1 (For member (5), it is 49.987 as in the figure 8). Then the integration in which the y is greater than Capacity and less than Demand represents the probability of the event: Demand is greater than Capacity. For easier verification, here still use the PDF of reduced tensile strength.

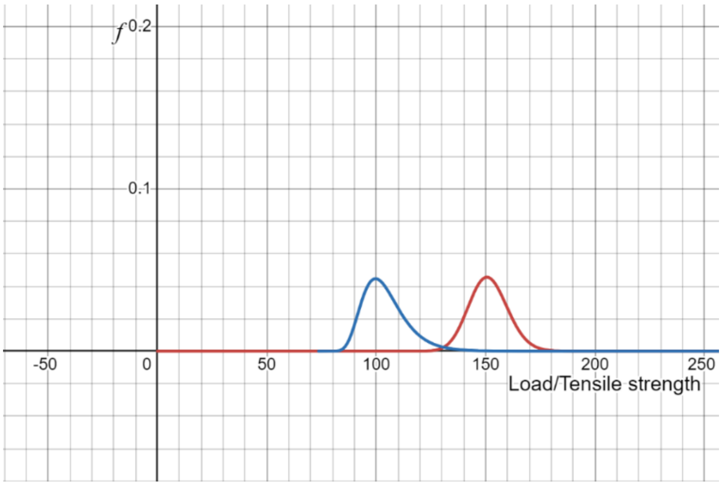

Figure 9. Diagrams of capacity and demand.

From the third line in the figure 9, it can prove that there is no problem with the Demand PDF. Also, the integral is 0.00248, which is very close to the simulated value of the code, which also indirectly proves the reliability of the code.

5. Conclusion

This study mainly studies the failure probability of members and the overall structure of a single span double slope roof steel truss in Northeast China under the combined action of multiple loads, mainly snow load, after laying solar photovoltaic panels. Through specific research, this article has drawn the following main conclusions:

(1) From the perspective of research methods, this study first searches relevant literatures, determines the expression of the capacity and load of any member in the truss structure under the set load, then compares the two through simulating a large number of samples, finally to determine its failure probability by Monte Carlo algorithm is feasible and has practical significance.

(2) From the perspective of results, the probability of failure of the structure in this study under the installation of rooftop photovoltaic panels and the snow load with a return period of 50 years is within the allowable range of engineering design.

(3) Under the premise of complying with engineering design requirements, laying solar photovoltaic panels on the steel structure truss of a single span double slope roof has almost no significant impact on the failure probability of the structure.

However, in terms of defects, the research conducted in this study has the following shortcomings:

(1) When determining the load action, this study first assumes that some uncertain loads are fixed values, which is logically redundant and requires additional explanation in the following text.

(2) For practical engineering, the section selection of building steel is too large, which is manifested in the results that the demand is too small compared to the capacity.

(3) When calculating the overall failure probability of the structure, all members use the failure probability of member (5), which can lead to a higher failure probability of the structure.

In the following research, the study will try to reduce the strength of structural materials to achieve a balance between safety and economic benefits. Afterwards, the failure probability of all members and the overall structure in the structure will be recalculated to verify the rationality of material selection.

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