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Research on UAV formation positioning based on passive positioning

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Abstract. In this paper, passive positioning technology in UAV formation is studied, and how to achieve effective positioning through signal transmission when UAV deviates is analyzed. The research shows that when four UAVs with transmitting signals are deployed, the positioning accuracy of feature point FY08 is very high, indicating that the positioning of UAVs can be effectively realized by setting four signal transmitting UAVs (including two new UAVs) when the location is unknown.In a typical formation of ten drones, nine are evenly distributed around the circumference, with the central drone being the FY00. By building a passive reception model, this paper discusses how to use the signals of FY00 and two other UAVs for positioning. When the drone deviates and receives signals from FY00, FY01 and another unknown drone, up to three drones are required to transmit signals to ensure effective positioning.The positioning model is derived by cosine law, and the linear distance between the deviating drone and the center drone is solved by geometry and sine theorem. Finally, the advantages of four signal transmitting UAVs in positioning accuracy were verified by measuring deviation and variance, and a conclusion was drawn that effective positioning of UAVs was achieved by signal transmitting under the condition of deviation.

Keywords: Formation flight, Sine law, Cosine law, Least squares method.

1. Introduction

Recent advances in pure azimuth passive localization have gained prominence in UAV formation flying due to their high precision and strong interference resistance. Xie et al. proposed a method using a polar coordinate system and the sine law to correct UAV positional deviations, enabling precise positioning in circular and conical formations [1]. Tang et al. developed a model integrating analytical geometry and an enhanced damped Gauss-Newton algorithm for real-time coordinate correction through an error-based iteration [2]. Zhu et al., applying dynamic programming, explored optimal UAV configurations but acknowledged challenges in complex electromagnetic environments [3]. Zhang et al.'s experiments validated the effectiveness of triangulation and polar coordinates in achieving uniform "circular" formation among nine UAVs by adjusting positions and using specific UAVs as signal sources [4]. This research underscores the suitability of pure azimuth passive localization in UAV formations. Designating a few UAVs as signal transmitters reduces external electromagnetic emissions, with the majority receiving signals for azimuth information, achieving precise positioning.

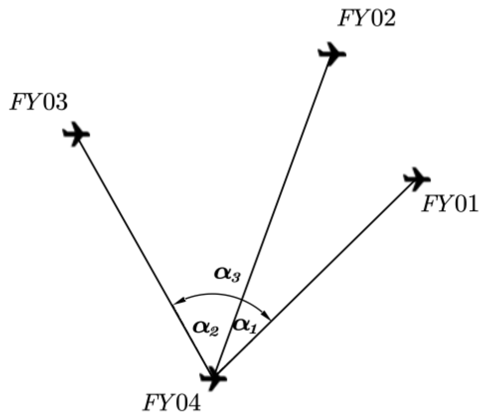

Figure 1. Schematic Diagram of Azimuth Information Received by UAVs.

In a typical UAV formation of ten drones, nine labeled FY01 to FY09 are evenly distributed along the circle's circumference, and one labeled FY00 is at the center[5]. All drones maintain the same altitude. This paper explores localizing drones using signals from FY00 and two others, and how additional signals can ensure accurate localization when drones deviate[6].This study establishes a passive reception model to determine deviated drone positions. When a drone deviates and receives signals from FY00, FY01, and another unknown drone, the question is how many additional drones must emit signals for effective localization. This scheme helps nine drones return to a uniform distribution by having up to three drones (including the central one) emit signals each time. This method can be applied to other formations, like conical ones, for position adjustment using pure azimuth passive localization[7].

2. UAV Formation Flight Positioning Model

2.1. UAV Positioning Model Based on the Sine Law

When establishing a positioning model for UAVs that passively receive signals, it is assumed that the positions of the signal-emitting UAVs are known and without deviation. In a circular formation flight maintaining constant height, the central UAV (FY00) acts as the pole in a polar coordinate system. The line connecting FY00 and another UAV (FY01) defines the positive direction, as shown in Figure 2[8].

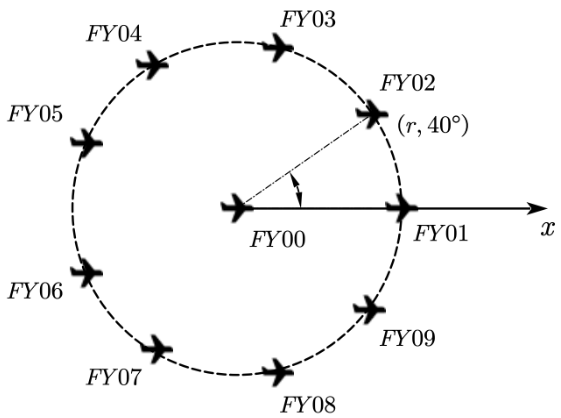

Figure 2. Drone formation flight positioning polar coordinate system.

In a circular UAV formation, positional deviations place the UAV on the line between its standard position and the formation's center, occurring either outside or inside the standard position[9].

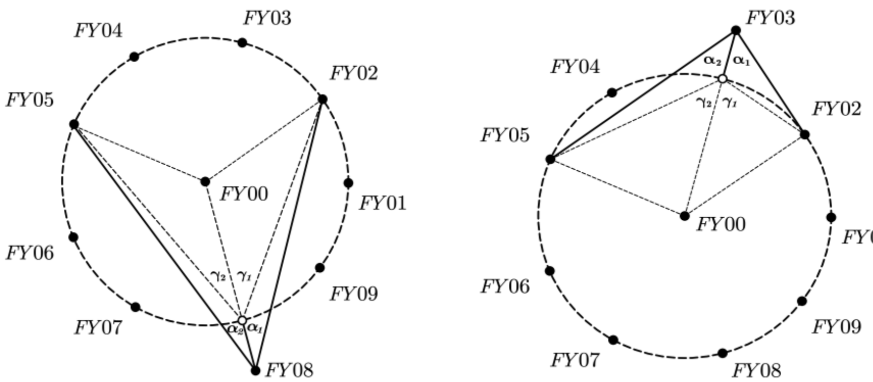

Figure 3. Drone with extroversion horizontal deviation.

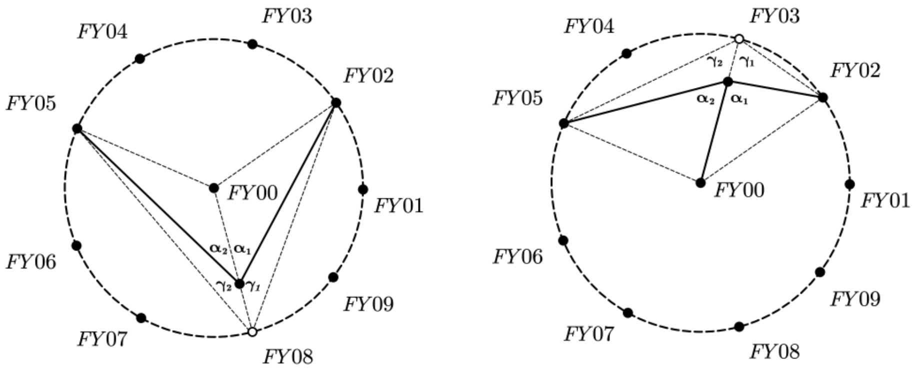

Figure 4. Drone with inward horizontal deviation.

The Angle measured between the UAV with horizontal deviation and the two signal launching UAVs is α_1 and α_2 respectively[10]. Meanwhile, since UAVs should meet uniform distribution in formation flight, the standard positions of each UAVs are known, that is, the Angle between the signal launching UAVs with any number and the standard positions of other UAVs is a fixed value, denoted as γ_1 and γ_2.

After the measurement of each Angle, the linear distance between the UAV with horizontal deviation and the pole (centered UAV) is denoted as *d*.

According to the sine theorem:

$$\frac{d}{\sin(2\gamma_1 - \alpha_1)} = \frac{r}{\sin\alpha_1} \tag{1}$$

$$d = \frac{\mathrm{r}\sin(2\gamma_1 - \alpha_1)}{\sin\alpha_1} \tag{2}$$

The polar coordinates of the deviation UAV are: $\left(\frac{r\sin(2\gamma_1-\alpha_1)}{\sin\alpha_1}, 40(n-1)^\circ\right)$

The Angle formed by the connection between the UAV receiving signals, the UAV in the center of the circle and the other two UAV transmitting signals is respectively β_1, β_2; the distance between the UAV receiving signals and the center of the circle is *d*; the Angle of the connection between the receiving m_1 and the center of the circle is α_1; the Angle of the connection between the receiving m_2 and the center of the circle is α_2 [11]. The connection Angle between m_1 and m_2 of the two transmitting signals receiving UAVs located on the circular formation is α_3.

When $\alpha_3 = \alpha_1 + \alpha_2$ and the short arc formed by the other two transmitting UAVs does not include FY01. Since the 9 UAVs are evenly distributed in the same circle, the Angle degree θ between any 2 unbiased UAVs and the center of the circle is:

$$\theta = \frac{360}{9}(m_2 - m_1) \tag{3}$$

If the maximum included Angle of the direction information received by the UAV receiving the signal is an acute Angle, the conditions must be met as follows: $n < m_1$ or $n > m_2$; If the maximum included Angle of the direction information received by the UAV receiving the signal is a non-acute Angle, that is, the condition must be met as: $m_1 < n < m_2$. The two cases are shown in Figure 5.

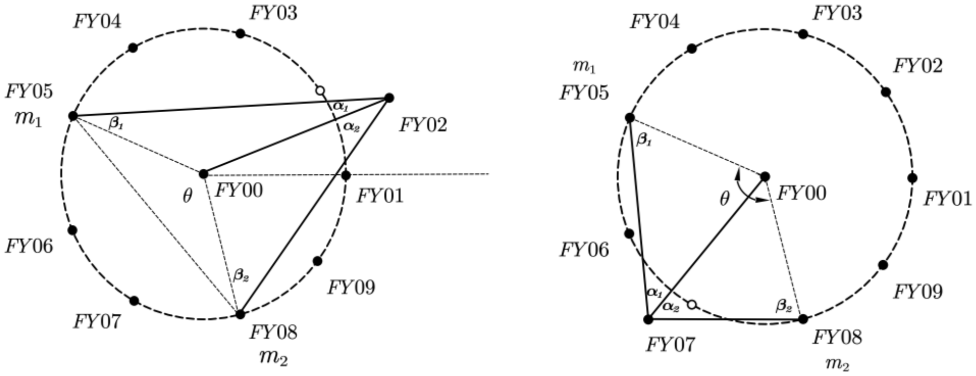

Figure 5. A drone with arbitrary deviation and the maximum included Angle is acute and non-acute.

The coordinates of FY02 are (d, ω), and the equations are derived:

$$\begin{cases} \dfrac{r}{\sin\alpha_1} = \dfrac{d}{\sin\beta_1} \\ \dfrac{r}{\sin\alpha_2} = \dfrac{d}{\sin\beta_2} \\ \alpha_1 + \alpha_3 + \beta_1 + \beta_2 = \theta \\ \omega = \pi - (m_1 - 10)40° + \alpha_1 + \beta_1 \end{cases} \tag{4}$$

The position (d, ω) of the UAV receiving the signal is solved as follows:

$$\begin{cases} d = \frac{r\sin\beta_1}{\sin\alpha_1} \\ \omega = [40(m_1 - 1) - 180 + \alpha_1 + \beta_1]° \end{cases} \tag{5}$$

Where β_1 is:

$$\beta_1 = \arcsin\sqrt{\frac{\left(\frac{\sin\alpha_1}{\sin\alpha_2}\right)^2[\sin(\theta-\alpha_1-\alpha_2)]^2}{\left[1+\left(\frac{\sin\alpha_1}{\sin\alpha_2}\right)^2\cos(\theta-\alpha_1-\alpha_2)^2\right]+\left(\frac{\sin\alpha_1}{\sin\alpha_2}\right)^2[\sin(\theta-\alpha_1-\alpha_2)]^2}} \tag{6}$$

Similarly, it is deduced that the coordinate of FY07 is (d, ω), and the position (d, ω) of the UAV receiving the signal is solved as follows:

$$\begin{cases} d = \frac{r\sin\beta_1}{\sin\alpha_1} \\ \omega = [40(m_1 - 1) - (180 - \alpha_1 - \beta_1)]° \end{cases} \tag{7}$$

$$\beta_1 = arcsin\sqrt{\frac{\left(\frac{sin\,\alpha_1}{sin\,\alpha_2}\right)^2[sin(\theta-\alpha_1-\alpha_2)]^2}{\left[1+\left(\frac{sin\,\alpha_1}{sin\,\alpha_2}\right)^2 cos(-\theta-\alpha_1-\alpha_2)^2\right]+\left(\frac{sin\,\alpha_1}{sin\,\alpha_2}\right)^2[sin(\theta-\alpha_1-\alpha_2)]^2}} \quad (8)$$

According to the above method, continue to calculate the position (d, ω) and β_1 of the receiving signal UAV when $\alpha_1 = \alpha_3 + \alpha_2$ and $\alpha_2 = \alpha_1 + \alpha_3$ are used.

Finally, the polar position of the UAV with horizontal deviation is as follows: $(\frac{r\sin(2\gamma_1-\alpha_1)}{\sin\alpha_1}, 40(n-1)°)$.

The measured Angle between the UAV with arbitrary deviation and the two signal transmitting UAVs is α_1 and α_2 respectively, and the Angle between the two signal transmitting UAVs is α_3. The size relationship between the three angles is discussed, and the following results are obtained:

Table 1. The polar coordinates of UAVs corresponding to the size relationship of various angles.

Angle relation	UAV polar coordinate positioning	remarks
$\alpha_3 = \alpha_1 + \alpha_2$	$(\frac{r\,sin\,\beta_1}{sin\,\alpha_1}, [40(m_1-1)-180+\alpha_1+\beta_1]°)$	no span, the maximum included Angle is acute
	$(\frac{r\,sin\,\beta_1}{sin\,\alpha_1}, [40(m_1-1)+180-\alpha_1-\beta_1]°)$	no span, the maximum Angle is non-acute
$\alpha_1 = \alpha_3 + \alpha_2$	$(\frac{r\,sin\,\beta_1}{sin\,\alpha_1}, [40(m_1-1)-180+\alpha_1+\beta_1]°)$	α_1 is the maximum Angle
$\alpha_2 = \alpha_1 + \alpha_3$	$(\frac{r\,sin\,\beta_1}{sin\,\alpha_1}, [40(m_1-1)+180-\alpha_1-\beta_1]°)$	α_2 is the maximum Angle

2.2. Validation of Positioning Model for UAVs with Arbitrary Deviations

In the case of the UAV with arbitrary deviation, based on the derivation of the original sine theorem, the positioning model is tested from a new Angle relationship, and the two derivation conclusions are compared and brought into the special standard position for simulation experiment. The verification legend is shown in Figure 7:

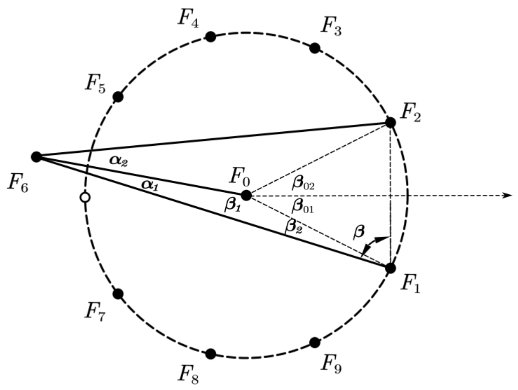

Figure 6. Drones with arbitrary deviations.

Wherein, the standard radius of the circular formation is γ, and the linear distance between the UAV with arbitrary deviation and the UAV at the center of the circle is *d*.

Received:

$$\begin{cases} \beta_1 = 360° - \beta_{01} - \theta \\ \beta_2 = \theta + \beta_{01} - 180° - \alpha_1 \\ \beta = \frac{\beta_{01}}{2} - \frac{\beta_{02}}{2} - \alpha_1 + \theta - 90° \end{cases} \quad (9)$$

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According to the sine theorem, the following equations can be obtained:

$$\begin{cases} \frac{d}{\sin\beta_2} = \frac{r}{\sin\alpha_1} \\ \frac{\sin\alpha_2}{r} = \frac{\sin(\theta-\beta_{02})}{|FF_2|} \\ \frac{\sin\alpha_3}{|FF_2|} = \frac{\sin\beta}{|FF_2|} = \frac{\sin(\frac{\beta_{01}}{2}-\frac{\beta_{02}}{2}-\alpha_1+\theta-90°)}{|FF_2|} \\ |FF_2| = 2r^2 - 2r^2\cos(\beta_{01}+\beta_{02}) \end{cases} \tag{10}$$

Therefore, according to this Angle relationship, the polar coordinate positioning of the UAV with arbitrary deviation can be deduced as follows: $(\frac{r}{\sin\alpha_1}\sin(\theta+\beta_{01}-180°-\alpha_1), \theta)$.

It is known that the polar coordinate of the standard point corresponding to the FY03 UAV in the circular formation is $(r, 80°)$, and this specific standard point is brought into these two different deduction equations for testing.

According to the relation $\alpha_1\alpha_2\alpha_3$, the receiving signal angle, whether the short arc formed by two transmitting signals includes another short arc, and the maximum angle of received direction information are divided into six categories. Using geometry and the sine theorem, the linear distance *d* between the deviated UAV and the central UAV is solved, and the polar coordinate positioning of the deviated UAV is deduced. Error analysis is conducted by adjusting the derivation, and the polar coordinates of the standard position on the circumference are tested.

2.3. UAV Positioning Model Based on the Cosine Law

When using the unknown location of the transmitting signal UAV for positioning and attitude correction of the deviated UAV, it is crucial to determine the position relationship between the transmitting UAV and the known UAVs FY00 and FY01. Given the polar coordinate system's weaker positioning effect, the central UAV (FY00) in the circular formation is taken as the origin of a new rectangular coordinate system, with the direction connecting FY00 and the standard-positioned signal transmitting UAV (FY01) as the positive X-axis[12].

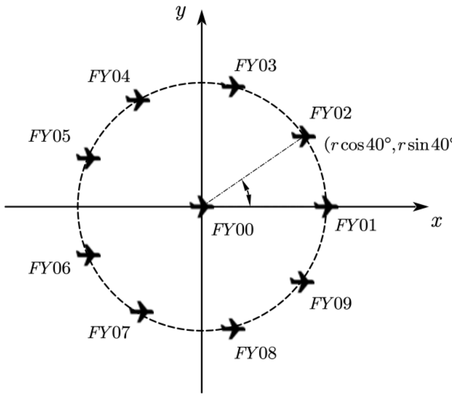

Figure 7. UAV flight positioning cartesian coordinate system.

In the case that there are three signal transmitting UAVs to achieve positioning, the position of the UAVs receiving signals is set as $F(x, y)$, and a standard point $P_i(i = 2,3, \ldots, 9)$ of the UAVs is randomly selected on the circumference, then the rectangular coordinate of P_i is $r\cos\omega_i, r\sin\omega_i$. The UAVs FY00, FY01, FY02 and any point $P_i(x_i, y_i), i = 2,3, \ldots, 9$ among the remaining eight standard points are selected each time, and these 4 points are the transmitting signal positions. The cosine values of the three included angles of $\alpha_1\alpha_2\alpha_3$ corresponding to the remaining 6 standard points are substituted into equation (11) to obtain the position $F(x, y)$ of the corresponding 6 anchor points.

In the case of four signal emitting drones, according to the law of cosine, we can deduce:

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$$\begin{cases} f_1(x,y,\alpha_1) = |FP_0|^2 + |FP_1|^2 - r^2 - 2|FP_0||FP_1| \cos\alpha_1 = 0 \\ f_2(x,y,\omega_i,\alpha_2) = |FP_0|^2 + |FP_i|^2 - r^2 - 2|FP_0||FP_i| \cos\alpha_2 = 0 \\ f_3(x,y,\omega_j,\alpha_3) = |FP_0|^2 + |FP_j|^2 - r^2 - 2|FP_0||FP_j| \cos\alpha_3 = 0 \\ f_4(x,y,\omega_i,\alpha_1,\alpha_2) = |FP_1|^2 + |FP_i|^2 - |P_1P_i|^2 - 2|FP_1||FP_i| \cos(\alpha_1+\alpha_2) = 0 \\ f_5(x,y,\omega_j,\alpha_1,\alpha_3) = |FP_1|^2 + |FP_j|^2 - |P_1P_j|^2 - 2|FP_1||FP_j| \cos(\alpha_1+\alpha_2+\alpha_3) = 0 \\ f_6(x,y,\omega_i,\omega_j,\alpha_2,\alpha_3) = |FP_i|^2 + |FP_j|^2 - |P_iP_j|^2 - 2|FP_i||FP_j| \cos(\alpha_3-\alpha_2) = 0 \end{cases} \tag{11}$$

The most critical factor to determine whether effective positioning can be achieved by using four signal transmitting UAVs is the minimum total deviation variance between the fixed point located by four aircraft and the original standard point[13].

According to the standard sample variance formula:

$$S^2 = \frac{1}{n-1}\sum_{i-1}^{n}(x_i - \overline{x})^2 \tag{12}$$

Where (x_n, y_n) is the position of each standard point on the circular formation, and $\bar{X}$ is the distance difference when all fixed points are precisely located at the standard point, and the value is 0.

Find the mean of all distance deviation variances:

$$\overline{S^2} = \frac{1}{6}\sum_{n=1}^{6} {S_n}^2 \tag{13}$$

Taking the minimum $\overline{S^2}$ as the objective function, and adjusting the variable equation derived from the cosine law to a non-homogeneous nonlinear equation as the constraints, the following single-objective nonlinear optimization model is established:

$$\min \overline{S^2} = \frac{1}{6}\sum_{n=1}^{6} {S_n}^2$$

$$s.t.\begin{cases} {S_i}^2 = \frac{1}{5}\sum_{n=1}^{6}(\sqrt{(x-x_n)^2+(y-y_n)^2} - \bar{X})^2 \\ f_1(x,y,\alpha_1) = 0 \\ f_2(x,y,\omega_i,\alpha_2) = 0 \\ f_3(x,y,\omega_j,\alpha_3) = 0 \\ f_4(x,y,\omega_i,\alpha_1,\alpha_2) = 0 \\ f_5(x,y,\omega_j,\alpha_1,\alpha_3) = 0 \\ f_6(x,y,\omega_i,\omega_j,\alpha_2,\alpha_3) = 0 \\ \omega_i = (40n)^\circ, n = 1,2,\ldots 8 \\ \omega_j = (40m)^\circ, m = 1,2,\ldots 8 \wedge m \neq n \\ i = j = 2,3,\ldots 9 \wedge i \neq j \end{cases} \tag{14}$$

Sequential quadratic programming (SQP) is an indirect algorithm that uses objective functions and constraints to construct augmented objective functions to solve nonlinear optimization problems, that is, to transform constrained nonlinear optimization problems into unconstrained nonlinear optimization problems. Compared with other nonlinear quadratic programming algorithms, the sequential quadratic programming algorithm has the advantages of better convergence, higher computational efficiency and stronger boundary searching ability[14].

The objective function of nonlinear optimization needs to be optimized using Taylor expansion:

$$\min f(x) = \frac{1}{2}K^T \lim_{x\to\infty} \nabla^2 f(X^2)K + \nabla f(X^k)^T K$$

$$s.t.\begin{cases} \nabla\alpha_i(x^k)^T K + \alpha_i(x^k) \leq 0, (i = 1,2,\ldots m) \\ \nabla b_j(x^k)^T K + b_j(x^k) \leq 0, (j = 1,2,\ldots n) \end{cases} \tag{15}$$

To solve the quadratic programming problem, the local optimal solution K^* obtained for the first time is taken as the next search direction of the original optimization objective, and the constraint conditions of the original optimization objective are searched in this search direction, and an approximate optimal solution x^{k+1} can be obtained[15].

At the same time, given the precision and termination criterion H, if the approximate optimal solution x^{k+1} meets this criterion, it is taken as the optimal solution of the original problem, and the corresponding $f(x^{k+1})$ represents the final optimization target result:

$$\begin{cases} x^{k+1} = X^k, x^{k+1} \leq H \\ k = k+1, x^{k+1} > H \end{cases} \tag{16}$$

For all possible anchor points of four signal transmitting UAVs, the most representative anchor point No. 8 is extracted, that is, the standard position data of UAVs numbered FY08 in the original formation. As shown in Table 2

Table 2. Positioning deviation results of four transmitting signal UAVs for FY08 number position.

Common UAV	3rd and 4th launch signal drones	Measured distance deviation /m	Measured variance
FY00、FY01 transmit signals	FY02、FY03	1.2855	0.1912
	FY02、FY04	1.1342	
	…	…	
	FY06、FY09	0.6841	
	FY07、FY09	0.6840	

Through the measurement variance of FY08 number anchor point of four transmitting signal drones, it can be clearly found that the positioning accuracy of feature point FY08 is very high when four transmitting signal drones are deployed

Therefore, it can be concluded that four UAVs should be set up to transmit signals in order to accurately and effectively locate the deviated UAVs when the location of each transmitting signal is unknown.

3. Conclusion

This study contributes to the field of UAV formation passive positioning by demonstrating the effectiveness of using four signal-transmitting UAVs for high-precision localization. Our proposed method significantly improves the accuracy and reliability of passive localization, even when UAVs deviate from their standard positions. Experiments show that deploying four signal transmitters significantly enhances accuracy, especially in scenarios where UAVs deviate, thereby reducing positioning errors. Reasonable signal transmission combinations and strategies are crucial, suggesting the need for flexible adjustments in practical applications. Compared to active positioning, passive positioning reduces electromagnetic emissions and detection risks, making it particularly suitable for military and covert operations. Future research could focus on two main areas: first, applying passive positioning in different UAV formations to validate the method's effectiveness under various conditions; second, developing AI-driven adaptive algorithms to enhance UAV positioning in complex environments. These efforts will further enhance the robustness and versatility of the proposed method.

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