

Quantum Ant Colony Algorithm for Solving the Traveling Salesman Problem: A Theoretical and Practical Analysis

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Abstract. The quantum ant colony algorithm (QACO) is explored as a solution to the traveling salesman problem (TSP), targeting inefficiencies such as slow convergence and local optima entrapment found in traditional ant colony optimization (ACO) methods. By integrating quantum computing elements, specifically quantum rotation gates and qubits, into the ACO framework, QACO demonstrates enhanced efficiency and faster convergence. This paper delves into the foundational principles of quantum computing and their application to refine ACO, effectively solving the TSP. An in-depth analysis is provided, showcasing QACO's advantages over traditional ACO through empirical examples. These examples illustrate QACO's potential to improve path planning for applications within the rapidly growing e-commerce and logistics sectors. The integration of quantum mechanisms facilitates a more dynamic pheromone update process, which significantly reduces the likelihood of premature convergence and enhances solution quality in complex optimization scenarios. This advancement indicates a promising direction for future research in algorithmic optimization in both theoretical and practical applications.

Keywords: TSP, Quantum computing, ACO.

1. Introduction

As industries such as e-commerce and logistics continue to expand, the importance of efficient path planning algorithms has become increasingly evident. Among the various problems in this field, the Traveling Salesman Problem (TSP) stands out due to its complexity and practical relevance. Traditionally, the ant colony optimization (ACO) method has been widely adopted for such tasks, drawing inspiration from the foraging behavior of ants to find optimal paths through probabilistic and pheromone-based techniques. However, despite its popularity, ACO suffers from limitations like slow convergence speeds and a tendency to get stuck in local optima, which can severely hinder its effectiveness in more complex or larger scale applications [1].

Current Research Status: To overcome the shortcomings of traditional ACO, advancements in quantum computing have paved the way for a more robust solution through the development of the Quantum Ant Colony Optimization (QACO). This novel approach integrates quantum mechanics with ACO, particularly employing quantum rotation gates and quantum bit (qubit) representations to enhance pheromone updating processes. This integration not only accelerates convergence but also provides a broader exploration capability, thereby increasing the likelihood of reaching the global optimum. Recent studies, such as those by Li Yueguang et al. and M. Garcia de Andoin, have demonstrated the potential

of QACO in solving not only the TSP but also other complex combinatorial optimization problems with more efficiency than its classical counterpart [2-4].

Research Content of This Paper: This paper delves into the foundational principles of quantum computing and explores its integration with the traditional ACO framework to address the TSP more effectively. Through theoretical analysis and empirical examples, the paper evaluates the performance enhancements that QACO offers over traditional ACO. It highlights the dynamic pheromone update process enabled by quantum mechanisms, which significantly mitigates issues of premature convergence and improves the quality of the solutions found. Furthermore, the practical applications of QACO in fields like e-commerce and logistics are discussed, illustrating its potential to revolutionize path planning tasks in various industries. By providing a comprehensive review and analysis of QACO, this study contributes to ongoing efforts to optimize complex problem-solving techniques in operational research and computer science.

2. Relevant theories

2.1. Traditional ant colony algorithms

Ant Colony Optimization (ACO) is a nature-inspired metaheuristic frequently used to address various optimization challenges [5].

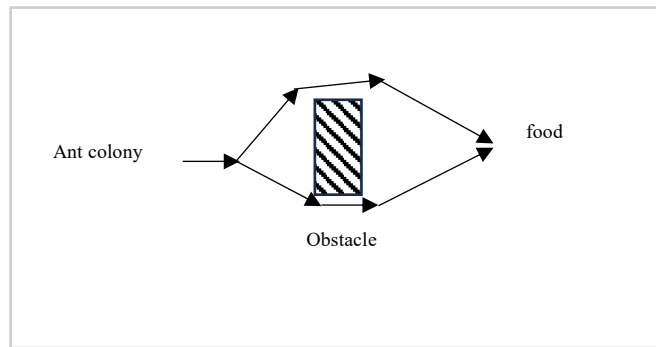


Figure 1. ACO schematic diagram (Photo credit: Original).

As illustrated in Figure 1, Ants start from the starting point. When they encounter obstacles, they will try to find a relatively short path to cross the obstacles and leave pheromones. Since the path below is relatively short, more ants will choose the path below, and thus leave more pheromones on this path. Subsequent ants are more inclined to select the path below [6].

Ants start from the starting point and choose the next city based on a probability formula as follows formula (1) [7]:

$$P_{ij}^k = \frac{(\tau_{ij})^\alpha (\eta_{ij})^\beta}{\sum_{l \in J^k} (\tau_{il})^\alpha (\eta_{il})^\beta} \quad (1)$$

Where:

τ_{ij} represents the pheromone concentration between city i and city j .

η_{ij} is the heuristic function, which indicates the attractiveness of the path from i to j and is often inversely proportional to the distance:

$$\eta_{ij} = \frac{1}{d_{ij}} \quad (2)$$

Parameters α and β determine the significance of pheromones and the heuristic function, respectively.

After all ants have completed one round of path construction, Pheromones on each path are updated according to this formula: (3) [8]:

$$\tau_{ij} \leftarrow (1 - \rho)\tau_{ij} + \sum_{k=1}^m \Delta \tau_{ij}^k \quad (3)$$

Where:

ρ represents the pheromone evaporation rate.

$\Delta \tau_{ij}^k$ is the pheromone increment added by ant k on the path from city i to city j , defined as:

$$\Delta \tau_{ij}^k = \frac{Q}{L_k} \quad (4)$$

Q is a constant, and L_k represents the length of the path completed by ant k .

2.2. The traveling salesman problem

The Traveling Salesman Problem (TSP) is an NP-hard problem, commonly used as a benchmark to analyze the performance of metaheuristic algorithms [9]. It is a classic problem in operations research and computer science. It involves a salesman who must iterate over a given number of cities, visiting each city exactly once before returning to the starting point. The goal is to determine the shortest possible route that minimizes the total distance or cost of the trip [10].

2.3. Quantum ant colony algorithm

QACO is an innovative optimization algorithm that combines traditional ACO with quantum computing. The core of QACO is to introduce quantum rotation gates in quantum computing into the ACO, use quantum bit encoding to generate quantum pheromones, and update pheromones through quantum rotation gates [11]. The algorithm flow of QACO is shown in the figure 2 [12]:

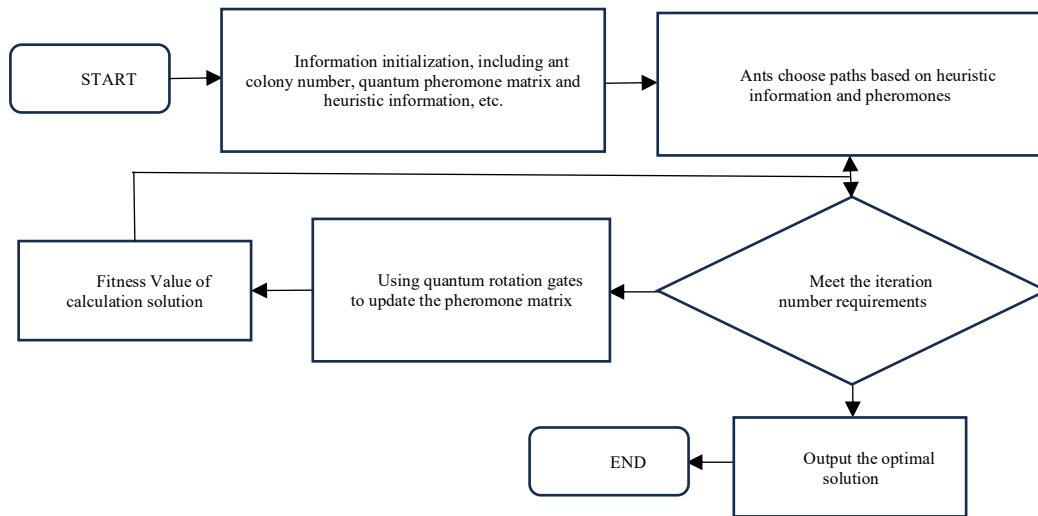


Figure 2. Implementation process of QACO (Photo credit: Original).

3. System analysis and applications

3.1. Mathematical model

3.1.1. Qubit encoding. Compared to traditional bits which represent only two states 0 and 1, qubits consist of two possible states $|0\rangle$ and $|1\rangle$. A qubit $|\psi\rangle$ can be represented as:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \quad (5)$$

α and β are the probability amplitudes of the quantum state $|\psi\rangle$, respectively, and α and β are complex numbers that satisfy formula (6) [13]:

$$\alpha^2 + \beta^2 = 1 \quad (6)$$

3.1.2. Quantum rotation gate. Different from the method of updating pheromones through formula mentioned above, quantum ant colonies update quantum pheromones through quantum revolving gates. The method is as follows:

$$\begin{bmatrix} \alpha_{t+1} \\ \beta_{t+1} \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \alpha_t \\ \beta_t \end{bmatrix} \quad (7)$$

Where:

α_t and β_t are the quantum state coefficients for path (i, j) in iteration t.

θ is the rotation angle that adjusts the pheromone update, directing the search towards the optimal solution.

The quantum rotation gate allows for dynamic adjustment of the pheromone concentration.

The principle diagram of the quantum revolving door is shown in Figure 3:

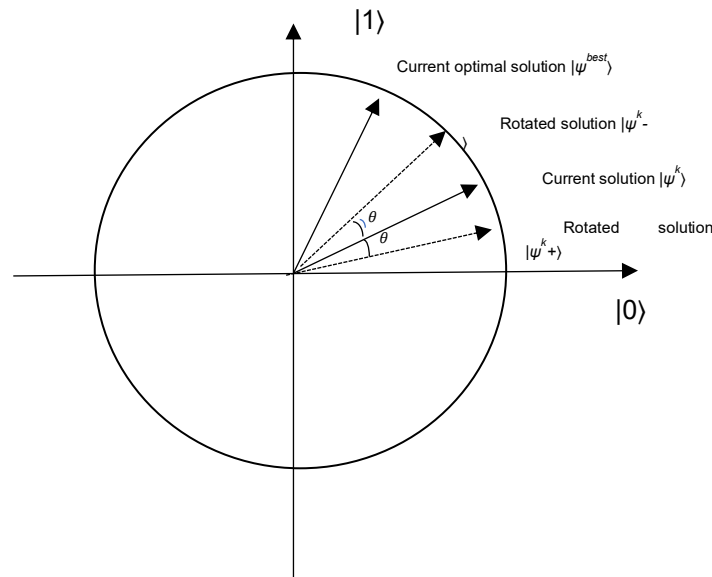


Figure 3. Principle diagram of quantum rotation gate (Photo credit: Original).

As shown in Figure3, the ant currently searching is the kth ant, the solution that needs to be rotated is $|\psi^k\rangle$, there are two rotation directions can be selected. One rotation direction approaches the optimal solution $|\psi^{best}\rangle$, while the other moves away from it. But the ideal situation is that all solutions approach the optimal solution, This enhances the speed of finding the optimal solution. Therefore, the choice of rotation angle θ is very important [14].

The rotation angle is selected according to formula (8):

$$\theta_{ij} = \Delta\theta_{ij} \times \text{sgn}(x) \quad (8)$$

$$x = (\alpha_{ij} \times \beta_{ij})(L^k - L^{best})(R^k[i, j] - R^{best}[i, j]) \quad (9)$$

In formula (9), x represents the magnitude of the rotation angle. $\text{sgn}(x)$ is a sign function that determines the direction of the rotation angle. x is the independent variable of the symbolic function. L^k and L^{best} represent the path length explored by current ant and the optimal path length searched by all ants, respectively. $R^k[i, j]$ and $R^{best}[i, j]$ are the path length searched by current ant and the optimal

path matrix (i, j) 's value that searched by all ants, respectively. The rotation angles used in different situations are different and need to be selected according to the actual situation.

3.2. Algorithm efficiency evaluation

To assess the effectiveness of the QACO, and its efficiency advantage over the traditional ACO, Li Xiang et al. designed a simulation experiment in the python 3.7 environment and compared the results. The experimental parameters are established as outlined in Table 1 [15]:

Table 1. Experimental parameter settings.

parameter	Ant colony size	Number of nodes	Pheromone Inducing Factor α	Expectation Heuristic Factor β	Pheromone volatility factor ρ	Maximum number of iterations
	200	50	1.0	0.5	0.5	1000

After taking the average value of multiple experiments, compared it with the classic ant colony algorithm. The comparison results are established as outlined in Table 2:

Table 2. Comparison experiment results.

Algorithm	Known optimal solution	Average evolutionary generations	Optimal solution	Worst solution	Average value
ACO	3823	74.1	3825	3826	3825.1
QACO	3823	44.9	3824	3910	3859.6

Table 2 shows that the QACO is significantly better than the classical ACO in global optimization and convergence speed.

3.3. Applications of QACO

Owing to its superior performance and faster convergence compared to traditional ACO, the quantum ant colony algorithm is widely applied to various optimization problems. For example, using QACO algorithms to optimize distributed database models and reduce the expense of query connections in distributed databases [16]. Using QACO algorithms to solve the two-stage permutation flow shop problem with batch processing machines to improve the work efficiency of large-scale operations in practical work [17]. The QACO can also be used in the field of wireless sensor networks to find low-latency paths between source nodes and target nodes and improve network utilization [18]. In addition, QACO have also been widely used in traffic path planning, supply chain management, intelligent manufacturing and other fields.

4. Challenges

Although QACO has been widely used in addressing optimization problems and practical applications, it still has the following limitations that need to be addressed [19]:

The efficiency of pheromone coding is not high, and the quantum gate rotation strategy is not adaptable enough.

The core logic of the QACO is to introduce the concepts of quantum coding and quantum revolving door into the traditional ant colony algorithm to improve its performance. However, the QACO itself is still just an improved ACO, and the inherent defects of the ACO itself still exist.

Although the quantum ant colony algorithm shows good results in simple optimization problems, if the quantum bits are converted into classical bits through simple projection measurement and transformation of the solution space, the algorithm will not be able to show its superiority in solving multi-parameter optimization and high-precision calculation problems.

At present, the basic mathematical research on quantum computing is still incomplete, especially in the aspects of parameter design and convergence analysis, which requires more attention from scholars.

5. Conclusion

This study has introduced and evaluated the Quantum Ant Colony Optimization (QACO) algorithm as an advanced solution to the Traveling Salesman Problem (TSP), a longstanding and complex issue in operations research and computer science. By integrating quantum computing elements, specifically quantum rotation gates and qubits, into the traditional Ant Colony Optimization (ACO) framework, the QACO demonstrates significantly enhanced efficiency and convergence speeds. Through a detailed analysis involving empirical examples, the paper highlighted the improved dynamic pheromone update process enabled by the quantum mechanisms. This process effectively addresses the shortcomings of slow convergence and frequent entrapment in local optima that are often observed in the traditional ACO approach. The quantum-enhanced algorithm not only streamlines the computation process but also expands the possibilities for more complex problem solving in various industrial applications. Looking forward, the potential of QACO in broader applications suggests a rich area for future research. The current study's success invites further exploration into other combinatorial and high-dimensional optimization problems where QACO's capabilities could be similarly beneficial. Future studies should aim to refine the quantum pheromone update mechanisms to ensure even greater efficiency and applicability across diverse scenarios. Additionally, considering the rapid advancements in quantum computing technology, ongoing research will also need to focus on integrating these developments to continually enhance the QACO framework. Investigating the adaptation of QACO to other challenging domains such as network design, logistics optimization, and artificial intelligence could provide valuable insights and contribute significantly to the fields of quantum computing and optimization algorithms.

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