

# *An Adaptive Event-Triggered Predictive Control Method*

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**Abstract:** This paper studies an adaptive event-triggered (AET) control method for sampled-data systems to enhance communication efficiency and conserve resources. Initially, a dynamic event-triggering mechanism is introduced to minimize redundant data transmissions while maintaining system performance. Then, a state feedback control law is designed by optimizing the event-triggered threshold condition, with the system's stability verified using a Lyapunov function. The gain parameters are computed through a cone complementarity linearization algorithm, ensuring computational efficiency and robustness. Additionally, the proposed approach addresses the trade-off between communication resource utilization and control performance by dynamically adjusting the triggering conditions based on system states. This ensures a reduction in redundant transmissions without compromising system reliability. Finally, this method provides a systematic and effective framework for improving communication efficiency, with potential applications in industrial automation and resource-constrained networked control systems.

**Keywords:** Adaptive control, Model predictive control, Event-triggered transmission scheme.

## 1. Introduction

With the development of event-triggered control theory, it finds broad application across different communication scenarios, such as Multi-Agent Systems [1] and switched systems [2]. On the one hand, it effectively improves communication efficiency; but on the other hand, it has limitations. Traditional event control methods often struggle to utilize event transmission effectively when communication resources are limited, resulting in constrained savings in communication bandwidth. In practice, AET control methods based on model predictive control offer promising solutions to these issues [3]. However, when selecting trigger condition thresholds, it is important to consider how to increase the rate of change in the initial state while slowing it down as stability is approached. Therefore, we propose an AET control method based on a sampling error system.

The control schemes presented in [4,5] are static and lack the capability to dynamically adjust triggering constraints. In contrast, the AET control method proposed in this paper has several advantages: it can adaptively adjust trigger constraints, dynamically respond to sampling errors, fully exploit the benefits of event triggering under limited communication resources, and maintain expected control performance while conserving more communication bandwidth. Compared to [6], the threshold constraint condition presented in this paper can reach and converge to a stable state faster, minimizing the volume of sampled data that must be transmitted. Furthermore, this paper's primary contributions are as follows:

- 1) This study introduces a new AET control approach grounded in the sampling error system. Unlike the methods presented in [4][5], the proposed adaptive trigger control method effectively maintains the advantages of event-triggered under limited communication resources, allowing for greater savings in communication bandwidth. Furthermore, it guarantees expected control performance while reducing transmission data.
- 2) The trigger conditions are further optimized compared to [6], enabling faster convergence to a stable state and minimizing the amount of data that needs to be transmitted.

Within this work,  $R$  signifies the set of real numbers,  $R^n$  is space of real  $n$ -dimensional vector,  $R^m$  is space of real  $m$ -dimensional vector,  $L_2$  represents space of signals:  $L_2 = \{\|a(k)\|_2 < \infty\}$ ,  $N$  stands for the set of natural numbers,  $\|a(k)\|_2$  refers to  $a(k) \in R^n: \sqrt{\sum_{k=0}^{\infty} \|a(k)\|^2}$ .

The organization of this paper is detailed below. Section II covers the system dynamics, assumptions, lemmas, and problem formulation. Section III outlines the main findings, encompassing the AET method, controller design, stability analysis, and solutions for gain parameters. Section IV provides the conclusion of the paper.

## 2. Foundations and Problem Statement

This section provides the system's dynamic description, relevant assumptions and lemmas, as well as the problem formulation.

### 2.1. System Dynamics

Considering the sampled data error, the state space model expression of the system is given by

$$\begin{cases} \dot{x}(t) = Ax(t) + B(t) + B_1 \omega(t), \\ y(t) = Cx(t), \end{cases} \quad (1)$$

where  $x(t) \in R^n$  and  $u(t) \in R^m$  are state control vectors and control input vectors,  $\omega \in L_2[0, \infty)$  is an external perturbation. The matrices  $A$ ,  $B$ , and  $B_1$  are all constant coefficients matrices, each having appropriate dimensions. The system's initial state is given as  $x(0) = 0$ .

In this article, the controller's computational delay and the delay during the transmission from the controller to the actuator are neglected.

Here, some necessary assumptions and lemma are given.

**Assumption 1**

The sensors are activated by time, with system states sampled at a fixed interval  $h > 0$ , and all transmitted packets are provided with timestamps.

**Assumption 2**

Data transmission is determined by an event-triggered scheme. Upon the acquisition of the control data, it dispatches a timestamp synchronized with the measured value into the control loop.

**Assumption 3**

A zero-order holder (ZOH) generates the control input of the system. When the latest control input has not been received, ZOH is employed to maintain the current control input state. The holding time  $t \in \Omega$  is defined as  $[t_k h + \eta_1, t_{k+1} h + \eta_1]$ , where  $\eta_1$  is the stable transmission time-delay existing from the sensor to the controller.

**Lemma 1** [7]:

Take into account the following system where

$$\begin{cases} \dot{x}(t) = Ax(t) + A_1x(t - \tau(t)) + B_1\omega(t), \\ z(t) = C_0x(t) + C_1x(t - \tau(t)), \\ t \in [t_k, t_{k+1}), \tau(t) = t - t_k. \end{cases}$$

Here,  $x(t) \in R^n$  represents the state vector,  $\omega(t) \in R^{n_\omega}$  represents the disturbance, and  $z(t) \in R^{n_z}$  is the controlled output. Matrices  $A, A_1, B_1, C_0$  and  $C_1$  have constant values with appropriate dimensions. Admit conditions that  $\dot{\bar{V}}(t) + z^T(t)z(t) - \gamma^2 \omega^T(t)\omega(t) < 0$ , where  $\bar{V}(t) = V(t, x_t, \dot{x}_t)$ . Almost for all  $t, \forall \omega \neq 0$  and a prescribed  $\gamma > 0$ , then the following results hold. For every nonzero  $\omega \in L_2[0, \infty)$  and under the zero initial condition,  $J = \int_0^\infty [\beta^2 x^T(s)x(s) - \gamma^2 \omega^T(s)\omega(s)]ds < 0$ .

## 2.2. Problem Formulation

For given scalars  $\gamma > 0$  and  $\beta > 0$ , set the performance index  $J$  to be:

$$J = \int_0^\infty [\beta^2 x^T(s)x(s) - \gamma^2 \omega^T(s)\omega(s)]ds. \quad (2)$$

We devise an event-triggered state feedback control law in the format of form:  $u(t) = Kx(t_k h)$ ,  $t \in \Omega$  ensuring that the corresponding closed-loop system attains internal asymptotic stability. and  $J < 0$  for the initial condition  $x_0 = 0$  and all nonzero  $\omega \in L_2[0, \infty)$ .

As can be seen from the expression in (8), a portion of sampled data stays untransmitted during the intervals between two transmission instances. With the aim of incorporating the event-triggered transmission scheme to determine the necessity of transmitting the current sampled data, an effective approach is presented. This approach considers the error of the sampled data at each sampling moment, resulting in the division of the ZOH holding interval  $\Omega$  into subintervals.

$$\Omega_{n,k} = [t_k h + nh + \eta_1, t_k h + nh + \eta_1 + h) \text{ i.e. } \Omega = \cup \Omega_{n,k}, n = 0, 1 \dots t_{k+1} - t_k - 1.$$

Define

$$\eta(t) \triangleq t - m_{n,k}h, t \in \Omega_{n,k} \quad (3)$$

$$e(m_{n,k}h) \triangleq x(m_{n,k}h) - x(t_k h) \quad (4)$$

with  $m_{n,k}h = t_k h + nh, n = 0, 1, 2 \dots, t_{k+1} - t_k - 1$ .

In light of the meaning of  $\eta(t)$ , it turns out to be a piecewise linear function satisfying the following conditions.

$$\dot{\eta}(t) = 1, \eta_1 \leq h + \eta(t) \leq \eta_2, t \in \Omega_{n,k} \text{ and } t \neq m_{n,k} + \eta_1. \quad (5)$$

Then the control law is represented as

$$u(t) = K[x(t - \eta(t)) - e(t - \eta(t))], t \in \Omega_{n,k} \quad (6)$$

From (1) and (6), the sampled-data error dependent system is of the following form.

$$\dot{x} = Ax(t) + BKx(t - \eta(t)) - BKe(m_{n,k}h) + B_1\omega(t), t \in \Omega_{n,k}. \quad (7)$$

Remark 1: By the definition of  $\eta(t)$  and  $e(m_{n,k}h)$ , it can be noted that the calculation  $e(m_{n,k}h)$  occurs solely at the sampling instant  $t_k h + nh$ . Thereby, it's not essential to conduct continuous measurement.

Remark 2: Model (7) notably includes a specific case where  $\eta_1 = 0$ . From equation (5), it is evident that  $\eta_2$  is influenced by the sampling period as well as the transmission delay. When  $\eta_2$  is determined, it serves the purpose of balancing the sampling period with the permissible transmission delay.

The primary initiatives carried out in this paper are enumerated below.

- 1) Propose an AET mechanism framework and conduct a stability analysis of the designed controller.
- 2) Provide the LMI method for solving the controller gain parameters.

### 3. Main Results

#### 3.1. Adaptive Event-Triggered Mechanism

Assuming that data transmission depends on predefined conditions rather than occurring at fixed time intervals, this approach determines the timing of the next data transmission. In each control cycle, the sampled data is transmitted conditionally by an AET generator, while the control quantity is continuously adjusted by the model predictive controller. Compared to the schemes proposed in [4][5], the main advantage of the suggested AET predictive control scheme shown in Fig.1 is its ability to dynamically adjust the threshold. The increment is faster at the initial moment, allowing the system to converge more quickly as it approaches the steady state. Building on this description, the adaptive event generator determines the next transmission time as

$$t_{k+1}h = t_kh + \min_{n \in N} \{e^T(i_kh)\Phi e(i_kh) > \sigma(t_kh)x^T(t_kh)\Phi x(t_kh)\} \quad (8)$$

where  $\Phi > 0$  stands for the weight coefficient matrix,  $h$  denotes the system sampling period,  $t_k (k = 0, 1, 2, \dots)$  are some integers such that  $\{t_0, t_1, t_2, \dots\} \subset \{0, 1, 2, \dots\}$ ;  $e(i_kh)$  represents the current sampling moment data  $x(i_kh)$  and the latest transmission data  $x(t_kh)$ , that is

$$e(i_kh) \triangleq x(i_kh) - x(t_kh), \quad i_kh \in (t_kh, t_{k+1}h]. \quad (9)$$

Moreover,  $\sigma(t_kh)$  in (8) is governed by the following adaptive rule.

$$\sigma(t_{k+1}h) = \max \left\{ \underbrace{1 - \alpha \tanh[\beta(|x(t_{k+1}h)| - |x(t_kh)|)]}_{\lambda}, \sigma_m \right\} \quad (10)$$

where  $\tanh(\cdot)$  represents the hyperbolic tangent function,  $0 < \alpha$  and  $0 < \beta$  are given constants to adjust the output of  $\tanh(\cdot)$ ,  $\sigma_m$  is the predefined lower bound of  $\sigma(t_kh)$ . In this research, we have  $\sigma(0) = \sigma_m$ .

The transmission events are influenced by the error  $e(i_kh)$ , the latest transmitted states  $x(t_kh)$ , and the adjustable threshold  $\sigma(t_kh)$ .

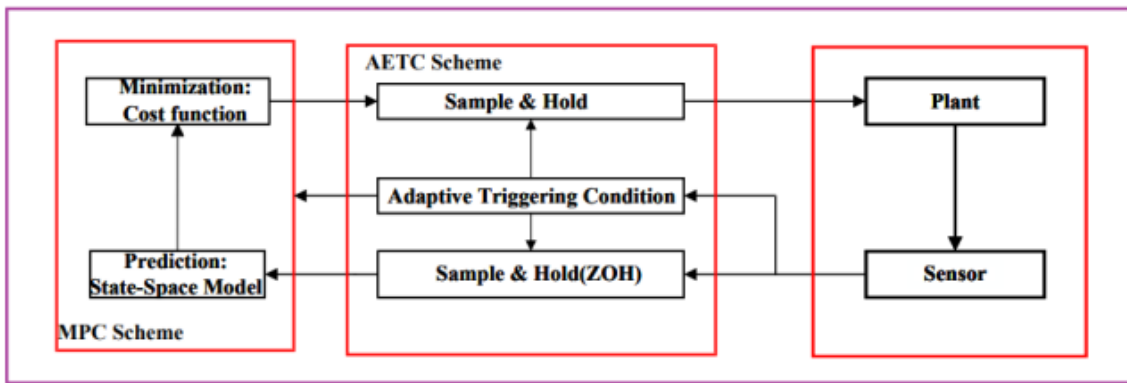


Figure 1: Structural diagram of proposed AET-MPC

Figure 2 provides an example illustrating the core concept of the proposed scheme, where  $t_kh$  represents the triggered transmission instants. Clearly, not all data will be transmitted, but only the data that meets the constraint conditions will be transmitted. For instance, the data at time  $0, h, 3h, 6h$  and  $9h$  are transmitted, while the data at the rest of times remain untransmitted.

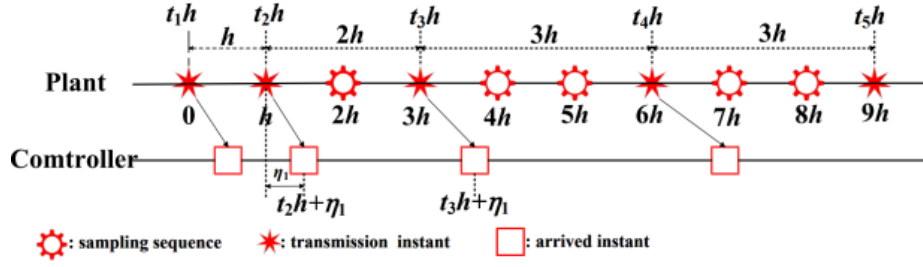


Figure 2: Specific instance of the time evolution of the sampling and transmission sequences.

Note that the function  $\tanh(\cdot)$  has both an upper and lower bound, denoted as  $\tanh(\cdot) \in (-1, 1)$ . This paper leverages these properties to dynamically adjust the threshold  $\sigma(t_k h)$ . For example, when  $\|x(t_{k+1}h)\| > \|x(t_k h)\|$ , we can achieve  $0 < \lambda < 1$  and  $\sigma(t_{k+1}h) < \sigma(t_k h)$ , which means using a smaller  $\sigma(t_{k+1}h)$  to establish a faster communication frequency, thereby reducing the error between  $\|x(t_{k+1}h)\|$  and  $\|x(t_k h)\|$ . Conversely, by setting a slower communication frequency  $\sigma(t_{k+1}h)$ , more communication bandwidth can be conserved.

Remak 3: The event-triggered threshold  $\sigma$  is a pre-selected constant, and the parameters  $\sigma(t_k h)$  can be dynamically adjusted according to the adaptive rules (10), depending on the current  $x(t_{k+1}h)$ , latest transmission data  $x(t_k h)$ ,  $\alpha$ ,  $\beta$  and the simultaneous adjustment  $\sigma_m$ . In addition, if  $\sigma_m$  is sufficiently close to zero, this implies that  $t_{k+1} = t_k h + h$  and all sampled data are transmitted with a constant sampling period  $h$ . In this case, the proposed adaptive communication scheme will degenerate into the general scheme [8][9]. More generally, if  $\alpha \equiv 0$ , the scheme will degenerate into the constant communication scheme [10][11].

### 3.2. System Stability Analysis and Parameter Solving

This subsection attaches importance to analyzing the system's stability and providing ways to determine the parameters in event-triggered solution schemes.

Consequently, focus on the ensuing Lyapunov function.

$$V(t, x_t) = V_1(t, x_t) + V_2(t, x_t), \quad t \in \Omega_{n,k}, \quad (11)$$

where  $x_t = x(t + \theta)$ ,  $\forall \theta \in [-\eta_2, 0]$ . Then, one gets

$$V_1(t, x_t) = x^T(t) P x(t) + \int_{t-\eta_2}^{t-\eta_1} \int_s^t \dot{x}^T(o) S_2 \dot{x}(o) do ds + \eta_1 \int_{t-\eta_1}^t \int_s^t \dot{x}^T(o) S_1 \dot{x}(v) do ds + \sum_{i=1}^2 \int_{t-\eta_i}^t x^T(o) P_i x(o) do,$$

where  $P > 0$ ,  $P_i > 0$ ,  $S_i > 0$  ( $i = 1, 2$ ) and

$$V_2(t, x_t) = (\eta_2 - \eta(t)) [\zeta^T(t) \hat{R} \zeta(t) + \int_{t-\kappa}^t \dot{x}^T(o) S_3 \dot{x}(o) do].$$

with

$$\zeta^T(t) = [x^T(t), x^T(t - \kappa)], \quad \kappa = \eta(t) - \eta_1.$$

$$\hat{R} = \begin{bmatrix} R_1 & R_2 - R_1 \\ * & R_1 - R_2^T - R_2 \end{bmatrix}, \quad S_3 > 0, \quad R_1 > 0 \quad \text{and} \quad R_2 \quad \text{being chosen such that} \quad V(t, x_t) \geq 0.$$

In what follows, the forms of  $V_1$  and  $V_2$  will be utilized to determine whether the system is stable.

Theorem 1: For given some constants  $\gamma > 0$ ,  $\beta > 0$ ,  $\sigma > 0$ ,  $\eta_1 > 0$ ,  $h > 0$  and a matrix  $K$  under an event-triggered transmission scheme (8), the system (7) from  $\omega$  to  $x$  is finite-gain  $L_2$ -stable with gain less than  $\gamma/\beta$ . If positive matrices  $P$ ,  $R_1$ ,  $\Phi$ ,  $P_1$ ,  $P_2$ ,  $S_i$  and appropriately sized matrices  $R_2$  and  $L_i$  ( $i = 1, 2, 3$ ) exist, and the conditions to follow are met.

$$\begin{bmatrix} P_1 + hR_1 & * \\ hR_2^T - hR_1 & hR_1 - hR_2^T - hR_2 \end{bmatrix} > 0. \quad (12)$$

$$\begin{bmatrix} E + \psi + \psi^T + (1-\rho)h\Sigma & * \\ F_{21}^\rho & -F_{22} \end{bmatrix} < 0, \rho = 0,1 \quad (13)$$

with

$$E = \text{diag}\{P_1 + P_2 - R_1 + \beta^2 I, -P_1, 0, -P_2, -\gamma^2 I, R_2^T + R_2 - R_1\} + 2PI_1 I_2 + \sigma I_4 \Phi I_4^T - S_1 I_5 I_5^T + 2I_1(R_1 - R_2)I_3^T,$$

$$F_{21}^\rho = \text{col}\{\eta_1 S_1 I_2, hS_2 I_2, hL_{\rho+1}^T hS_3 I_2 + \rho hL_3^T\},$$

$$F_{22} = \text{diag}\{S_1, hS_2, hS_2, hS_3\},$$

$$\psi = [L_3 \ L_2 \ L_1 - L_2 \ 0 \ -L_1 \ -L_3 \ 0],$$

$$\Sigma = 2I_1 R_1 I_2 + 2I_3(R_2^T - R_1^T)I_2,$$

where

$$I_1 = [I \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T,$$

$$I_2 = [A \ 0 \ BK \ -BK \ 0 \ B \ 0],$$

$$I_3 = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ I]^T,$$

$$I_4 = [0 \ 0 \ I \ -I \ 0 \ 0 \ 0]^T,$$

$$I_5 = [I \ -I \ 0 \ 0 \ 0 \ 0 \ 0]^T.$$

*Proof:* When the time derivative of  $V(t, x_t)$  in (11) is computed along the trajectory of system (7), it leads to

$$\begin{aligned} \dot{V}(t, x_t) = & 2x^T(t)P\dot{x}(t) - \int_{t-\kappa}^t \dot{x}^T(o)S_3\dot{x}(o)do - \zeta^T(t)\hat{R}\zeta(t) + \eta_1^2 \dot{x}^T(t)S_1\dot{x}(t) - \eta_1 \int_{t-\kappa}^t \dot{x}^T(o)S_3\dot{x}(o)do + h\dot{x}^T(t)S_2\dot{x}(t) \\ & - \int_{t-\eta_2}^{t-\eta_1} \dot{x}^T(s)S_2\dot{x}(s)ds + (\eta_2 - \eta(t))\{2\zeta^T(t)\hat{R}[\dot{x}^T(t), 0]^T + \dot{x}^T(t)S_3\dot{x}(t)\} + \sum_{i=1}^2 [x^T(t)P_i x(t) - x^T(t-\eta_i)P_i x(t-\eta_i)], \\ & t \in \Omega_{n,k}. \end{aligned} \quad (14)$$

From (10), it is clear that the minimum value of  $\sigma(m_{n,k}h)$  is  $\sigma_m$ . If the currently sampled data isn't transmitted, the following relationship holds.

$$e^T(m_{n,k}h)\Phi e(m_{n,k}h) < \sigma_m x^T(t_k h)\Phi x(t_k h) = \sigma_m (x(t-\eta(t)) - e(m_{n,k}h))^T \Phi (x(t-\eta(t)) - e(m_{n,k}h)), t \in \Omega_{n,k} \quad (15)$$

By applying the Newton-Leibniz formula and introducing matrices  $L_i$  ( $i=1, 2, 3$ ) of suitable dimensions to handle the integral terms in (14), the following result is derived from equations (14) and (15).

$$\dot{V}(t, x_t) < \xi^T(t) \Delta_0 \xi(t) - \beta^2 x^T(t) + \gamma^2 \omega^T(t) \omega(t). \quad (16)$$

with

$$\xi^T(t) = [x^T(t), x^T(t-\eta_1), x^T(t-\eta(t)), e^T(m_{n,k}h), x^T(t-\eta_1), \omega^T(t), x^T(t-\kappa)],$$

$$\Delta_0 = E + (\eta_2 - \eta(t))\Delta_1 + (\eta(t) - \eta_1)\Delta_2 + \psi + \psi^T + \eta_1^2 I_2^T S_1 I_2 + hI_2^T S_2 I_2,$$

$$\Delta_1 = L_1 S_2^{-1} L_1^T + I_2^T S_3 I_2 + \Sigma,$$

$$\Delta_2 = L_2 S_2^{-1} L_2^T + I_2^T S_3 I_2 + \Sigma,$$

where  $E$ ,  $\psi$ ,  $I_2$  and  $\Sigma$  are defined in Theorem 1.  $\square$

Through the Lyapunov function in (11) and equations (13) to (16), it can be concluded that system (7) with  $\omega(t)=0$  is asymptotically stable under zero initial conditions. Furthermore, this result is supported by (12). Applying Lemma 1, the system is shown to be  $L_2$  finite gain stable with the gain constrained by  $\gamma/\beta$ .

**Theorem 2:** Given some constants  $\gamma > 0$ ,  $\beta > 0$ ,  $\sigma > 0$ ,  $\eta_1 > 0$  and  $h > 0$  under the event-triggered transmission scheme (8), the system (7) remains finite-gain stable from  $\omega$  to  $x$  with a gain less than

$\gamma / \beta$ , and the state feedback controller gain is expressed as  $K=YX^{-1}$ , if there exist positive matrices  $X, \tilde{R}_1, \tilde{\Phi}, \tilde{P}_i, \tilde{S}_i$  and a scalar  $\lambda > 0$ , and suitable dimensioned matrices  $\tilde{R}_2 > 0$  and  $\tilde{L}_i > 0$  ( $i = 1, 2, 3$ ), in a way that the conditions outlined are fulfilled.

$$\begin{bmatrix} X+h\tilde{R}_1 & * \\ h\tilde{R}_2^T-h\tilde{R}_1 & h\tilde{R}_1-h\tilde{R}_2^T-h\tilde{R}_2 \end{bmatrix} > 0, \quad (17)$$

$$\begin{bmatrix} \tilde{E}+\tilde{\psi}+\tilde{\psi}^T & * \\ \tilde{F}_{21}^\rho & -\tilde{F}_{22}^\rho \end{bmatrix} < 0, \rho=0, 1, \quad (18)$$

where

$$\begin{aligned} \tilde{E} &= \text{diag}\{\tilde{P}_1+\tilde{P}_2-\tilde{R}_1, 0, -\tilde{\Phi}, \tilde{P}_2, -\gamma^2 I, \tilde{R}_2+\tilde{R}_2-\tilde{R}_1\}-\tilde{S}_1 I_5 I_5^T+2I_1 \tilde{L}_2+2I_1(\tilde{R}_1-\tilde{R}_2)I_3^T+\sigma I_4 \tilde{\Phi} I_4^T, \\ \tilde{F}_{21}^0 &= \text{col}\{\eta_1 \tilde{I}_2, \sqrt{h} \tilde{L}_2, \sqrt{h} \tilde{L}_1^T, \sqrt{h} \tilde{L}_2, \sqrt{2} \tilde{I}_2, h \tilde{R}_1 I_1^T, h(\tilde{R}_2-\tilde{R}_1)I_3^T, \beta X\}, \\ \tilde{F}_{22}^0 &= \text{diag}\{X\tilde{S}_1^{-1}X, X\tilde{S}_2^{-1}X, X\tilde{S}_3^{-1}X, X\lambda^{-1}X, \lambda I, \lambda I, I\}, \\ \tilde{F}_{21}^1 &= \text{col}\{\eta_1 \tilde{I}_2, h \tilde{L}_2, h \tilde{L}_2^T, h \tilde{L}_3^T\}, \\ \tilde{F}_{22}^1 &= \text{diag}\{X\tilde{S}_1^{-1}X, hX\tilde{S}_2^{-1}X, h\tilde{S}_2, h\tilde{S}_3\}, \\ \tilde{\psi} &= [\tilde{L}_3 \ \tilde{L}_2 \ \tilde{L}_1-\tilde{L}_2 \ 0 \ -\tilde{L}_1 \ -\tilde{L}_3 \ 0], \\ \tilde{I}_2 &= [AX \ 0 \ BY \ -BY \ 0 \ B_1 \ 0]. \end{aligned}$$

*Proof:* Define  $X=P^{-1}$ ,  $X\Phi X=\tilde{\Phi}$ ,  $XP_i X=\tilde{P}_i$ ,  $X\tilde{R}_i X=\tilde{R}_i$  ( $i=1, 2$ ),  $XS_j X=\tilde{S}_j$ ,  $XL_j X=\tilde{L}_j$  ( $j=1, 2, 3$ ) and  $Y=KX$ . For any scalar  $\lambda > 0$ , it yields

$$\Sigma \leq h^2 I_1 R_1 \lambda R_1 I_1^T + 2I_2 \lambda^{-1} I_2 + h^2 I_3 R^T \lambda^{-1} R I_3^T \quad (19)$$

where  $R=R_2-R_1$ . Then multiply both sides of (12) with  $\text{diag}(X, X)$ . For  $\rho=0$ , apply  $\text{diag}(X, X, X, X, X, X, I, X, I, X, I)$  on both sides of (13) and for  $\rho=1$ , use  $\text{diag}(X, X, X, X, X, X, I, X, I, I, X, X)$ .

Employing the Schur complement and equation (19), (17) and (18), thereby completing the proof.

Nonlinear terms  $X\tilde{S}_i X$  and  $X\lambda^{-1}X$  render the matrix inequality (18) non-convex, preventing direct resolution via the MATLAB LMI Toolbox. To this end, a variable matrix  $M_j^T > 0$  ( $j=1, 2, 3, 4$ ) is introduced as

$$X\tilde{S}_i^{-1}X \geq M_i, \quad X\lambda^{-1}X \geq M_4 \quad (20)$$

Let  $N_j=M_j^{-1}$ ,  $Z_{4+i}=\tilde{S}_i^{-1}$  ( $i=1, 2, 3$ ),  $N_8=\lambda^{-1}I$  and  $N_9=X^{-1}$ . Then (18) can be replaced by

$$\begin{bmatrix} N_i & N_9 \\ * & N_{4+i} \end{bmatrix} \geq 0, \quad i=1, 2, 3, 4. \quad (21)$$

Thus, it becomes possible to subsequently transform the original non-convex minimization problem into a minimization problem constrained by LMI conditions.

$$\begin{cases} \min \text{tr}(\sum_{j=1}^4 N_j M_j + \sum_{i=1}^3 N_{4+i} \tilde{S}_i + N_8 \lambda I) \\ \text{subject to: (17), G, (21) and} \\ \begin{bmatrix} M_j & I \\ * & Z_j \end{bmatrix} \geq 0, \begin{bmatrix} X & I \\ * & N_9 \end{bmatrix} \geq 0, \\ \begin{bmatrix} \tilde{S}_i & I \\ * & N_{4+i} \end{bmatrix} \geq 0, \begin{bmatrix} \lambda I & I \\ * & N_8 \end{bmatrix} \geq 0, \end{cases} \quad (22)$$

where  $G$  is obtained from (18) by substituting  $X\tilde{S}_iX$  and  $X\lambda^{-1}X$  with  $M_j$  and  $M_4$  in (22), respectively. The minimization problem discussed is solvable using CCL [12].

To derive a practical solution, the following algorithm is introduced to systematically identify the optimal parameters  $h, \sigma, \Phi$  and  $K$ .

Define:

$$\begin{cases} \min f(\sigma) \\ \text{subject to: } \sigma \in (0, 1) \end{cases} \quad (23)$$

where

$$f(\sigma) = \frac{\text{the quantity of the trasmitted sampled-data}}{\text{the quantity of all the sampled-data}}$$

To achieve a feasible solution, the following algorithm is proposed to obtain the optimal parameters  $h, \sigma, \Phi$  and  $K$ .

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**Algorithm 1 Determination of Parameters  $h, \sigma, \Phi$  and  $K$ .**

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- 1: Initialize  $k:=1, f^1:=1$ , set a smaller initial value  $\sigma$  and give  $\varepsilon$  as the step increment of  $\sigma$ ;
  - 2: For given  $\beta$  and  $\gamma$ , solve (22) to judge whether a feasible solution exists. If a feasible solution exists, proceed to the step 3; otherwise, adjust the constant  $\beta$  and  $\gamma$  and restart from the step 2;
  - 3: Applying CCL solve (22) to obtain the parameters  $K, h$  and  $\Phi$ . If  $f(\sigma) < f^k$ , set  $k = k+1$ , update  $h^k := h, \sigma^k := \sigma, \Phi^k := \Phi, K^k := k$  and  $f^k := f(\sigma)$ . Else keep the current values of  $h^k, \sigma^k, \Phi^k, K^k$  until  $f(\sigma) < f^k$ ;
  - 4: Update  $\sigma := \sigma + \varepsilon$ , if  $\sigma \in (0, 1)$ , then output  $k, h, \sigma, \Phi$  and exit. Otherwise, return to the step 3.
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Remark 4: In this analysis, it is assumed that all system states are fully observable and can be utilized for state feedback control. If the states of the system cannot all be measured, state estimation can be performed through a sampled control system. Similarly, if  $\eta_1 = 0$  this can also be solved using a defined Lyapunov function (11).

Remark 5: The AET scheme proposed in this paper builds upon the static event-triggered scheme, utilizing the threshold  $\sigma$  from the static event-triggered scheme as the initial value for the adaptive scheme. Consequently, the selection and determination of other parameters remain consistent with those in [13].

## 4. Conclusion

The paper has proposed an AET control method based on a sampling error system. The AET mechanism is implemented to allow the system to dynamically modify the trigger threshold, reducing the transmitted sampling data and conserving bandwidth. Compared to traditional event-triggered mechanisms, the proposed approach has not only preserved the benefits of event-triggered control under constrained communication resources but also optimized bandwidth usage while maintaining the desired control performance. This approach has offered valuable insights for enhancing event-triggered performance and improving communication transmission efficiency.

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