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Solution to the problem of setting up medical points and road maintenance in mountainous areas based on optimization algorithms

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Abstract. Assume there are 100 villages in a mountainous area. To improve the medical standards of the local residents, three medical points need to be established in several villages, and the roads need to be repaired. This paper modifies the Floyd algorithm to derive all the shortest paths between nodes. Based on the Prim algorithm, a minimum spanning tree model is constructed to find the road maintenance method with the lowest cost; a nested greedy algorithm is used to construct an optimization model under multiple variables, effectively solving the problem of setting up medical points and road maintenance in mountainous areas.

Keywords: Floyd algorithm, minimum spanning tree problem, Prim algorithm, bi-objective optimization, greedy algorithm.

1. Introduction

To advance the construction of the rural medical and health service system, improve the basic health conditions in mountainous areas, enhance service capacity, and promote the accessibility and equality of public health and basic medical services for residents in mountainous areas, a plan to build medical points in these areas is proposed. However, due to the scattered villages, complex terrain, and dilapidated roads, the problem lies in how to select the locations of medical points and road repairs to minimize the total distance for villagers to access medical points and the total mileage of road repairs.

2. Model Establishment and Solution

To solve the above problems, assume the following mathematical model. Assume there are 100 villages in a mountainous area, and several medical points need to be established to facilitate medical treatment for villagers. Figure 1 shows the location coordinates and optional road connections of these 100 villages. Three medical points will be established among the 100 villages, and partial road repairs will be carried out as needed. It is assumed that villagers will choose the repaired roads to seek medical treatment.

2.1. Nearest Medical Points

2.1.1. Data Preprocessing

Figure 1 is a structure consisting of a vertex set $V = (v_1, v_2, \cdots, v_n)$, an edge set $E = (e_1, e_2, \cdots, e_m)$, and the determined association relationships between vertices and edges, denoted as $G = (V, E)$:

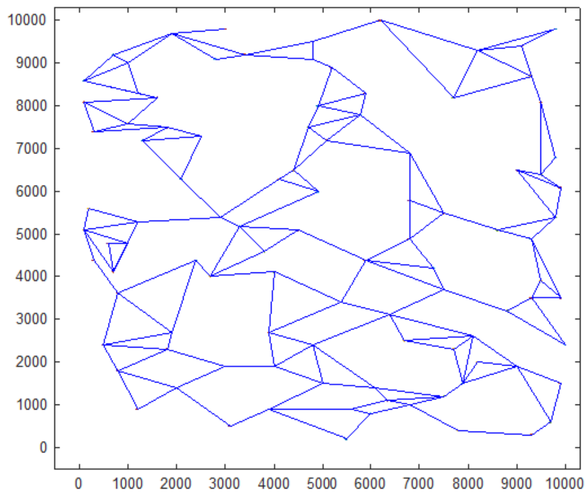

Figure 1. Location and road connection diagram of the 100 villages

Thus, the relationships in the village location distribution map can be represented by an incidence matrix and an adjacency matrix:

(1) Incidence matrix

$$R = (r_{ij})_{n\times m}(n = 100\ is\ the\ number\ of\ vertices, m \text{ is the number of sides})$$

$$r_{ij} = \begin{cases} 1 & Exists\ v_k \in V, \text{Let } e_j = v_i v_k \\ 0 & Other\ Conditions \end{cases}$$

(2) Adjacency matrix $\mathrm{X} = (\mathrm{X_{ij}})_{100\times 100}$, where

$$x_{ij} = \begin{cases} 1 & exists\ e_k \in V, which\ makes\ e_k = v_i v_j \\ 0 & other\ conditions \end{cases}$$

$$X = \begin{bmatrix} x_{11} & x_{12} & x_{13} & \vdots & x_{1j} \\ x_{21} & x_{22} & x_{23} & \vdots & x_{2j} \\ x_{31} & x_{32} & x_{33} & \vdots & x_{3j} \\ \cdots & \cdots & \cdots & \ddots & \vdots \\ x_{i1} & x_{i2} & x_{i3} & \cdots & x_{ij} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & \vdots & 0 \\ 0 & 0 & 1 & \vdots & 0 \\ 0 & 0 & 0 & \vdots & 0 \\ \cdots & \cdots & \cdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

Using the given data, the initial matrix DX of the lengths of all edges in the adjacency matrix can be programmed.

2.1.2. Establishment of the Shortest Path Model Based on Floyd Algorithm

Model idea: Use the Floyd algorithm to iterate and obtain the shortest path matrix $D = (d_{ij})_{100\times 100}$, and d_{ij} represents the shortest path between village i and village j, and then use the exhaustive method to find the positions of the three medical points that minimize the total distance [1].

Specific steps are as follows:

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Based on the previous processing, we represent villages as points in a matrix, with elements in the initial matrix (i, j) representing the distance from node i to node j. If there is no edge between two points, the corresponding element is marked as infinity.

		1	2	……	j	……	n
	1	-	d_{12}	……	d_{1j}	……	d_{1n}
	2	d_{21}	-	……	d_{2j}	……	d_{2n}
$D_0 =$	……	……	……	……	……	……	……
	i	d_{i1}	d_{i2}	……	d_{ij}	……	d_{in}
	……	……	……	……	……	……	……
	n	d_{n1}	d_{n2}	……	d_{nj}	……	-

The node sequence matrix R_{ij} here is equivalent to a route table, and $r_{ij} = j$ indicates that it only needs to pass through node j from node i to node j. Let row k be the pivot row and column k be the pivot column. For each element in matrix DX_{k-1} (the matrix completed in the previous step), perform the following operations:

If condition $d_{ik} + d_{kj} < d_{ij}$ is met, where $(i \neq k, j \neq k, i \neq j)$, perform the following steps:

- Step 1: Replace the element in matrix d_{ij} with $d_{ik} + d_{kj}$to obtain matrix DX_k.
- Step 2: Replace the element r_{ij} in matrix k with R_{k-1} to obtain matrix R_k.
- Step 3: Repeat the steps until no more replacements are needed.

After multiple iterations, the final shortest path matrix $D = (d_{ij})_{100\times100}$ and node sequence matrix R_{ij} are obtained, and d_{ij} indicates the shortest path between village i and village j, and $r_{ij} = j$ indicatesnthe nodes needed to pass from node i to node j.

2.1.3. Model Solution

Based on the shortest path matrix, exhaust all possible situations, and use MATLAB programming to select the three medical points that minimize the total distance, which are 10, 50, and 57. The minimum distance and the total mileage $S_{2=}95365.5$ of road repairs are calculated.

Table 1. Starting and Ending Points of Roads to be Repaired for Problem 1

Serial Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Start	2	4	5	5	6	6	7	8	9	10	11	12	13	14	15	17	18
End	5	8	2	16	1	9	13	4	6	3	14	22	7	11	26	21	25
Serial Number	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
Start	20	20	22	22	23	24	25	26	28	29	30	31	32	32	36	37	37
End	19	30	12	17	29	28	18	15	24	23	20	32	31	34	41	27	33
Serial Number	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51
Start	37	38	38	41	41	42	43	45	45	46	46	47	48	49	50	50	52
End	38	37	39	36	49	35	45	40	43	48	51	53	46	44	54	61	42
Serial Number	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68
Start	52	55	57	58	59	59	60	60	65	66	66	68	68	68	69	71	72
End	59	47	55	60	52	56	58	70	66	62	65	63	67	69	68	74	75
Serial Number	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85
Start	73	74	74	75	75	76	77	77	80	81	83	84	86	87	87	89	90
End	64	71	73	72	78	84	79	80	77	83	81	76	87	86	95	92	93

Table 1. (continued).

Serial Number	86	87	88	89	90	91	92	93	94	95	96	97
Start	90	91	91	92	92	93	94	95	95	96	96	97
End	97	88	96	85	89	90	82	94	99	91	100	98

The road map from villages to the corresponding medical points is as follows:

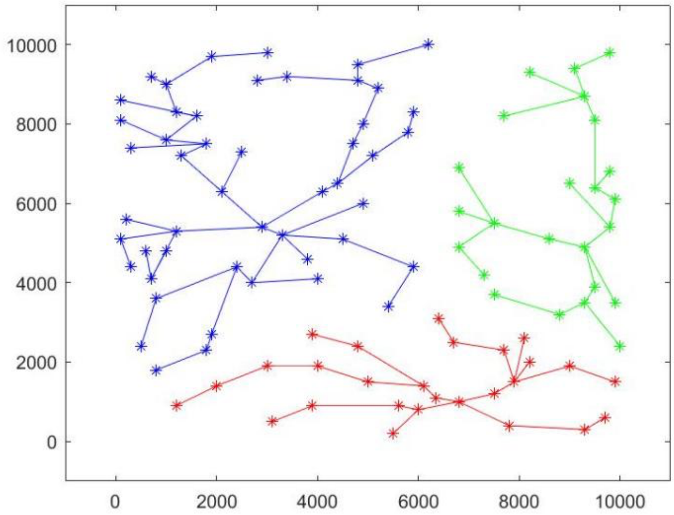

Figure 2. Road usage diagram for villages to corresponding medical points

2.2. *Minimum Road Maintenance Cost*

There are many spanning trees for Figure 1, with the sum of the path lengths of all edges as the total weight. Different spanning trees have different total weights, and the goal is to find the path distribution with the smallest total weight, i.e., the minimum spanning tree.

2.2.1. *Establishment of the Minimum Spanning Tree Model Based on Prim Algorithm*

Based on the provided village locations and the distances between adjacent points, digitize the village and road distribution map into a weighted graph G(V, E). All vertices are in set V, and set S stores the visited vertices, initially empty. Each time, choose the vertex u with the minimum shortest distance from the V-S set, visit it, and add it to set S. Then, use vertex u as the intermediary point to optimize the shortest distances from u to all vertices v in set S. Repeat until set S contains all vertices. According to the cut theorem, the visited points and unvisited points are cut [2]. Specific steps are as follows:

Step 1: Initially visit vertex 10.

Step 2: Mark vertex 10 as visited.

Step 3: Traverse the edges marked with vertex 10. If the other end of the edge has already been visited, it is not a cross-edge and is skipped. If not visited, the edge forms a cross-edge and is added to the minimum heap.

Step 4: Get the minimum weight from the index heap as an edge of the minimum spanning tree.

Step 5: Repeat the steps until all vertices are visited.

2.2.2. *Model Solution*

After obtaining the minimum spanning tree through MATLAB programming, since the villagers only need to choose one medical station, there is no need to connect the three medical stations (10-50, 50-57, 57-10) directly. Therefore, removing the longest edge from the 10-50 path and the longest edge from

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the 10-57 path will minimize the length of the roads needing repair. By calculating the total length of the roads to be repaired and deriving the path network available for villagers to use, the code from problem 1 can be repeated to determine the total distance villagers will travel on repaired roads to reach the medical stations.

Table 2. Start and End Points of Roads Needing Repair for Problem 2

Serial Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Start	10	11	10	12	22	11	9	6	17	22	24	21	16	5	24	31	32
End	11	14	12	22	17	9	6	1	21	24	28	16	5	2	31	32	34
Serial Number	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
Start	14	15	26	20	20	19	13	7	18	18	23	34	44	49	41	49	60
End	15	26	20	30	19	13	7	18	25	23	29	44	49	41	36	60	58
Serial Number	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51
Start	60	10	54	50	61	63	68	68	50	38	38	37	25	35	42	52	52
End	70	3	50	61	63	68	69	67	38	37	39	33	35	42	52	59	48
Serial Number	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68
Start	48	46	59	15	8	37	69	86	87	95	95	94	70	76	76	72	75
End	46	51	56	8	4	27	86	87	95	99	94	82	76	84	72	75	78
Serial Number	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85
Start	84	88	91	72	66	66	30	45	45	47	55	47	57	64	73	74	74
End	88	91	96	66	65	62	40	43	47	55	57	53	64	73	74	71	80
Serial Number	86	87	88	89	90	91	92	93	94	95	96	97	98				
Start	80	77	80	81	83	89	92	89	97	90	97	96	10				
End	77	79	81	83	89	92	85	97	90	93	98	100	90				

The road repair layout minimizing the repair cost is shown in the following figure:

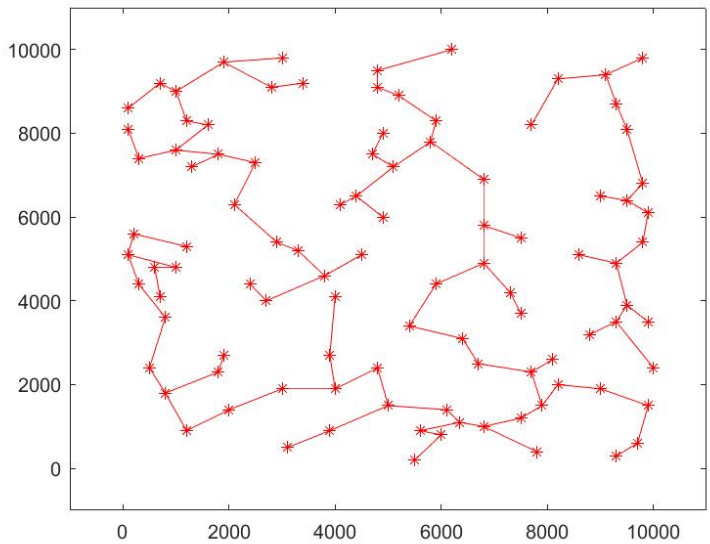

Figure 3. Road Maintenance Map

2.3. Section 2.3 Optimal Solution

To achieve a balance between these two aspects, a two-layer greedy algorithm is utilized to solve the multivariable problem [3]. The process begins by arbitrarily selecting three villages to serve as health care centers. The initial step is to determine the shortest path for S_1. Following this, akin to the Prim algorithm, optimization of the road network commences from a node, where nodes that minimize the sum of S_1 and S_2 are sequentially chosen to replace the current node. This results in the minimum value of $S_1 + S_2$ for a given distribution of health care centers. Subsequently, the process is iterated C_{100}^{3} times to calculate the value of $S_1 + S_2$ for any arbitrary placement of three health care centers, and the smallest value from these iterations is identified. This minimum value represents the optimal location for the health care centers and the sum of the two types of distances, $S_1 + S_2$. The specific steps are as follows:

2.3.1. Establishment of the Greedy Algorithm Optimization Model

Select any three villages to serve as medical points. Similar to the first question, initially determine the path that minimizes the total distance S1 from each village's residents to the medical points. Then, starting from any given point, assume that this point is connected to only two other nodes (Node 1 and Node 2). Record the minimum distance from this point to the medical station via Node 1 as S1' (where the first point passed is through Node 1), and the minimum distance from this point to the medical station via Node 2 as S2, $\Delta s_1 = |s_1 - s_2|$. Let the distance from this point to Node 1 be denoted as d1 and to Node 2 be d2. The impact on the total S2 is solely dependent on the difference in distances from this point to Nodes 1 and 2,$\Delta s_2 = |d_1 - d_2|$

Initially, each point's weight Q_i is set to 1. Compare the values of $Q_i * \Delta s_1$ and $Q_i * \Delta s_2$. If $Q_i * \Delta s_1 > Q_i * \Delta s_2$, choose node 2 for the next step, adding the point's weight to node 2's weight and recording node 2 and its weight in the path matrix. If $Q_i * \Delta s_1 < Q_i * \Delta s_2$, choose node 1 for the next step, adding the point's weight to node 1's weight and recording node 1 in the path matrix. By randomly determining three medical stations and iterating multiple times, the shortest path from each point to the medical stations:$S_1 + S_2$ can be found. After C_{100}^{3} iterations, you can obtain the corresponding medical points for any given setup. From these, you can identify the medical points that correspond to the shortest path to solve the problem.

2.3.2. Solution of the Model

The solution was obtained using MATLAB programming, and the results for S1 and S2 are summarized in the following table:

Table 3. Summary of Results

	S1 (m)	S2 (m)	S1+S2 (m)
The nearest medical point	316598.7	95365.5	411964.2
The lowest cost of road construction	558821.4	78515.6	637337

3. Comprehensive Evaluation and Extension of the Model

3.1. Analysis of the Model's Advantages

(1) **Advantages and Disadvantages of the Floyd Algorithm for Solving the Shortest Path Model**
The Floyd algorithm can directly determine the minimum distance between any two points and can handle negative edges, whereas Dijkstra's algorithm can only determine the minimum distance from all other points to one point [4][5].

(2) **Good Portability of the Minimum Spanning Tree Model**
A basic model for solving the minimum spanning tree problem has been established, which has good portability. The Prim algorithm used is less prone to producing cycles compared to the Kruskal algorithm, has a lower time complexity, reduces the difficulty of programming, and is more suitable for solving dense graphs as in this problem.
(3) **Multi-layer Greedy Algorithm for Progressive Solutions to Multi-objective Optimization Problems.** For such multi-variable cases, if dynamic programming is considered, the state decision space is too large, the computational effort is too high, and it is not possible to obtain a better solution in a short time. Although also based on the traversal idea, the greedy algorithm first searches in the best direction, so it can always obtain a better solution in a short time.

3.2. Analysis of the Model's Disadvantages

(1) The Minimum Spanning Tree Model is only suitable for undirected graphs and cannot be used when edges are vectors or have specified directions; the Prim algorithm is more difficult to understand; the Floyd algorithm has an increasing amount of redundant calculations with the increase of node numbers, the computational time complexity is high, and it can only be applied to specific problems.
(2) In the solution of this problem, due to the large number of nested layers, the large number of variables, and the complexity of the traversal cases, the computational effort of the greedy algorithm is large, and in the subsequent calculations, it is not possible to use the experience summarized from the previous medical stations, which greatly increases the time complexity.

3.3. Improvement of the Model

(1) The disadvantage of the Floyd algorithm is that with the increase of node numbers, redundant calculations also increase. By observing the Floyd algorithm, it is found that it calculates the distances from any node to other nodes through each node. Generally, the degree of the nodes in the network is not very large, so a large amount of redundant calculations are caused during the calculation. Therefore, eliminating unnecessary intermediate node path calculations can reduce the computational effort and effectively improve the computational efficiency of the Floyd algorithm [6].
(2) When solving the third question, the computational effort of the greedy algorithm is large, and machine learning can be used for optimization. The Shortest Path Finder (SPF) in the machine learning history is used to search for the shortest path between nodes in the connected graph. By simplifying the complex problem to the shortest path between nodes, the optimal path can be searched out.

3.4. Extension of the Model

(1) The Floyd algorithm can handle the task of finding the shortest path between any two nodes and can be applied to tasks that evaluate the importance of nodes.

When studying network attacks or defenses, it is necessary to sort the importance of network nodes. In the process of evaluating the importance of nodes, the more shortest paths that pass through the node, the more important the node is according to the betweenness centrality measure.

(2) The minimum spanning tree model built based on the Prim algorithm can solve this multicast tree problem.

The minimum spanning tree model can be applied to the layout of optical cables between cities, which not only connects every two cities with optical cables but also minimizes the cost.

4. Conclusion

Addressing the issue of the nearest medical points, this paper establishes a model for solving the shortest path problem based on the Floyd algorithm. Within 100 villages, three medical points are selected (10, 50, 57), and the total distance from each villager to the nearest medical point is minimized to $\mathbf{S_2} = \mathbf{316598.7m}$ meters. The roads that villagers travel are the ones that need to be repaired (see Table 2). Based on this, programming is used to calculate the total mileage of the roads to be repaired, $\boldsymbol{S_2} =$

$\mathbf{95365.5}\boldsymbol{m}$ meters, and a road map from each village to the corresponding medical point is drawn (see Figure 4).

For the lowest road construction cost, this paper constructs a minimum spanning tree model based on the Prim algorithm to find the minimum sum of weights for the minimum spanning tree. On the basis of the medical stations determined in the first question, the longest roads connecting the three medical stations (10-50, 50-57, 57-10) are removed, ultimately obtaining the road repair method that minimizes maintenance costs (see Table 3). At this time, $\mathbf{S_1 = 558821.4m}, \mathbf{S_2 = 78515.6m}$, and the road map of the roads that need to be repaired is shown (see Figure 6).

Addressing the optimal solution for both, finding the minimum value of s1 + s2 is a bi-objective optimization problem. This paper uses a two-layer greedy algorithm, first calculating the sum of the shortest distances when any three medical points are selected, and then iterating through C_{100}^{3} methods of selection. When calculating the sum of the shortest distances under a certain selection of medical points, the second layer of the greedy algorithm is used, ultimately determining the selection of the medical stations.

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