DOI: 10.54254/2755-2721/105/2024.TJ17855

Analytical Solution of Electromagnetic Force Lines in Rectangular Wave Guides for Transverse Electric Waves

Bixiao Fei

School of Physics, Xidian University, Xi'an, China

fbx1608940897@outlook.com

Abstract. The analytical solution of the electromagnetic field lines was derived, providing a foundation for generalizing conclusions and forming a coherent set of equations to describe the dynamics of electromagnetic waves. By applying Maxwell's equations, this paper analyzed the rectangular waveguide model, focusing on the guiding effect of the passive region on the propagation of electromagnetic waves. Additionally, the Helmholtz equation was used to describe the behavior of electromagnetic fields within the passive guided wave system. Through the analytical solution of the field lines, the distribution of electromagnetic forces could be determined through integration calculations. To enhance the understanding of these concepts, this paper utilized Python programming to simulate electromagnetic force lines, providing a visual representation of the discussed theories. This simulation not only deepened the understanding of electromagnetic principles but also demonstrated their importance in practical applications, contributing to the field of electromagnetism research. The results of the study emphasize the significance of analytical solutions in understanding electromagnetic phenomena and provide valuable insights for future research in related fields.

Keywords: Rectangular waveguide, electromagnetic field lines, analytical solution, maxwell's equations, helmholtz equation.

1. Introduction

Maxwell's equations, formulated by the British physicist James Clerk Maxwell, are a set of equations summarizing the fundamental laws of electromagnetic fields and are considered one of the most significant scientific achievements of 19th-century physics. While Maxwell is credited with this major breakthrough, the creation of these equations was also an inevitable result of scientific progress at the time [1,2,3]. With the successive establishment of Coulomb's Law, Biot-Savart Law, Ampere's Law, Ohm's Law, and Faraday's Law of Induction, it became clear that localized electromagnetic phenomena had been discovered, indicating that the conditions for forming a universal electromagnetic law that could unify various electromagnetic phenomena were ripe [4]. On December 8, 1864, Maxwell presented his summary of electromagnetic theory in a paper titled "A Dynamical Theory of the Electromagnetic Field" at the Royal Society of London, which was published in the Philosophical Transactions of the Royal Society in 1865. The third section of this paper, titled "General Equations of the Electromagnetic Field," was where Maxwell's equations were first introduced. These equations are the core and main achievement of the paper.

Radio communication is a common method of communication in daily life. It utilizes electromagnetic waves to transmit information through space, where modulated electrical signals are loaded onto radio waves, transmitted across space and ground, and finally received at the other end. Radio communication relies on the principles of electromagnetic waves to achieve wireless communication [5].

Maxwell's equations allowed him to successfully predict that light is a form of electromagnetic wave, possessing wave-like properties [6]. Therefore, light also exhibits electromagnetic wave phenomena such as refraction, reflection, scattering, diffraction, and absorption. This prediction promoted advancements in spectroscopic analysis, diffraction imaging, and atmospheric pollution measurements. Maxwell's equations provided the first complete description of electromagnetic theory, demonstrating the conversion between electric and magnetic fields. They have wide applications in everyday life and have profoundly influenced subsequent physical research. The significance of these equations for physics cannot be overstated [7,8].

This paper utilizes Maxwell's equations, the Helmholtz equation, and the equation for magnetic force lines to analyze and plot the distribution of electric and magnetic force lines in a rectangular waveguide, thereby providing a comprehensive analytical solution for electromagnetic force lines. In this paper, the analytical solution agrees with theoretical results when taken to the limit. The analysis indicates that the current study on the distribution of electric and magnetic field lines in rectangular waveguides, aimed at improving the analytical solution of electromagnetic field lines, is not thorough enough and requires further research and analysis.

2. Methods

2.1. Maxwell's equations and the Helmholtz equation

The position of Maxwell's equations in electromagnetism is analogous to Newton's laws of motion in classical mechanics. Electromagnetic theory, with Maxwell's equations at its core, is one of the proudest achievements of classical physics, revealing the perfect unification of electromagnetic interactions.

For guided electromagnetic waves, the Maxwell equations for a passive region are:

$$\nabla\times\boldsymbol{H}=j\omega\varepsilon\boldsymbol{E} \tag{1}$$

$$\nabla\times\boldsymbol{E}=-j\omega\mu\boldsymbol{H} \tag{2}$$

The Helmholtz equation, which governs the electromagnetic fields in a passive guided wave system, is:

$$\nabla^2\boldsymbol{E}+k^2\boldsymbol{E}=0 \tag{3}$$

$$\nabla^2\boldsymbol{H}+k^2\boldsymbol{H}=0 \tag{4}$$

2.2. Using the Helmholtz equation to solve for the various field components of the TE wave

For TE waves, $\boldsymbol{E}_z=0$, which is a property of transverse electric waves. The equation satisfying the boundary conditions of the guided wave system is:

$$\nabla_t^2 H_z + k_c^2 H_z = 0 \tag{5}$$

$$\nabla_t^2= \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}, k_c^2 = k^2 - k_z^2 \text{。} \tag{6}$$

After solving the Helmholtz equations, we obtain:

$$H_z(x,y,z) = H_0\cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-jk_z z} \tag{7}$$

The solution process is as follows:

$$H(x,y,z) = H_0(x,y)e^{-jk_z z} \tag{8}$$

DOI: 10.54254/2755-2721/105/2024.TJ17855

Let

$$H_0(x,y) = X(x)Y(y) \tag{9}$$

Therefore

$$\frac{X''(x)}{X(x)} + \frac{Y''(y)}{Y(y)} + k_c^2 = 0 \tag{10}$$

We make $k_c^2 = k_x^2 + k_y^2$, and

$$\frac{X''(x)}{X(x)} + k_x^2 = 0 \tag{11}$$

$$\frac{Y''(y)}{Y(y)} + k_y^2 = 0 \tag{12}$$

Finally we get

$$X(x) = c_1 \cos(k_x x) + c_2 \sin(k_x x) \tag{13}$$

$$Y(y) = c_3 \cos(k_y y) + c_4 \sin(k_y y) \tag{14}$$

The general solution of H_z is:

$$H_z(x,y,z) = (c_1 \cos(k_x x) + c_2 \sin(k_x x))(c_3 \cos(k_y y) + c_4 \sin(k_y y))e^{-jk_z z} \tag{15}$$

Based on the values of x and y on the waveguide wall H_z, it can be solved that

$$k_c^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \tag{16}$$

$$H_z(x,y,z) = H_0 \cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-jk_z z} \tag{17}$$

$$k_z = k\sqrt{1-\left(\frac{f_c}{f}\right)^2} = k\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2} \tag{18}$$

$$f_c = \frac{k_c}{2\pi\sqrt{\varepsilon\mu}} \tag{19}$$

In summary, due to the source-free Maxwell's equations, the various field components of the TE wave and H_z can be expressed as follows:

$$H_x(x,y,z) = j\frac{k_z}{k_c^2}\frac{m\pi}{a}H_0 \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-jk_z z} \tag{20}$$

$$H_y(x,y,z) = j\frac{k_z}{k_c^2}\frac{m\pi}{a}H_0 \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-jk_z z} \tag{21}$$

$$E_y(x,y,z) = j\frac{\omega\mu}{k_c^2}\frac{m\pi}{a}H_0 \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-jk_z z} \tag{22}$$

$$E_x(x,y,z) = j\frac{\omega\mu}{k_c^2}\frac{m\pi}{a}H_0 \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-jk_z z} \tag{23}$$

2.3. Discussion on a rectangular waveguide problem

For a rectangular waveguide with cross-sectional dimensions of 22.86 mm × 10.16 mm and air (vacuum) as the medium, we need to find the instantaneous values of the electric and magnetic field components of TE_{01}.

First, according to the source-free Maxwell's equations in the guided electromagnetic wave region

$$\nabla \times \boldsymbol{H} = j\omega\varepsilon\boldsymbol{E} \tag{24}$$

$$\nabla \times \boldsymbol{E} = -j\omega\mu\boldsymbol{H} \tag{25}$$

Next, based on the properties of the TE wave when m=0 and n=1, and similar to the field components in the method, the corresponding time-domain instantaneous form can be easily obtained

$$E_x(x,y,z) = A_1 \cos(k_x x)\sin(k_y y)\, e^{i(k_z z-\omega t)} \tag{26}$$

$$E_y(x,y,z) = A_2 \sin(k_x x)\cos(k_y y)\, e^{i(k_z z-\omega t)} \tag{27}$$

$$H_x(x,y,z) = A_3 \sin(k_x x)\cos(k_y y)\, e^{i(k_z z-\omega t)} \tag{28}$$

$$H_y(x,y,z) = A_4 \cos(k_x x)\sin(k_y y)\, e^{i(k_z z-\omega t)} \tag{29}$$

$$H_z(x,y,z) = A_5 \cos(k_x x)\cos(k_y y)\, e^{i(kz-\omega t)} \tag{30}$$

$$k_y A_1 - k_x A_2 = -i\omega\mu A_5 \tag{31}$$

$$ikA_1 = i\omega\mu A_4 \tag{32}$$

$$ikA_2 = -i\omega\mu A_3 \tag{33}$$

$$-k_x A_4 + k_y A_3 = 0 \tag{34}$$

$$k_x = \frac{m\pi}{a} \tag{35}$$

$$k_y = \frac{n\pi}{b} \tag{36}$$

$$k = \sqrt{\frac{4\pi^2}{\lambda^2} - k_x^2 - k_y^2} \tag{37}$$

If $n \neq 0$,

$$A_3 = \frac{m}{n}\frac{b}{a}H_0 \tag{38}$$

$$A_1 = c\mu H_0 \tag{39}$$

$$A_2 = -c\mu\frac{m}{n}\frac{b}{a}H_0 \tag{40}$$

$$A_5 = \frac{i}{k}\left(\frac{n}{b} + \frac{m^2 b}{na^2}\right)\pi H_0 \tag{41}$$

In which

$$A_4 = H_0 \tag{42}$$

Let n=1 and m=0, now calculate the equations for the electric and magnetic field lines based on the force line equations

$$\frac{dx}{ds} = u_x(x,y,z) \tag{43}$$

$$\frac{dy}{ds} = u_y(x,y,z) \tag{44}$$

$$\frac{dz}{ds} = u_z(x,y,z) \tag{45}$$

For TE_{01}

$$H_x = 0 \tag{46}$$

$$H_y = A_4 sin k_y y cos(kz-\omega t) \tag{47}$$

$$H_z = -|A_5| cos k_y y sin(kz-\omega t) \tag{48}$$

DOI: 10.54254/2755-2721/105/2024.TJ17855

So we get

$$\frac{dx}{ds} = 0 \tag{49}$$

$$\frac{dy}{ds} = A_4 sink_y ycos(kz - \omega t) \tag{50}$$

$$\frac{dz}{ds} = -|A_5| cosk_y ysin(kz - \omega t) \tag{51}$$

Here, by dividing $\frac{dy}{ds}$ by $\frac{dz}{ds}$, the introduced intermediate variables are eliminated, resulting in

$$\frac{dz}{dy} = \frac{|A_5|}{A_4} tank_y ycot(kz - \omega t) \tag{52}$$

By placing $\frac{dy}{ds}$ and $\frac{dz}{ds}$ on both sides of the equation and integrating, the expression for the magnetic field lines can be obtained

$$-\frac{|A_5|k}{A_4 k_y} ln\left|cosk_y\right| = ln|\cos(kz - \omega t)| + C \tag{53}$$

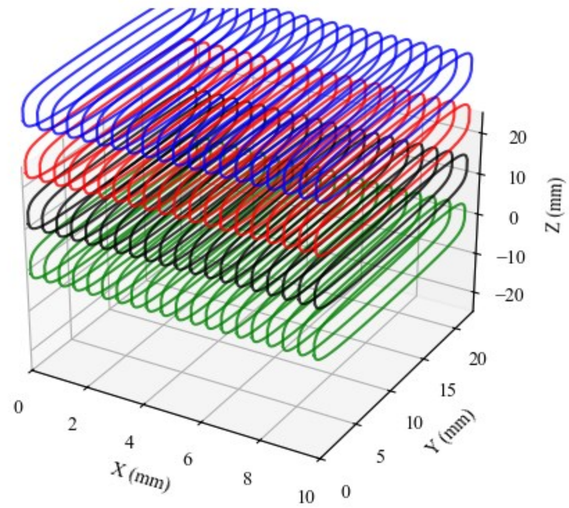
The electric field lines are similar, as it is a TE mode field with m=0 and n=1. Therefore, the entire electric field line is

$$E_x = A_1 cosk_x ysink_y ye^{i(kz - \omega t)} \tag{54}$$

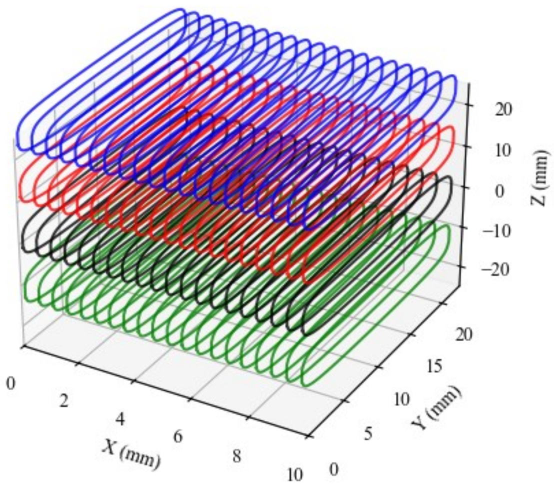
And it will appear as straight lines in space. By using m=0,n=1, we can plot the electric and magnetic field lines.

3. Simulation results

Using Python code, we simulated the analytical solution for the electromagnetic force lines. Figure 1 shows the simulation results for the electromagnetic force lines, where time is expressed in units of one electromagnetic wave period. Figures 1(a)-(d) display the changes in the magnetic force lines over time. It can be seen that the magnetic force lines are entirely circular, and the shape of the magnetic force line loops does not change over time, demonstrating the propagation characteristics of a lossless waveguide.

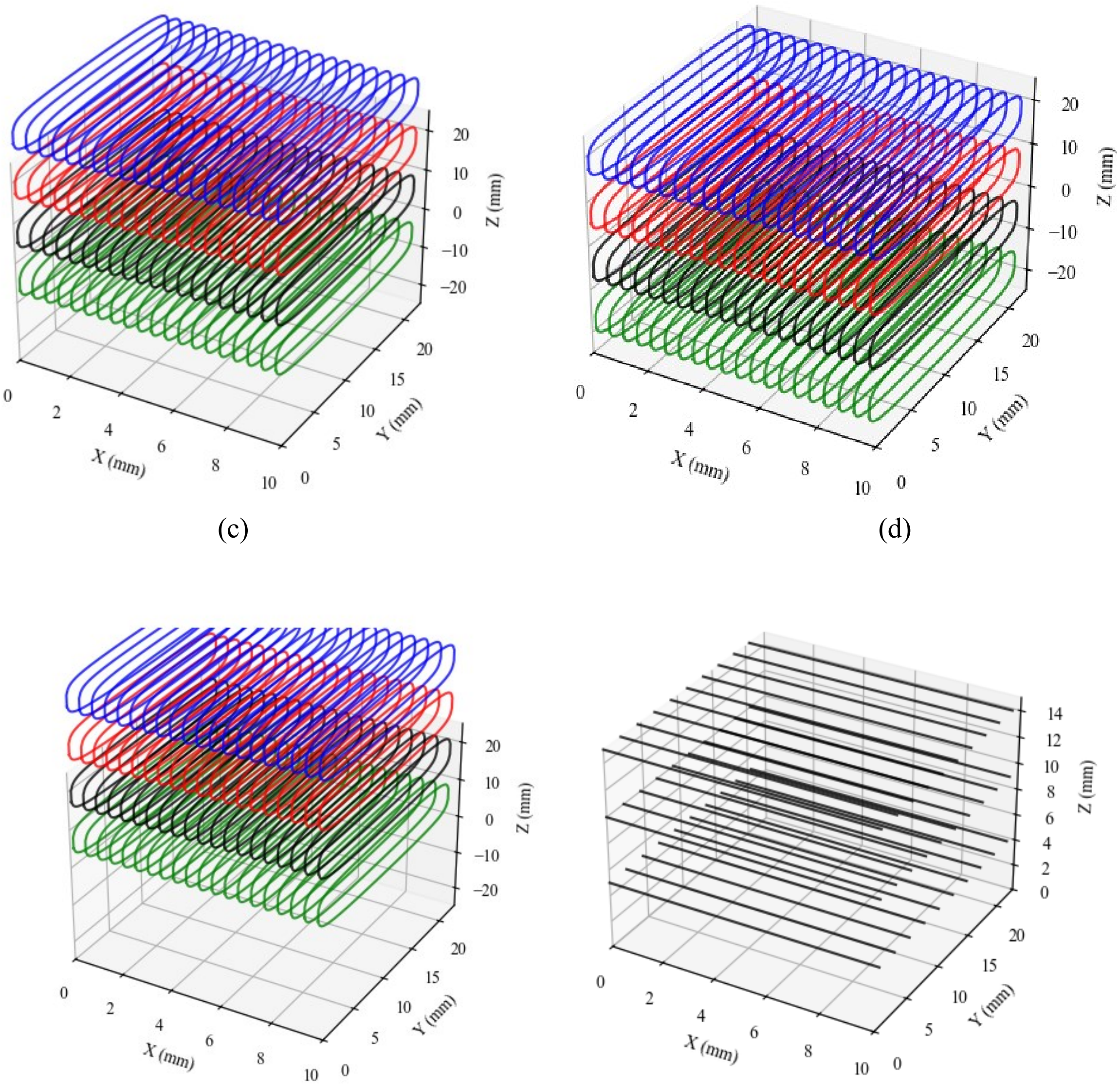


(a)



(b)

DOI: 10.54254/2755-2721/105/2024.TJ17855



(c) (d)

(e) (f)

Figure 1. The simulation results ()Photo/Picture credit : Original

Figure 1 (e) and (f) show the electric field lines in the waveguide. Since the propagating electromagnetic wave is in the TE01 mode, the wave vector of the electric field in the direction parallel to the waveguide is zero, so the electric field lines do not move with time. Additionally, there is only one mode of electric field lines in the cross-section of the waveguide, which points in a single direction. As a result, the electric field lines are always straight and perpendicular to the surface of the rectangular conductor.

4. Conclusion

This study conducted an in-depth investigation of the rectangular waveguide model for guided electromagnetic waves in electromagnetics. By solving the Helmholtz equation, the analytical method was used to calculate the components of the field, and the time-domain instantaneous forms of the field components were derived for specific cases. In solving the force line equations, the definition of force lines was applied, and the method of elimination was cleverly used to derive the equation for the

DOI: 10.54254/2755-2721/105/2024.TJ17855

magnetic force lines, laying the foundation for simulating the force line diagrams. Since the electric field lines are straight, they are simpler to handle compared to the closed-loop magnetic field lines.

Furthermore, this study explored the behavior characteristics of electromagnetic fields under different conditions, particularly the propagation modes in passive regions. Through detailed mathematical derivations and theoretical analyses, we not only validated the effectiveness of existing theories but also revealed some new phenomena. These findings are significant for understanding and optimizing the propagation of electromagnetic waves in waveguides. Additionally, the simulation of electromagnetic force lines using Python programming provided intuitive visual results, further enhancing the understanding of theoretical principles and demonstrating their potential value in practical applications. The research findings underscore the importance of analytical methods in electromagnetic research and provide valuable references for future studies in related fields.

References

[1] Mousavian, A., Anguelov, D., Flynn, J., & Kosecka, J. (2017). 3D bounding box estimation using deep learning and geometry. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition (pp. 7074-7082).

[2] Zhang, C., & Huang, C. (2020). Understanding complex waveguides with effective medium theory and electromagnetic simulations. Journal of Applied Physics, 127(12), 123307. DOI: 10.1063/1.5140256.

[3] Wu, Z., & Zeng, W. (2019). A comprehensive review on electromagnetic modeling of planar structures. IEEE Transactions on Microwave Theory and Techniques, 67(2), 705-719. DOI: 10.1109/TMTT.2018.2887140.

[4] Lee, J.H., & Park, H.J. (2018). Numerical solutions to Maxwell’s equations for waveguide structures using finite element method. Journal of Electromagnetic Waves and Applications, 32(5), 643-651. DOI: 10.1080/09205071.2018.1433490.

[5] Bock, W.J., & Bar, D.J. (2021). Advances in 3D electromagnetic field modeling for photonic devices. Optics Express, 29(8), 12345-12360. DOI: 10.1364/OE.429153.

[6] Feng, E. (2016). Electromagnetic Fields and Waves. Xi'an: Xi'an Jiaotong University Press, pp. 69-75.

[7] Tian, S., & Ding, P. (2015). Research and Discussion on the Teaching of Maxwell's Equations. Weekly Exam, (67), 159-160.

[8] Zhu, F. (2018). A Brief Overview and Application of Maxwell's Equations. China New Telecommunications, 20(15), 230-231.