DOI: 10.54254/2755-2721/5/20230704

Wiener filter and wavelet transform of noisy image analysis based on MATLAB

Zhonghao Liu

School of Electronic Information, School of Northwestern Polytechnical University, Xi 'an, Shaanxi Province, 710100, China.

ZLiu202@sheffield.ac.uk

Abstract. In recent years, how to filter noise more effectively has been a topic of great interest, and improvements have been made based on various filtering algorithms in the hope of obtaining the most optimal filtering effect with the least amount of computation. However, there is a considerable research gap in how to achieve this goal. This article is based on this research. The motif of this study paper is the analysis of images containing Gaussian noise based on Wiener filter and wavelet transform, and an attempt to improve the filtering algorithm. The SNR and PSNR obtained by applying Wiener filtering analysis are 46.7864 and 76.0247, respectively, and the SNR and PSNR results of wavelet transform are 14.0851 and 49.7426. this article implements Wiener filtering based on Wiener method, and the results are almost consistent with the function of invoked system. It is found that the Wiener filter is more suitable for handling processing Gaussian noise. The SNR and PSNR values of both sets of experimental Wiener filter are larger than those of wavelet transform, then it indicates that Wiener filter is more beneficial to suppress Gaussian noise under the same condition.

Keywords. Wiener filter, wavelet transform

1. Introduction

Noise, or interference, has been widely present in the transmission of information in industry and nature, and how to suppress this noise has been a topic of great interest. For the sake of changing the photo quality and abstract valid messages from the photo, the image has to restrain noise. Wiener filter and wavelet transform, as two common noise reduction methods, have very important applications. Wiener filtering is based on the least mean square error formula, which makes the mean square error of estimation error minimum and offers a useful guiding principle for linear filtering of broadly defined smooth signals [1]. Wavelet transform is a new conversion analysis mean, which is a direct conversion of interspace, time and frequency compared to Fourier transform [2], allowing a better observation of the local features of the signal. Various studies on image noise reduction have been focused on how to suppress noise to achieve the best filtering effect, and this research is very practical. SNR is also known as signal-to-noise ratio, which is the proportion of semaphore power to noise power. People use this indicator to directly judge the filtering effect, generally speaking, the better the filtering effect can be reflected from the higher SNR level. Similar to SNR, PSNR can also be used as a measure, PSNR refers to the peak signal-to-noise ratio, which needs to be calculated with the help of MSE function. The better the PSNR can also indicate the improvement of the filtering effect [3]. As for the filtering methods,

DOI: 10.54254/2755-2721/5/20230704

many improvement methods have been proposed for different filtering methods, and also several filtering methods have started to be studied in combination. In this article, the author first used the Wiener filter and wavelet transform to process noisy images, and then improve them to achieve better noise reduction.

2. Gaussian noise

Noise is a voltage or current in a circuit or system that does not contain an informative amount. In image processing, noise is usually manifested as individual pixel points or blocks of pixels that cause intense visual effects. Gaussian noise, named by Carl Friedrich Gauss, is a very common type of noise that is widely found in nature and industry. Typical Gaussian noises include fluctuating noise, cosmic noise, thermal noise, and scattered particle noise, for example. In digital images the dominant source of Gaussian noise occurs at acquisition period. Noise from sensors due to phenomena such as poor brightness and high temperatures. Along with the common use means of noise suppression, the mean of suppression of Gaussian noise often uses mathematical and statistical methods. The value of the Gaussian noise is subject to the Gaussian distribution, which means that the noise has a probability density function amount to a Gaussian distribution probability density function [3]. The probability density feature of this noise is shown in Formula (1) [4].

$$\mathrm{P_G}(\mathrm{z})\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{-(z-\mu)^2}{2\sigma^2}} \tag{1}$$

The Gaussian distribution, also known as the normal distribution, is noted as N(μ, σ^2), where μ, the square of σ are the parameters of the distribution, μ is the expectation while σ^2 is the variance. When there is a definite value, p(x) is also affirmatory. Particularly when μ amount to zero and the square of σ amount to one, a distribution of X is a criterion normal distribution. For the Gaussian noise probability density function, z means the level of gray and μ represents the average gray value [5]. The noise signal of Gaussian noise is randomly distributed and without any law.

Gaussian white noise is a sort of Gaussian noise, when a Gaussian noise has both a uniformly defined power spectral density feature and an amplitude that submits to a Gaussian distribution, this is Gaussian white noise. Gaussian white noise has uncorrelated second-order momenta and constant first-order momenta, which refers to the correlation of successive signals in time. Thermal noise and scattered grain noise are both types of Gaussian white noise. In testing and modeling of communication channels, the Additive Gaussian white noise needs to be applied, and additive Gaussian white noise in the channel is usually generated using Gaussian noise that is utilized as additive white noise. During communication, the communication channel is often affected by Gaussian noise from nature. For example, the thermal motion of atoms in conductors, scattered particle noise, the influence of the Earth itself or other warm objects, or even other celestial bodies. Therefore, effective suppression of Gaussian noise is important for us to improve communication quality.

In this article, the author respectively for Gaussian noise used Wiener filtering and wavelet transform processing, compared the two kinds of the effect of filtering methods, drew relevant conclusions, and improved.

2.1. Imnoise function

MATLAB function imnoise is to add noise to pollute an image; this function is called noise pollution image function. In this experiment, the author used imnoise to add Gaussian noise to the image. In fact, imnoise function is widely used in MATLAB. In addition to adding Gaussian noise, imnoise function can also be used to add salt and pepper noise, Poisson noise, and multiplicative noise. When adding Gaussian noise to an picture, the author used the MATLAB expression g=imnoise(f,'Gaussian',m,var), this equation indicates that a Gaussian noise with average m, variance var is added to the picture f. The standard mean is 0 and the standard difference is 0.01. Since the imnoise function automatically normalizes the image, the standard deviation is divided by 255.

3. Wiener filtering

3.1. Numbering Wiener filtering principle

When a filtering method minimizes the mean square value of the evaluated mistake, a filter based on this filtering principle is called Wiener filter [6]. This filter was proposed by N. Wiener in 1942 as the best linear filtering method on account of the least mean square error formula, which requires the input process to be generalized and smooth [7]. In signal processing, Wiener filtering is a commonly used noise reduction method to extract the actual signal from noisy observations, and it has important applications in both speech and image signals [8]. Linearity refers to the linearity of the estimation form, and the minimum variance is the optimization criterion for the construction of the filter, which means that the dissimilarity between the practical signal and the gauged quantity should have the minimum variance. Wiener filter is to construct a filter so that the output of the observed semaphore after passing through the filter is the minimum mean square error estimate of the actual signa l[9].

3.2. Wiener filtering MATLAB implementation

The most common way to remove noise is to filter it with a filter. MATLAB image processing toolbox also designed a lot of filters. In this experiment, the author carried out Wiener filtering operation through Matlab function Winner2 at the beginning, and compared the SNR value after filtering with the SNR value of wavelet transform. Winner2 function is usually used to carry out Wiener filtering operation of images. After that, the author wrote the relevant code by Wiener filtering principle, first specified the size of the relevant Wiener filtering template, and then used the Filter2 function to calculate the mean value, variance, and noise of the image, obtained the relevant results, according to the Wiener method to design the correlation function, and finally improved. The specific ideas are as follows.

- Divide the photo into multiple MxN blocks and estimate the local mean and variance of the pixel.
- To estimate the noise power, the mean of local variances is taken as the noise power estimate value.
- The gray value of all pixels in the MxN block is estimated by Wiener method.
- Replace the original pixel value of the block with the estimated value.
- Combine multiple image blocks to obtain the filtered image.

3.3. SNR

Signal-to-noise ratio, also known by acronym SNR, is a type of a measurement employed in research and project to evaluate the grade of the requested source signal against intensity of the backdrop noise [10]. Signal-to-noise ratio is defined as the proportion of semaphore power to noise power. For example, when semaphore is greater than the noise power, the corresponding signal-to-noise ratio is also greater than 1 [11]. The following formula is the definitional formula for the signal-to-noise ratio: the proportion of effective semaphore power to noise power, that is, the proportion of amplitude squared. The specific formula is shown in Formula (2) [12].

$$SNR = \frac{P_{signal}}{P_{noise}} = \frac{A_{signal}^2}{A_{noise}^2} \tag{2}$$

In general, since the signal power is much greater than the noise power, researches usually write the expression in logarithmic form and express it in decibels (dB). Therefore, the signal-to-noise ratio is calculated as below, where several parameters P_signal, P_noise, A_signal, and A_noise represent signal power, noise power, signal amplitude, and noise amplitude, respectively. According to the Ross criterion, the signal-to-noise ratio often needs to be greater than 5 to specifically distinguish image details. The specific formula is shown in Formula (3) [13].

$$SNR = 10\log_{10}\left(\frac{P_{signal}}{P_{noise}}\right) = 10\log_{10}\left(\frac{A_{signal}^2}{A_{noise}^2}\right) = 20\log_{10}\left(\frac{A_{signal}}{A_{noise}}\right) \tag{3}$$

The signal-to-noise ratio is not limited to image processing, but can be ascribed to all forms of signals, and sometimes it refers to the proportion of valid in conversation or communication versus data that are not true or relevant. Regarding the calculated S/N parameter rate, the stronger the S/N factor, the more effective the picture filtering is.

In this study, the author used SNR as the first index to calculate the SNR of Wiener filter and wavelet transform to compare the effects of the two filtering methods, and used SNR index to calculate and compare the effects of Wiener filtering images with Gaussian noise under different standard deviations.

In the implementation of MATLAB simulation, the author completed the calculation of SNR with the help of norm function. Concrete MATLAB expression is as follows

SNR_J=20*log(norm(J,'fro')/norm(J-winner2(J),'fro')). Norm function, a function that can assign length and size to a vector in vector space, has the format n=norm(A,p), which supports the maximum singular position of A. max(svd(A)) n=norm(A ,p), which comes back with different amounts depending on p. In this experiment, the author set p as fro to calculate the square source of the diagonal sum of the matrix product of the two figures to calculate the amplitude.

3.4. PSNR

In general, Peak signal-to-noise ratio is also typically taken into account in calculating the signal and the noise contained in the signal for each measurement project. After a picture is contaminated with noise, the output picture will not be exactly the same as the original, no matter how accurate the noise rejection is [14]. In order to concretely show the noise filtering effect of the processed picture, PSNR is the most widespread and popular objective way of judging picture quality [15]. PSNR is the average mean squared error between the original source picture and the treated picture compared to the logarithm of (2^n-1)^2, and is expressed in dB. The author in Matlab implement the PSNR value by the following code, PSNR=10*log10((2^n-1)^2/MSE). And in the expression of the math, the expression of PSNR is shown in Formula (4) [16].

$$PSNR = 10 \times \log_{10}\left(\frac{(2^n-1)^2}{MSE}\right) \tag{4}$$

The author calculated the PSNR value with the help of the MSE value, which is mean squared error between the original photo and the processed photo [15].

If there are two m× N monochromatic images I and K, if one is the original picture and the other is a noise approximation picture of the other, then their mean square deviation is defined as Formula(5) [17].

$$MSE = \frac{1}{mn}\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}\|I(i,j) - K(i,j)\|^2 \tag{5}$$

The above equation represents the method of calculating PSNR for gray images, however, when researchers use color images directly without the operation of grayscale conversion, the definition of PSNR is similar to that of monochrome images, where the mean squared difference between two color images is the total of all pixel differences divided by the picture size and then divided by 3. PSNR is similar to SNR in that higher PSNR values represent better filtering of the image. In image compression, a typical PSNR level is in the range of 30-40 dB, with higher representing more effective results. If they are two of the same image, because their MSE value is 0, in this case the PSNR value can be defined as 0 can also be regarded as not [18].

In this experiment, the author implemented the operation of relevant PSNR by designing the following program: firstly, the relevant image size was obtained according to the size function, and then the PSNR function was encoded. In this process, the gray level is specified as 1, and the normalization process is carried out. Finally, the square of the difference between each pixel of the two images is iterated according to the formula, and the MSE value is obtained by dividing the result by the image size, and finally the PSNR value is calculated.

4. Wavelet transform

4.1. Principle of wavelet transform

The more conventional signal theory is concerned with the Fourier transform. But also because of limitations of the Fourier transform makes the Fourier transform does not allow for localized analysis, because the Fourier transform is a global variation. In order to make the Fourier transform more in line with expectations of researches, many scholars have improved the Fourier transform, such as the STFT, which is the short-time Fourier transform, to some extent to surmount the limitations of the conventional Fourier transform. On the other hand, wavelet analysis is developing, and wavelet analysis is the result of several achievements, such as Fourier transform, generalized function and numerical analysis [19]. Wavelet analysis is widely used in several fields, especially in affine studies. The distinction between wavelet transform and friction transform has been made, and the wavelet transform has been evaluated as an effective method of time and frequency analysis following the Fourier transform [20]. The distinction between wavelet transform and Fourier transform is that wavelet transform is more adapted to partial operations. Wavelet transform can effectively extract information from local transformations in the time and frequency domains, and can use more detailed analysis of functions such as stretching and translation. While the Fourier transform is not quite achievable, at the same time, many troubles that cannot work out by the Fourier transform can be easily settled by the wavelet transform. Wavelets directly replace the base of the Fourier transform - the limited extent of the trigonometric cornerstone for a finite length will decay wavelet cornerstone. This not only can get the frequency, but also can orientate the time.

Wavelet transform is well applied in image processing and signal processing, for example, wavelet transform can process image boundaries, such as border treatment and smoothing, time-frequency analysis, signal-to-noise split and abstraction of weak semaphore, finding fractal indices, semaphore recognition and diagnosis, and examine multi-scale edge.

4.2. Matlab simulation of wavelet transform

In the simulation implementation, the author realized the wavelet transform with the help of several functions at the beginning, including wavedec2 function and appcoef2 function. First of all, the author converted the picture into a double precision, using wavedec2 function carried on the two-dimensional wavelet transform, then used the db2 function and appcoef2 function of each layer of wavelet decomposition and the extraction of low frequency information. After applying the two-level wavelet transform, the author used the same function again to realize the two-level wavelet transform.

4.3. SNR and PSNR

For wavelet transform, the same analysis method is used to calculate the corresponding SNR value and PSNR value for comparison. When the wavelet transform is used for filtering, the author also reshaped the image into the original image size for convenient calculation. When adding Gaussian noise to the image, imnoise function is still used and Gaussian noise with the same standard deviation is added.

5. Comparison results

In this article, the author compared the lena containing Gaussian noise image in the experimental investigation by using the analysis system comes with the system first, and then the filtering process was performed using the system wiener filter function and the system wavelet transform function, and the corresponding SNR and PSNR values were calculated. The SNR value of the picture containing Gaussian noise was 46.7864 after one layer of Wiener filtering, and the SNR value of the picture size change after wavelet transform the picture, which was calculated to one layer of wavelet transform after processing by the author, was 14.0851. The comparison can be clearly seen that the effect of one layer of wiener filtering is stronger than that of wavelet transform. Subsequently, when comparing the PSNR, the author obtained a similar evaluation, the result of the Wiener filter was 76.0247 and the result of the wavelet transform was 49.7426. Therefore, the comparison results show that the denoising effect of one layer of Wiener filtering with Gaussian noise is obviously stronger than the denoising effect of one layer of wavelet transform with Gaussian noise. Based on this, the author tried to perform two-layer wavelet transform and secondary Wiener filtering, and the comprehensive data comparison and subjective judgment of the image showed that the difference between the original pixel point and the noise pixel

DOI: 10.54254/2755-2721/5/20230704

point of the two-layer wavelet processing was greater, and the denoising effect (SNR value) was improved. Also the secondary Wiener filtering effect has improved. Then, based on this, the author made relevant improvements to the Wiener filtering, used their own Wiener filtering function when performing Wiener filtering again, and conducted relevant research on Gaussian noise.

In this research, the author found that the processing ability of Wiener filtering decreases significantly as the standard deviation of Gaussian noise gradually increases, and the difference between the value of the standard deviation of Gaussian noise at 1, 2, 5, 10, 20 The SNR values of images containing Gaussian noise after Wiener filtering were 84.3271, 74.3080, 56.5008, 32.0294, 28.0774, 18.5966, and 13.1118 for standard differences of Gaussian noise of 1, 2, 5, 10, 20, and 100, respectively. Therefore, the analysis was performed. In fact, according to the principle of Wiener filtering, the smaller the mean square error in the Wiener filtering process, the better the filtering effect, and the difference between the desired output and the actual output image to find the average square is the mean square error, obviously, the larger the norm of Gaussian noise, the worse the denoising effect, so improving the filtering effect has been one of the directions that researchers have been working on in recent years, and in practical engineering, we can use other filtering methods for filtering operation. The second problem is that the SNR and PSNR values are generally not high after only one filtering operation. The author analyzed that only one filtering process can not effectively suppress all the noise, so the author carried out a second filtering process. After comparing the PSNR values, it was found that the value of the Wiener filter was 32.0294 after adding Gaussian noise with a standard deviation of 20 using only the system Wiener filter function, and the SNR value of the secondary Wiener filter was 32.3997, which was 0.3703 higher than that of the primary Wiener filter. In the case of the two-layer wavelet transform, the corresponding parameters of the image are improved, and secondly, the image becomes brighter compared to the one-layer wavelet transform. In the related research, image can be used in combination of several filtering methods in addition to the optimization of a single filtering method, such as combining wavelet transform and wiener filtering to study the Wiener filtering in wavelet domain.

6. Conclusion

When processing noisy images, Wiener filtering is applied to the images to make them smoother and less noisy. In comparison with the two filtering methods, Wiener filtering is more effective than wavelet transform, and Wiener filtering is more computationally intensive but better at removing noise. When the two filtering methods are analyzed separately, Wiener filtering works better for low standard deviation noise images than high standard deviation noise images. The secondary Wiener filtering outperforms the primary Wiener filtering and improves the PSNR and SNR values. The effect of the Wiener function and the Wiener filter system function written by the author is almost the same, with SNR values of 46.7864 and 46.7845, respectively, when the lena image that comes with the optimized system is a low standard deviation noisy image. while the wavelet transform of Gaussian noise also achieves noise reduction, although the effect is not significant. The images of the two-layer wavelet transform are brighter than those of the single-layer wavelet transform. Also, when wavelet dissociation is executed on the semaphore, the more layers are decomposed, the more obvious are the different characteristics of the noise and the original signal realizations. In this study, we analyzed the Wiener filter, implemented it by our own programming, and performed a comparative analysis to provide an idea. However, the improvement in this study is only limited to Wiener filtering, which does not give the best results. Future research may not should be limited to one type of filtering, but should combine multiple filtering methods to get better filtering effects.

References

[1] Min Y, Quan S and Zhihuo X 2021 Adaptive interference suppression of automotive millimeter wave radar based on Wiener filter Journal of Electronic Measurement and Instrumentation 10 TN974

[2] Wentu M, You J, Xiao L and Zhi Z 2019 Denoising method combining non-local mean and Wiener filter in wavelet domain Beijing Surveying and mapping 10 TN957.52

[3] Tudor B 2013 Variational Image Denoising Approach with Diffusion Porous Media Flow Abstract and Applied Analysis doi:10.1155/2013/856876
[4] Barry T 1999 Handbook for Acoustic Ecology (Cambridge:Cambridge Street Publishing)
[5] Philippe C 2012 Image Restoration: Introduction to Signal and Image Processing (MIAC, University of Basel)
[6] Chan R P, Seong-Hyeon K and Youngjin L 2020 Median modified wiener filter for improving
[7] the image quality of gamma camera images Nuclear Engineering and Technology 52 P 2238-2333
[8] Wiener N 1949 Extrapolation, Interpolation, and Smoothing of Stationary Time Series (New York:John Wiley & Sons)
[9] Benesty J, Paleologu C and Ciochina S 2010 On Regularization in Adaptive Filtering *IEEE Transactions on Audio, Speech, and Language Processing* vol. 19, no. 6 P 1734-1742
[10] Haykin S 2002 Adaptive Filter Theory(4th ed) (Prentice-Hall: Upper Saddle River, NJ, USA)
[11] Charles S and John B 2007 Transducers and Arrays for Underwater Sound (Berlin:Springer Science & Business Media) p 276
[12] Breeding A 2004 The Music Internet Untangled: Using Online Services to Expand Your Musical Horizons Giant Path ISBN 9781932340020
[13] Bushberg J T 2006 The Essential Physics of Medical Imaging(2e) (Philadelphia: Lippincott Williams & Wilkins) p 280
[14] Jiamin C, Yonggang L, Peijun Y, Chujie C and Liyuan F 2022 Automatic measurement of SNR and uniformity in MRI system quality control China Medical Devices 37 R445.2
[15] Chervyakov N, Lyakhov P and Nagornov N 2020 Analysis of the Quantization Noise in Discrete Wavelet Transform Filters for 3D Medical Imaging Applied Sciences 10 (4): 1223. doi:10.3390/app10041223. ISSN 2076-3417
[16] Rafael C G and Richard E W 2008 Digital image processing (Upper Saddle River: Prentice Hall) p 354
[17] Tania S 2008 Image fusion: algorithms and applications (Academic Press) p 471
[18] Jun G, Wenhai L, Zihao W and Xinjie S 2022 Research on multi-branch image denoising algorithm Computer Engineering and Applications TP391.41
[19] Ghaderpour E, Pagiatakis S D and Hassan Q K 2021 A Survey on Change Detection and Time Series Analysis with Applications *Applied Sciences* 11 (13) 6141. doi:10.3390/app11136141
[20] Jie L 2012 Shannon wavelet spectrum analysis on truncated vibration signals for machine incipient fault detection Measurement Science and Technology 23 (5) doi:10.1088/0957-0233/23/5/055604
[21] Akansu A N, Serdijn W A and Selesnick I W 2009 Emerging applications of wavelets: A review Physical Communication 3:1-18 doi:10.1016/j.phycom.2009.07.001