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Damage inference of statically deterministic structural trusses is carried out by Bayesian updating

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Abstract. In the face of harsh environments, aging construction materials, and increased traffic, the performance of Bridges in service continues to deteriorate. To avoid the serious consequences caused by structural damage, it is of great significance to infer the damage of the truss structure and evaluate the reliability of the bridge. In this paper, a random sampling process is used to characterize the resistance degradation, and the historical load test information and Bayesian method are used to update the resistance of serving Bridges. The modeling of structural mechanics solver determines the maximum axial force on the damaged structural member by loading different nodes. Secondly, node displacement is calculated. Then, the displacement results are taken as new information as the parameters of prior distribution for Bayesian update, and finally, the maximum probability of structural damage is obtained. The results show that the probability of EA values between 1.34 and 1.38 is the greatest. The uncertainty and variability in the process of resistance degradation of reinforced concrete bridge members can be described more reasonably by the Bayesian updating process. By integrating the real resistance degradation information detected during bridge service into practical application, and using the research method in this paper to analyze and calculate, Bridges' future service status and remaining service life can be predicted more accurately.

Keywords: Bayesian updating , Statically determinate truss , Structure optimization , Fatigue residual life of bridge.

1. Introduction

Highways and bridges are important parts of China's traffic system, and the operation law of bridge structure is complex, with great uncertainty, difficult to quantify, etc. In order to avoid the serious consequences brought by structural damage, it is not enough to rely on structural design to ensure the safety of structureof structural damage, it is not enough to rely on structural design to ensure the structure's safety. How to fully excavate and timely update the latest status information of service structures and establish a set of practical and effective safety monitoring and estimation methods is of great significance for bridge structure health monitoring, status assessment and maintenance decision-making. It is a prerequisite and core issue to accurately describe the performance degradation process of in-service Bridges with a reasonable mathematical model.At present, researches on bridge reliability mainly focuses on reinforced concrete bridges. In the service process, reinforced concrete Bridges are often affected by factors such as harsh environmental erosion, natural aging of materials, vehicle load and improper maintenance measures, so their performance deteriorates over time. Liang Can et al. [1] proposed a time-varying reliability analysis method based on Dynamic Bayesian Network (DBN) to analyze the impact of component performance degradation on the reliability of composite steel plate girder Bridges. Chen Long et al. [2] used inverse Gaussian stochastic process and compound Poisson process, respectively to establish the models of bridge resistance degradation and vehicle load effect, established the time-varying reliability analysis method of bridge components based on the double stochastic process model of resistance load effect, and derived the corresponding time-varying reliability calculation formula. Han Yugang used random process to represent the resistance degradation, used historical load test information and combined with the Bayesian method to update the resistance of Bridges in service, obtained the resistance truncation distribution, and then proposed a method to calculate the time-varying reliability of Bridges. A concrete bridge was taken as an example to demonstrate the resistance renewal and time-varying reliability evaluation.

In this study unlike the traditional method of obtaining a fixed constant from a large number of data tests, this study utilizes Bayesian updating based on known conditions, i.e., a priori probability, and using uses the displacement response of the truss model under load experiments as the basis of updating, to carry out damage identification of the structure, and based on the efficiency of the damage identification, to carry out damage extrapolation of static structural trusses using Bayesian updating, as well as discussing and exploring optimization of the load experiments. Specifically, this paper uses the displacement response of the truss model as the basis for updating to perform damage inference for static structural trusses via Bayesian updating. Firstly, through the modeling of mechanics solver, the loads of different sizes and directions are applied to different nodes. Using the virtual work principle, the force of size 1 is applied to each node, the maximum axial force of the damaged rods is selected, and the node displacements required to be measured by the truss are determined. The loading mode of the truss is initially determined after analyzing and comparing. The computer is then used to find the magnitude and direction of the displacement of the requested node at this point in this loading mode. Once the displacement result is obtained, it is used as a parameter of the prior distribution, the likelihood function is obtained, and then a Bayesian update is performed to obtain a clearer and better posterior probability. The results are then computed for the structure based on the physical parameters of the parameters obtained from the Bayesian update, and a histogram of the updated posterior probability distributions is drawn to give the results of the Bayesian update. Finally, the obtained images are analyzed theoretically to conclude.

2. Methodology

2.1. Bayesian method

Bayesian method was proposed in the 1950s to solve the reliability evaluation problem of complex systems. It has obvious advantages in dealing with small sample data, and has achieved good evaluation results and economic benefits in weapons and aerospace. Bayesian updating is a statistical method based on Bayes' theorem, which is used to gradually update the probability estimate of an event as new evidence emerges on the basis of existing prior knowledge. So when this report have some prior results and receive new data, Bayesian updating can adjust probability estimates based on those data.

The distinction between the Bayesian method and classical statistical theory lies in considering the parameters themselves as random variables, such as the element's lifetime, and obtaining a posterior distribution representing these parameters' probability distribution interval. The Bayesian method employs prior distributions to derive precise probability estimates even with limited sample sizes. Compared to the classical approach, one advantage of the Bayesian method is its ability to incorporate diverse forms of prior information, including reliability test data, historical data, expert knowledge, and simulation test results. This reduces reliance on field test sample sizes while maintaining assessment accuracy requirements. It is important to note that the Bayes method does not aim for reduced information usage but rather aims to fully leverage all available information during product testing processes (such as reliability test data, historical data, expert knowledge, and simulation information), making it more applicable in practical engineering.

Bayesian updating is a simple and fast process for predicting predicted samples, and it is also effective for multi-classification problems. In addition, if the assumption of distribution independence is true, Bayesian classification works very well, slightly better than logistic regression, and the calculation requires less sample size.

2.2. Related concept

Bayesian updating involves four key concepts: prior probability, posterior probability, likelihood function, and evidence factor. The prior probability represents the estimated probability before conducting a Bayesian update. The likelihood function is the complementary concept to probability. Whereas probability calculates the chance of an event based on known conditions, likelihood assesses the inverse probability based on the occurrence of that event. The evidence factor denotes the observed probability of one event happening when estimating the likelihood of another event occurring.

The Bayesian formula for continuous distribution is commonly used in reliability data analysis because the distribution is clearly continuous over the possible intervals for components' operational failure rates and demand failure rates . Let {X1... Xn} represents a sample that follows a distribution with a likelihood function L, continuous parameters, and a prior density g (θ). Then the posterior distribution density is:

$$f(\theta|x_1...x_n)=\frac{L(x_1...x_n|\theta)g(\theta)}{\int L(x_1...x_n|\theta)g(\theta)d\theta} \tag{1}$$

This is the Bayes formula for continuous functions.

In the Bayes formula (1), the treatment of prior distribution g (θ) is a key problem, which is always solved bu the natural conjugate distribution. The comparative study of lognormal distribution Bayesian updating methods\shows that the likelihood function of running failure rate is Poisson distribution, and its conjugate distribution is Gamma distribution [3-5]. The likelihood function of the demand failure rate is a binomial distribution and its conjugate distribution is a Beta distribution.

Its probability density function is expressed as follows:

$$g(\lambda)=\frac{\beta^{\alpha}}{\Gamma(\alpha)}\lambda^{\alpha-1}e^{-\beta\lambda} \quad (\lambda,\alpha,\beta>0) \tag{2}$$

$$g(\gamma)=\frac{\Gamma(\delta+\rho)}{\Gamma(\delta)\Gamma(\rho)}\gamma^{\delta-1}(1-\gamma)^{\rho-1} \quad 0\le\gamma\le 1,\delta>0,\rho>0 \tag{3}$$

The characteristic of the conjugate distribution is that the prior distribution and the posterior distribution belong to the same distribution family [6-7]. According to the Bayes principle, the posterior distribution of formula (2) is as follows:

$$f(\lambda)=\frac{(\beta+T)^{\alpha+r}}{\Gamma^{\alpha+r}}\lambda^{\alpha+r-1}e^{-(\beta+T)\lambda} \tag{4}$$

R indicates the cumulative number of running failures during the running of the device;

T represents total operating hours of the device during operation.

Be transformed intoλ~Gamma(α+r,β+T)。

By the same principle, the posterior distribution of formula (2) can be obtained as:

$$f(\gamma)=\frac{\Gamma(\delta+\rho+n)}{\Gamma(\delta+r)\Gamma(\rho+n-r)}\gamma^{\delta+r-1}(1-\gamma)^{\rho+n-r-1} \tag{5}$$

r represents cumulative number of required failures;

n represents cumulative number of demands.

Be transformed intoγ~Beta(δ+r,ρ+n-r)

2.3. Likelihood function

Site information such as field test data, monitoring data and observation information generally needs to be mathematically described by likelihood function, and then integrated into Bayesian analysis [8,9].

The likelihood function is proportional to the probability of occurrence of a certain site information Z, given that the uncertainty parameter X takes a certain implementation value x:

$$L(x) \propto P(Z|X = x) \quad (6)$$

P represents the probability of an event happening

Z represents site information event

Measurement errors caused by insufficient accuracy of test devices and instruments, improper operation of test personnel or random test effects in the test process are inevitable [10,11].

If the measurement error ε_i is regarded as an additional quantity, then the measured value of the geotechnical parameter can be expressed as the simulated value plus the measurement error, that is, the group i field or laboratory test data $x_i^m = x + \varepsilon_i$, then the likelihood function is

$$L(x) = f_e(x_i^m - x) \quad (7)$$

f_ε is the probability density function of the measurement error ε_i

If the measurement error ε_i is regarded as a multiplicative quantity, the measured value of geotechnical parameters can be expressed as the parameter simulation value multiplied by the measurement error. $x_i^m = x\varepsilon_i$,Then the corresponding likelihood function is

$$L(x) = f_\varepsilon(\frac{x_i^m}{x}) \quad (8)$$

It is generally assumed that ε_i of different tests are independent of each other and follow a normal distribution of mean 0 and standard deviation σ_{ε_i}, then the corresponding likelihood function can be expressed as:

$$L(X) = k_1 \prod_{i=1}^{n_d} exp[-\frac{(x_i^m - x)^2}{2\sigma_{\varepsilon i}^2}] \quad (9)$$

2.4. Principle of virtual work

It is difficult to calculate the virtual work of the force F corresponding to the virtual displacement δr in the virtual displacement state, especially when the virtual displacement δr is moving in a rigid plate in a general plane. Shu Kaiou et al. [10] adopted the following methods to subtly solve this problem.

Let the virtual angular displacement of the rigid plate where the force F is located in the virtual displacement state be $\delta\theta$, and point O is the displacement center of the rigid plate, as shown in Figure 1. Then the virtual work done by F on δr is equal to the virtual work done by MO(F) on $\delta\theta$ on the moment of F on O.

$$F \cdot \delta r = M_O(F) \cdot \delta\theta \quad (10)$$

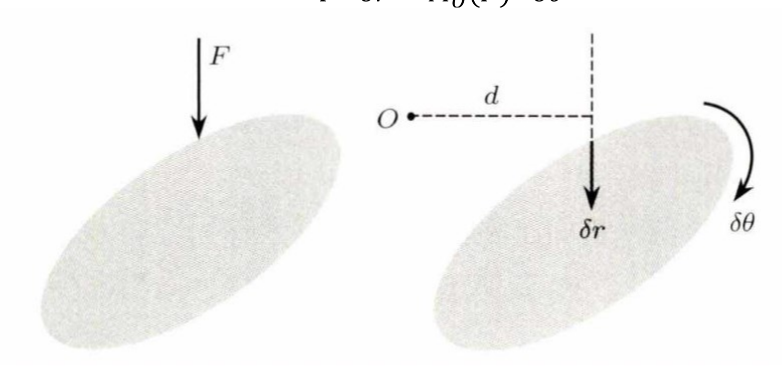

Figure 1. Conversion of virtual work.

The virtual work done by force F on δr, $W = F \cdot \delta r$, is $W = F \cdot \delta r = Fd \cdot \delta\theta$ due to $\delta r = d \cdot \delta\theta$, and note that the moment of force F with respect to point O, MO(F) = Fd, is $F \cdot \delta r = M_O(F) \cdot \delta\theta$.

By converting the force as virtual work to the moment as virtual work, it can avoid calculating the dotted line displacement corresponding to each force on the same rigid plate one by one, but the displacement center and virtual angular displacement of the rigid plate must be determined first. However, to obtain the dotted line displacement, the rigid plate's displacement center and virtual angular

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displacement must be determined first, so the virtual work conversion technique can often simplify the calculation of virtual work.

3. Specific experiments

3.1. Research target

The object of this paper is to model a static truss bridge with a span of 60 meters. Its forces are characterized by the fact that the internal forces in the structure are only axial forces, without bending moments and shear forces. This force characteristic reflects the main factors of the actual structure, and the axial force is called the main internal force of the truss. The truss consists of top chord, bottom chord and web. The form of the belly bar is further divided into oblique belly bar, straight belly bar. The total height of this truss is 15 meters. The top chord consists of two rods with a length of 15 meters connected by an articulation with a total length of 30 meters. The lower chord bar consists of four bars with a length of 15 meters connected by articulation, and its total length is 60 meters. The upper and lower chord bars are connected at each end by two diagonal web bars angled at 45° to the horizontal. The three points of articulation of the upper chord bar and the three points of articulation in the center of the lower chord bar are each connected by three straight web bars. The articulation points at the ends of the upper chord bar and one at the midpoint of the lower chord bar are each connected by two diagonal web bars (Table 1). This study damaged the third link from left to right in the subject's lower chord bar. Its post-injury EA obeys a uniform distribution U(0.90,1.98) (Figure.2).

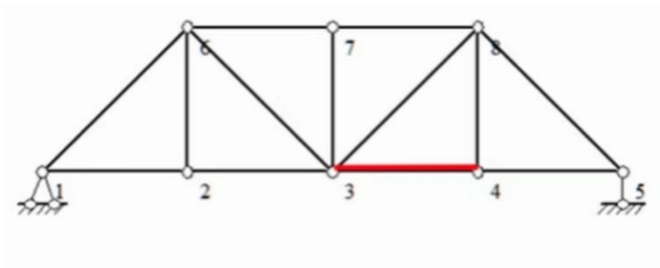

Figure 2. Research target.

Table 1. Known rod stiffness.

Rods (nodes)	EA/10^5kN·m^2
1-2	1.8
2-3	1.71
8-5	1.98
4-5	1.8
1-6	1.98
6-7	1.89
7-8	1.89
2-6	1.62
3-6	1.53
3-7	1.539
3-8	1.52
4-8	1.62

3.2. Computational process

According to the principle of virtual work and the displacement formula(Formula 10), it can be seen that the denominator of the summation for calculating the displacement of a point is Np, the product of N and the length of the rod.According to the principle of virtual work and the displacement formula(Formula 10). It is known that the summation numerator for calculating the displacement at a point is the product of Np, N and the length of the rod. In the previously mentioned equations, the EA values of the rods other than the damaged rods remain unchanged. In order to optimize the damage update for this study, there should be a significant change in the displacement of the measured points with a small change in the EA of the damaged rod. In the case of a constant length of the bar, the value of Np for the segment should be maximized with respect to the value of N to make the difference in the displacement of the measured points under different EA more obvious.

The object of study is modeled using a structural mechanics solver and different sizes of concentrated loads are applied at different nodes respectively. Similarly using the principle of virtual work, a force of magnitude 1 in different directions is applied at each point. Attempts were made to apply a concentrated load of 100kN at the rightmost end of the top chord and to measure the displacement of the bottom chord at the second articulation point from the right. At this point the denominator of the variable term of the above equation is maximized. Finally, the loading scheme and measurement scheme for the subjects in this study were derived.

3.3. Bayesian updating process

After determining the method for finding the displacement, an EA is derived as an unknown using the formula. The formula is (2653.725+843.75/EA)/100000. Bayesian updating of the displacement results as new information as prior distribution parameters. and results are computed based on Bayesian updating of physical parameters for the structure. The prior function for Bayesian updating is obtained based on a uniform distribution with a total area of one.

A computer is utilized to derive the displacement measurements at the points required to be measured for the bar in the identified loading mode. Taking EA values that are uniformly distributed between 0.90 and 1.98. Calculated values are calculated from the different EA values, and the difference is obtained by subtracting the calculated values from the measured values. From the material, it is known that the standard deviation of the damage of the study object is 5mm. The likelihood function is obtained from equation (9).

Finally, the posterior function is obtained by probability calculation using the accepted samples. From equation (5), The product of the prior function and the likelihood function is then summed and multiplied by the product of the two above and multiplied by the index value of the data taken, and the two are divided to give the posterior probability. The indexing value is 0.01. Table 2 shows the probability values corresponding to some of the EA values. The EA values in the table are taken uniformly in the range (0.9, 1.98). The second column in the table shows the difference between the calculated and measured values of displacement.

Table 2. Probability values corresponding to some EA values.

EA		difference	Likelihood	Prior		chess *0.01	Post
0.90	0.035912	0.003218	8.053E-07	0.925925926	7.4565E-07	210.345135	3.54489E-09
0.91	0.035809	0.003115	2.9696E-06		2.7497E-06	210.345135	1.30721E-08
0.92	0.035708	0.003014	1.0216E-05		9.4593E-06	210.345135	4.49705E-08
0.93	0.03561	0.002916	3.2904E-05		3.0467E-05	210.345135	1.44841E-07
0.94	0.035513	0.002819	9.9551E-05		9.2177E-05	210.345135	4.38216E-07
0.95	0.035419	0.002725	0.00028382		0.0002628	210.345135	1.24936E-06
0.96	0.035326	0.002632	0.00076478		0.00070813	210.345135	3.36652E-06
0.97	0.035236	0.002542	0.00195318		0.0018085	210.345135	8.59777E-06
0.98	0.035147	0.002453	0.00474032		0.00438919	210.345135	2.08666E-05
0.99	0.03506	0.002366	0.01096026		0.01014839	210.345135	4.82464E-05
1.00	0.034975	0.002281	0.02419949		0.02240694	210.345135	0.000106525
1.01	0.034891	0.002197	0.05113687		0.04734895	210.345135	0.000225101

Using the uniformly distributed EA values as horizontal coordinates and the posterior function as vertical coordinates, a histogram is obtained. The bar graph obtained is Figure 3.

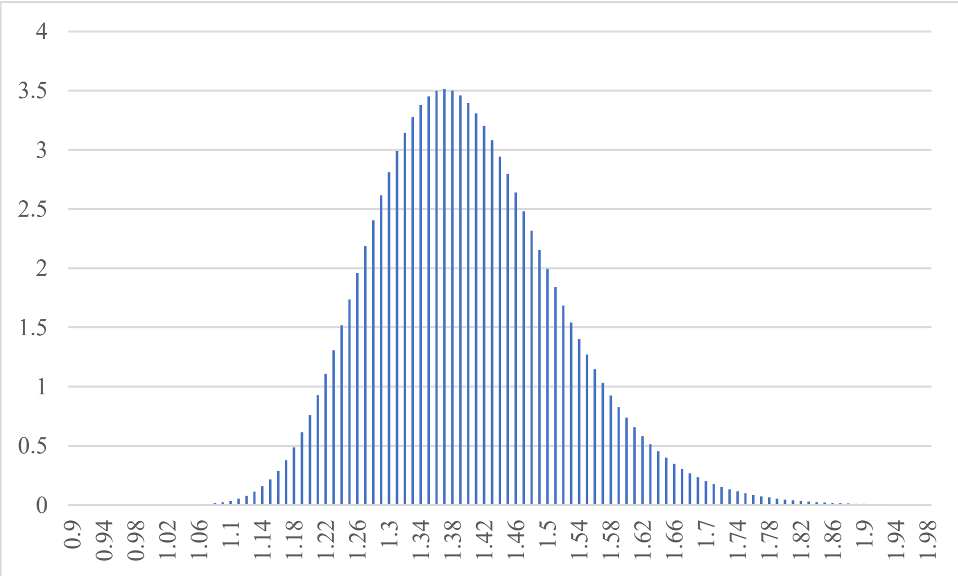

Figure 3. Plot of results after Bayesian update.

As can be seen from the histogram, the posterior probability of damaged rods in this truss is greatest for EA values between 1.34 and 1.38.

4. Conclusion

In this paper, damage is inferred from a static structural truss with a span of 60 meters. Load this truss again. To calculate and measure the change in displacement at a point.The two results are compared and calculated, a Bayesian update using knowledge of probability. The uniformly distributed EA values were eventually updated to a bar chart with significant differences. The probability-based Bayesian updating method gives the posterior distribution of parameter probability densities through the structure of the measured data. The maximum probability of damage to the structure is obtained from the calculated. The result of this is that the probability of an EA value between 1.34 and 1.38 is maximized.

The posterior distribution of the parameters can be estimated more accurately by means of Bayesian updating. It can help to categorize and estimate the probability of things under uncertainty for unknown

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situations such as building damage. It allows events to arrive at reasonable results despite incomplete data and information. Bayesian updating is better for solving practical problems than traditional statistics.

Future developments can be directed mainly towards nonlinear phase damage recognition. Most of the current research methods focus on linear phase damage identification, while there is room for development of techniques and methods for nonlinear phase damage identification. Although the research on nonlinear identification has performed well in various testing and simulation experiments, it has rarely been applied to practical engineering problems.

Moreover, in today's social construction projects, a series of factors that cause structural damage, such as the environment in which the project operates, are subject to constant change. Therefore it is imperative to study the real-time monitoring of the structural condition of the structure and the judgment of structural damage. This is important to extend the life of the building.

Authors Contribution

All the authors contributed equally and their names were listed in alphabetical order.

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