DOI: 10.54254/2755-2721/101/20240921

Uphill dynamics: A spring-mass model analysis of sloped walking

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Abstract. This study explores the biomechanics of uphill running by adjusting and analyzing the spring-mass model. Modifications specific to slope conditions were made to the equations of motion. Despite facing challenges in code execution, an in-depth investigation of the model's physical characteristics was conducted through both static and dynamic state analyses. The findings aid in the theoretical understanding of motion on sloped surfaces and provide valuable experience for subsequent research.

Keywords: Spring Mass Model,Uphill Walking,Matlab

1. Introduction

Understanding the biomechanics of human movement across different terrains is crucial for advancements in fields ranging from sports science to bionic robot design. The spring-mass model has long been a cornerstone of running biomechanics analysis, offering valuable insights into the elastic and dynamic characteristics of human gait . [1] [2] While widely applied to the study of motion on flat ground, the mechanics of movement on slopes have been less explored.

Slopes present unique mechanical demands and energy consumption challenges for movement. Animals moving uphill must overcome increased gravitational potential energy, typically resulting in slowed speeds and gait adjustments to extend ground contact time, thereby necessitating additional work. [3] Exploring uphill running through the lens of the spring-mass model allows for a deep dive into the roles of leg stiffness, ground reaction forces, and energy utilization during inclined movement. These studies are not only academically intriguing but also have significant practical implications. For athletes, insights into the mechanics of uphill running can inform training programs aimed at performance enhancement and injury prevention. [4] For engineers, this knowledge assists in designing better assistive devices to navigate the challenges posed by sloped surfaces. [5]

This report extends the traditional spring-mass model to examine the biomechanics of uphill running. We delve into how adjustments in leg stiffness and attack angle—two core parameters of the model—can elucidate the adaptations necessary for effective uphill motion. Our work aims to reveal the subtle changes in dynamics and kinematics experienced by runners on inclines, providing a detailed account of the involved energy dynamics.

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2. Description of the Model Code

The spring-mass model [6] is a simplified physical model used to simulate biological motions, such as running and jumping. It abstracts the moving body into a point mass m connected to the ground via an ideal spring with stiffness k. This model captures key dynamical characteristics of running and jumping, including ground reaction forces and the body's bouncing motion.

2.1. Model Parameters Definition

This section of the code defines the fundamental parameters of the model: the natural length of the spring l_0, mass m, spring stiffness k, acceleration due to gravity g, and the landing angle of the leg α_0. These parameters are the key variables in the simulation, collectively determining the dynamic behavior of the model.

Table 1. Fundamental parameters of the spring-mass model

Parameter	Description
l_0	Natural length of the spring
m	Mass of the moving body
k	Spring stiffness, quantifying the spring's resistance
g	Acceleration due to gravity, the constant acceleration
α_0	Landing angle of the leg,

2.2. Mathematical Expressions

2.2.1. Equations of Motion During the flight phase (i.e., the aerial phase), the model's equation of motion is solely influenced by gravity:

$$\ddot{y} = -g$$

where g represents the acceleration due to gravity, and $\ddot{y}$ is the acceleration of the point mass in the vertical direction.

In the stance phase, when in contact with the ground, the dynamics of the model are described by the following equations, taking into account the compression and restoration of the spring:

$$\ddot{x} = \frac{k}{m}(x_1 - x)$$

$$\ddot{y} = \frac{k}{m}(y_1 - y) - g$$

Here, x and y denote the positions of the point mass in the horizontal and vertical directions, respectively; x_1 and y_1 are the natural length positions of the spring when it is not under force; k is the spring stiffness, and m is the mass.

2.2.2. Leg Landing Angle (α_0) The leg landing angle affects the position of the foot contact and the initial compression of the spring, thereby influencing the dynamics of running or jumping. [7] The angle α_0 is expressed in radians and is converted from degrees as follows:

$$\alpha_0 = \alpha_0 \times \frac{\pi}{180}$$

2.3. Code Description

- **`spring_mass_running.m`**: Simulates the dynamics of flight and landing phases using the `ode45` solver, with specific transition conditions to control different phases and plot the motion trajectory.

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2.3.1. Event Function

- **apex**: Identifies the motion's apex, where vertical velocity is zero.
- **take_off**: Detects the take-off moment when the spring length equals its natural length.
- **touch_down**: Uses the landing angle α_0 to calculate the landing moment, pinpointing the simulated foot's initial contact with the ground.

3. Physical Model Analysis

To extend the application of the spring-mass model to uphill phases, it is necessary to consider the impact of slope on the model. [8] The incline affects the components of gravity as well as the motion of the mass along the slope.

3.1. Modifying the Gravity Component

On flat ground, gravity acts only in the vertical direction (i.e., in the y-direction) with a magnitude of mg. However, during the uphill phase, gravity can be decomposed into two components: one along the slope downwards and another perpendicular to the slope. Let the slope angle be denoted by β, where β is the angle between the slope and the horizontal plane. Positive values of β indicate an uphill slope, while negative values indicate a downhill slope. The component of gravity along the slope downwards, g_{down}, is given by $g\sin(\beta)$. The component of gravity perpendicular to the slope, $g_{\text{perpendicular}}$, is given by $g\cos(\beta)$.

3.2. Modifying the Equations of Motion

Flying Phase (EoM_{air})

During the flying phase, the mass does not contact the ground, thus only gravity acts on the mass. The vertical component of gravity remains unchanged, but the component along the slope downwards needs modification. [9] The modified equations of motion for the flying phase (EoM_{air}) are:

$$\dot{x} = q_3, \quad \dot{y} = q_4,$$
$$\ddot{x} = -g\sin(\beta), \quad \ddot{y} = -g\cos(\beta).$$

Standing Phase (EoM_{ground})

During the standing phase, the mass contacts the ground, and the applied spring force depends on the relative position between the mass and the point of contact on the ground. Due to the slope, the direction of the spring force is also affected. [10] The modified equations of motion for the standing phase (EoM_{ground}) are:

$$\dot{x} = q_3, \quad \dot{y} = q_4,$$
$$\ddot{x} = q_1\omega^2\left(\sqrt{q_1^2 + q_2^2} - l_0\right) - g\sin(\beta),$$
$$\ddot{y} = q_2\omega^2\left(\sqrt{q_1^2 + q_2^2} - l_0\right) - g\cos(\beta).$$

3.3. Touchdown and Takeoff Conditions

Touchdown Condition (touch_down)

For uphill motion, the touchdown condition considers the projection length of the mass along the slope rather than just its vertical height. This condition determines if the mass has reached the predefined contact point on the slope. Given the slope angle β, the touchdown condition is expressed as:

$$\text{impact} = (q_2 - l_0\sin(\alpha_0 + \beta))$$

where α_0 is the angle between the mass and the vertical line (in the absence of a slope), and β is the slope angle. This modification takes into account the mass's touchdown position on the slope. [11]

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Takeoff Condition (take_off)
The takeoff condition evaluates if the spring has returned to its uncompressed length l_0, also considering the slope's effect. Because of the slope, the actual spring length must be measured in the slope's direction. Hence, the takeoff condition might not need significant adjustments based on the slope since it is grounded on the geometric condition of the spring length. However, calculating the takeoff angle and direction may need to reflect the slope's influence.

$$\text{impact} = \sqrt{q_1^2 + q_2^2} - l_0$$

This ensures that the mass takes off when the spring reaches its natural length, with elongation or compression relative to the ground contact point considered under uphill conditions.

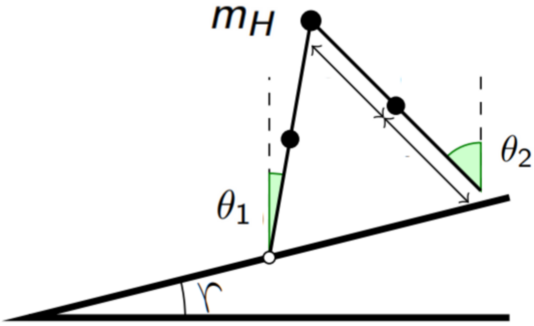

Figure 1. Spring mass uphill model physical model

4. Model Modification Section

Based on the original spring-mass running model code, this study aims to modify the model for uphill running scenarios. Firstly, an incline was incorporated, and the original model's initial vertical position y was set to 1 meter, which does not align with uphill running as the runner's "foot" should be at the spring's natural length upon landing. [12] Therefore, we adjusted the initial value of y to the spring's uncompressed length l_0. Additionally, the original model's horizontal velocity $\dot{x}$ was maintained at 5 m/s to simulate the runner's initial momentum uphill.

4.1. Adjustment of Initial Conditions

The dynamic equation EoM_air remained unchanged as aerial motion is not affected by the incline, being solely influenced by vertical gravity. However, the EoM_ground equation underwent significant modification to calculate the spring force and gravity components on the incline. We recalculated the spring's elongation and angle based on the incline angle, ensuring that the spring force is applied along the actual direction of spring extension. [13] Moreover, we introduced the influence of the incline angle on the gravity components, which is a critical factor in uphill running.

```
function output = EoM_ground(~,q)
   global g m k l0 slope_angle
   output = zeros(4,1);
   spring_length = sqrt(q(1)^2 + q(2)^2);
   theta = atan2(q(2), q(1));
   delta = theta - slope_angle;
   spring_extension = max(spring_length - l0, 0);
   F_spring = k * spring_extension;
   F_spring_x = F_spring * cos(theta);
```

```
    F_spring_y = F_spring * sin(theta);
    g_x = g * sin(slope_angle);
    g_y = g * cos(slope_angle);
    output(1) = q(3);
    output(2) = q(4);
    output(3) = (F_spring_x / m) - g_x;
    output(4) = (F_spring_y / m) - g_y;
end
```

4.2. Optimization of Event Handling Functions

Event functions were also adjusted to correctly handle different phases of the running cycle within the simulation. Specifically, the touch_down function was modified to detect when the runner's "foot" touches the slope surface, and the take_off function was used to determine when the leg's spring extends to its natural length, prompting the runner to jump. [14]

```
function [impact,terminate,direction] = touch_down(~,q)
    global l0 alpha0 slope_angle
    foot_x = q(1) + l0 * cos(alpha0) * cos(slope_angle);
    foot_y = q(2) - l0 * cos(alpha0) * sin(slope_angle);
    impact = foot_y;
    terminate = 1;
    direction = -1;
end
```

5. Simulation Results

Regrettably, despite modifications made to the code based on the physical model, issues encountered during the actual simulation led to discrepancies between the anticipated and the resulting images. Initially, the spring-mass model's motion on flat ground was expected to exhibit oscillatory behavior. However, the introduction of an incline significantly altered the motion characteristics of the model.

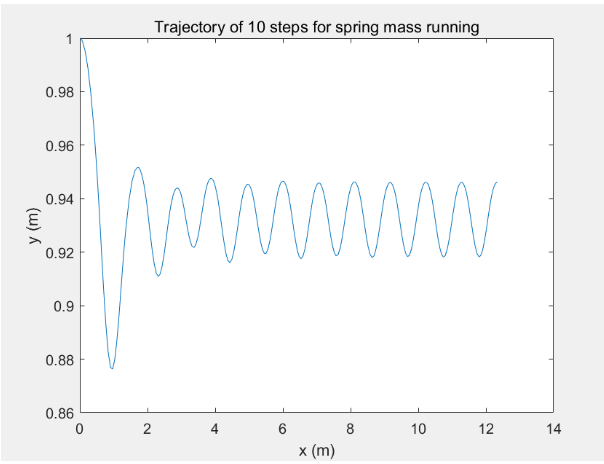

Figure 2. Original spring mass model image

Theoretically, the presence of an incline results in a component of gravity acting along the slope, subjecting the model to an additional downward force during uphill movement. This force not only accelerates the model's descent along the slope but may also alter the natural length and elasticity coefficient of the spring, thereby affecting the entire system's vibrational mode. Consequently, it can

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be anticipated that the model's uphill motion will display oscillatory characteristics distinct from those observed on flat terrain, with these characteristics becoming increasingly complex as the incline steepens.

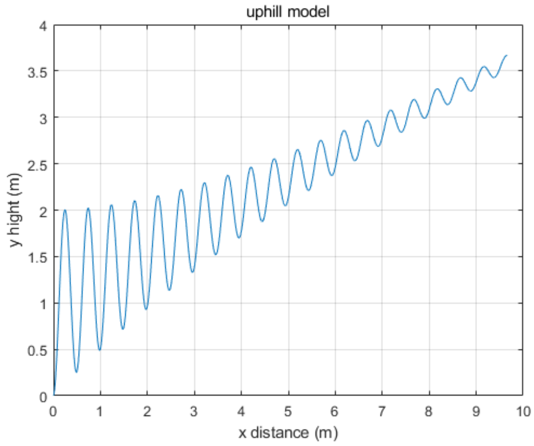

Figure 3. Images of Erroneous Execution 1

Although there are still some issues with the code execution in this study, we firmly believe that the goal pursued is correct. When dealing with the complex scenario of uphill running, despite increased challenges, this model can still operate as it does on flat ground, similar to the spring mass model. The changes in different parameters within the system have a predictable regular effect on its stability.

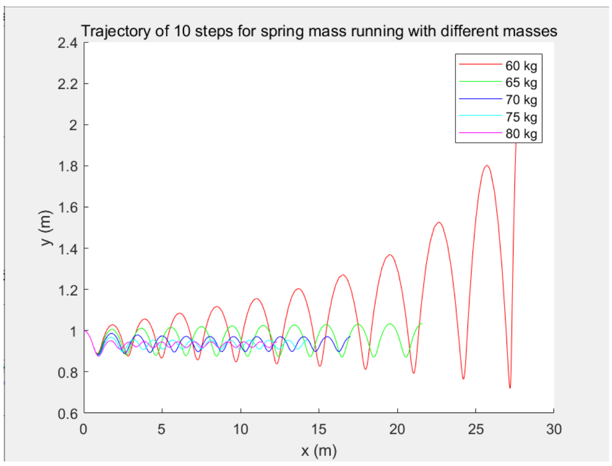

Figure 4. spring mass model in different mass

This section also includes images that illustrate errors in operation, specifically highlighting issues with image inaccuracies and the exponential increase in numerical values. These mistakes will offer important lessons for subsequent studies.

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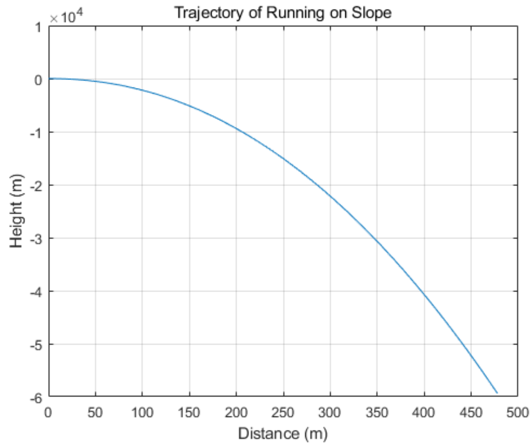

Figure 5. Images of Erroneous Execution 1

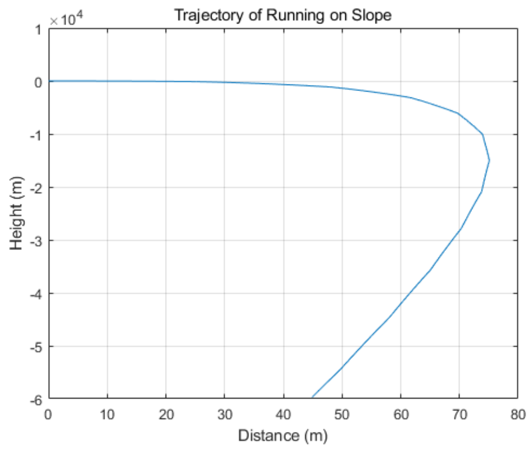

Figure 6. Images of Erroneous Execution 2

6. Physical and Mathematical Analysis

Given the issues encountered in the code execution of this study, we have decided to shift our perspective and continue our investigation into the spring mass model during uphill motion from the fundamental physical and mathematical aspects. This analysis is divided into two parts: static analysis and dynamic analysis. [15]

6.1. Static Analysis

On a slope with an angle θ, a mass m is attached to an ideal spring with a stiffness k. When the mass is subjected to the force of gravity, the spring will compress according to its stiffness and the applied force. After establishing the equilibrium condition for the spring-mass system, the spring compression x can be calculated using the well-known equilibrium force equation :

$$kx = mg\sin(\theta)$$

where k is the spring stiffness, m is the mass, g is the acceleration due to gravity, and θ is the slope angle. This approach is consistent with classical mechanics as described in Hibbeler (2016). [16] Solving this equation, we obtain the spring compression x:

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$$x = \frac{mg\sin(\theta)}{k}$$

As our goal is to analyze the motion at different slopes, we can set up a table to display the change in spring compression x at different slopes (θ) under the conditions $m = 80$ kg, $k = 23000$ N/m, $g = 9.81$ m/s^2:

Table 2. Spring compression x at different slopes.

Parameter	Description
Slope Angle (θ in degrees)	Spring Compression (x in meters)
5	2.97×10^{-3}
10	5.93×10^{-3}
15	8.83×10^{-3}
20	1.167×10^{-2}
25	1.44×10^{-2}

6.2. Dynamic Analysis

In the dynamic scenario, we consider the process of a runner moving on a slope. The dynamic model accounts for the conversion of the runner's kinetic energy and potential energy (both spring and gravitational). When the runner's "foot" contacts the ground, the spring compresses, storing energy; as the runner pushes off, this energy converts into kinetic and gravitational potential energy. [17]

Assumptions:

- The runner's velocity before ground contact $v_{\text{down}} = 5\,\text{m/s}$.
- Slope angle $\theta = 10°$.
- Mass $m = 80\,\text{kg}$.
- Spring stiffness $k = 23000\,\text{N/m}$.
- Gravitational acceleration $g = 9.81\,\text{m/s}^2$.

Calculating Spring Compression x: First, calculate the spring compression when it contacts the ground:

$$x = \frac{m \cdot g \cdot \sin(\theta)}{k} = \frac{80 \cdot 9.81 \cdot \sin(10°)}{23000}$$

Conservation of Energy: The conservation of energy law is used to calculate the velocity of the runner after pushing off the ground:

$$\frac{1}{2}mv_{\text{down}}^2 + mgh_{\text{down}} = \frac{1}{2}kx^2 + \frac{1}{2}mv_{\text{up}}^2 + mgh_{\text{up}}$$

Assuming h_{down} and h_{up} are based on the runner's horizontal distance d and slope angle θ, and simplifying:

$$\Delta h = d\sin(\theta)$$

For simplification, assuming $h_{\text{down}} \approx h_{\text{up}}$and neglecting their contribution, we get:

$$v_{\text{up}} = \sqrt{v_{\text{down}}^2 + \frac{2kx^2}{m} - 2g\Delta h}$$

This study delves into the application of the spring-mass model in uphill running through static and dynamic analyses. By calculating and analyzing the spring compression and post-push-off velocity at different slopes, we have arrived at the following key insights:

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(i) **Spring Compression Increases with Slope:** As the slope increases, the spring compression gradually increases. This phenomenon indicates that steeper slopes require the leg's spring system to provide greater force to overcome the gravitational component.

(ii) **Velocity After Push-off Changes with Slope:** The analysis shows a slight decrease in velocity after push-off as the slope increases, reflecting the increased energy needed to overcome gravity on slopes.

(iii) **Assistance to the Spring-Mass Model:** This study demonstrates the effectiveness and applicability of the spring-mass model in simulating the dynamics of uphill running, providing important insights for understanding human kinematics and developing efficient training methods.

(iv) **Application of the Model in Uphill Running Research:** The spring-mass model serves as a powerful tool for understanding how runners adapt to various inclines and offers a scientific basis for designing adapted training plans and running shoes.

7. Conclusion

This study, based on the spring-mass model, delves into the physical properties of the original code, extending and optimizing it with the aim of exploring its application during uphill running. Despite encountering some issues with the initial code implementation and subsequent revisions, the principal direction of the research remains sound. The paper conducts a thorough analysis of the model in both static and dynamic states, revealing that as the slope increases, so does the compression of the spring; concurrently, the velocity after push-off slightly decreases with an increase in slope. These findings highlight the additional load borne by the leg's spring system when facing varying slopes and the increased energy required to overcome gravity and changes in energy conversion efficiency on inclines. This report provides preliminary experimental insights for the research on the spring-mass model, laying a foundation for further in-depth studies.

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