

# Fractionally spaced frequency domain equalization system based on FQPSK modulation

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**Abstract.** Severe multi-path effects in the process of high-speed data transmission cause inter-symbol interference (ISI). The single carrier frequency domain equalization (SC-FDE) method combining the benefits of Feher-patented quadrature phase shift keying (FQPSK) modulation is raised to solve this problem, including the constant envelope, high spectrum efficiency, and insensitivity to nonlinear distortion. Furthermore, a fractionally spaced frequency domain equalization (FS-FDE) system based on FQPSK modulation is modeled based on the proposed algorithm. Simulation results confirm the superior performance of the proposed model on erasing ISI, comparing to the symbol-spaced FDE (SS-FDE) system based on FQPSK modulation and the FS-FDE system based on QPSK modulation.

**Keywords:** multi-path effect, FQPSK, SC-FDE, fractionally spaced equalization.

## 1. Introduction

With the vigorous development of wireless communication technology, unmanned communication terminals are required to transmit voice, high-speed video, real-time data interaction, and other diversified services. With the increase of services and data transmission capacity, frequency band resources are increasingly strained, putting higher requirements for unmanned communication systems' frequency spectrum and power utilization ratio.

Feher-patented quadrature phase shift keying (FQPSK) [1-4], first proposed by Kato and Feher in 1983, is a modulation system proposed after weighing these two aspects and has been widely concerned in the deep space communication field. The modulated signal has the characteristics of the continuous phase, high spectral efficiency, and quasi-constant envelope. It is insensitive to nonlinear distortion, which makes the communication system use a high-efficiency nonlinear power amplifier, reduce hardware cost, and improve the efficiency of the power amplifier.

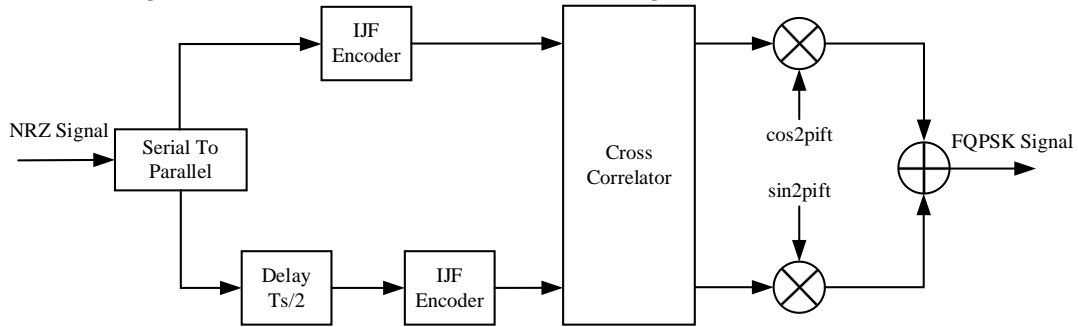
In the high-speed data transmission system, the wireless channel is complex and variable, resulting in the apparent multi-path effect of the wireless channel, which will cause inter-symbol interference (ISI) that will seriously affect the performance of the communication system. Therefore, the single carrier frequency domain equalization (SC-FDE) algorithm [5,6] is introduced to eliminate ISI. Compared with the single carrier time domain equalization (SC-TDE), fast Fourier transform (FFT) / inverse FFT (IFFT)

is used to process in the frequency domain, which reduces the computational complexity. Compared with orthogonal frequency division multiplexing (OFDM), it has the characteristics of a low peak-to-average power ratio (PAPR), insensitivity to frequency offset and synchronization, and higher overall efficiency, which is beneficial to reducing device volume and power dissipation [7].

Since FQPSK is a nonlinear modulation with memory, a fractionally spaced frequency domain equalization (FS-FDE) system based on FQPSK modulation is proposed. In contrast to the conventional system, fractionally spaced equalization is not sensitive to sampling timing deviation. The system has better demodulation performance due to the retention of the memory of FQPSK modulation during equalization. It's found that the FS-FDE system based on FQPSK has a superior ability to resist multi-path effects.

## 2. FQPSK modulation based on trellis coded

The predecessor of FQPSK modulation is inter-symbol interference and jitter-free offset QPSK (IJF-OQPSK). IJF-OQPSK modulation firstly performs IJF encoding for the in-phase (I) branch and orthogonal (Q) branch, respectively. Then it conducts offset-QPSK (OQPSK) modulation [2]. After IJF encoding, a cross-correlation arithmetic unit is added to FQPSK to reduce 3 dB envelope fluctuation. The schematic diagram of FQPSK modulation is shown in Figure 1.



**Figure 1.** The schematic diagram of FQPSK modulation.

The waveform transformation process of FQPSK can be interpreted as the trellis coded. The cross-correlation operation of FQPSK modulation is changed into mapping the input data of I and Q branches directly in every full symbol interval [8]. By defining  $d_{I,n}$  and  $d_{Q,n}$  as the polar signals input by I and Q branches respectively, the definition of mapping signals  $D_{I,n}$  and  $D_{Q,n}$  are shown in equation (1), and their value range is  $\{0,1\}$ .

$$D_{I,n} = \frac{1-d_{I,n}}{2}, \quad D_{Q,n} = \frac{1-d_{Q,n}}{2} \quad (1)$$

The index values  $i$  and  $j$  of the modulated waveform are represented by the binary-coded decimal (BCD) representation, which are shown as

$$\begin{aligned} i &= I_3 \times 2^3 + I_2 \times 2^2 + I_1 \times 2 + I_0 \\ j &= Q_3 \times 2^3 + Q_2 \times 2^2 + Q_1 \times 2 + Q_0 \end{aligned} \quad (2)$$

where

$$\begin{aligned} I_0 &= D_{Q,n} \oplus D_{Q,n-1}, \quad Q_0 = D_{I,n+1} \oplus D_{I,n} \\ I_1 &= D_{Q,n-1} \oplus D_{Q,n-2}, \quad Q_1 = D_{I,n} \oplus D_{I,n-1} = I_2 \\ I_2 &= D_{I,n} \oplus D_{I,n-1}, \quad Q_2 = D_{Q,n} \oplus D_{Q,n-1} = I_0 \end{aligned} \quad (3)$$

$$I_3 = D_{I,n}, \quad Q_3 = D_{Q,n}$$

The values of  $i$  and  $j$  are  $0, 1, 2, \dots, 15$ . The 16 kinds of baseband waveform are defined as  $s_i(t)$ ,  $i = 0, 1, \dots, 15$ , with the time interval of  $-\frac{T_s}{2} \leq t \leq \frac{T_s}{2}$ , where  $T_s$  is the symbol period. Furthermore,  $A$  is the correlation factor, the value range is  $[\frac{1}{\sqrt{2}}, 1]$ . When  $A = \frac{1}{\sqrt{2}}$ , the envelope fluctuation of modulated signal is close to 0 dB. Thus, the set of all the signal waveform of I and Q branches are shown by

$$\begin{aligned} s_0(t) &= A, \quad -\frac{T_s}{2} \leq t \leq \frac{T_s}{2} & s_8(t) &= -s_0(t) \\ s_1(t) &= \begin{cases} A, & -\frac{T_s}{2} \leq t \leq 0 \\ 1 - (1 - A) \cos^2 \frac{\pi t}{T_s}, & 0 \leq t \leq \frac{T_s}{2} \end{cases} & s_9(t) &= -s_1(t) \\ s_2(t) &= \begin{cases} 1 - (1 - A) \cos^2 \frac{\pi t}{T_s}, & -\frac{T_s}{2} \leq t \leq 0 \\ A, & 0 \leq t \leq \frac{T_s}{2} \end{cases} & s_{10}(t) &= -s_2(t) \\ s_3(t) &= 1 - (1 - A) \cos^2 \frac{\pi t}{T_s}, \quad -\frac{T_s}{2} \leq t \leq \frac{T_s}{2} & s_{11}(t) &= -s_3(t) \\ s_4(t) &= A \sin \frac{\pi t}{T_s}, \quad -\frac{T_s}{2} \leq t \leq \frac{T_s}{2} & s_{12}(t) &= -s_4(t) \\ s_5(t) &= \begin{cases} A \sin \frac{\pi t}{T_s}, & -\frac{T_s}{2} \leq t \leq 0 \\ \sin \frac{\pi t}{T_s}, & 0 \leq t \leq \frac{T_s}{2} \end{cases} & s_{13}(t) &= -s_5(t) \\ s_6(t) &= \begin{cases} \sin \frac{\pi t}{T_s}, & -\frac{T_s}{2} \leq t \leq 0 \\ A \sin \frac{\pi t}{T_s}, & 0 \leq t \leq \frac{T_s}{2} \end{cases} & s_{14}(t) &= -s_6(t) \\ s_7(t) &= \sin \frac{\pi t}{T_s}, \quad -\frac{T_s}{2} \leq t \leq \frac{T_s}{2} & s_{15}(t) &= -s_7(t) \end{aligned} \quad (4)$$

The baseband waveform of I and Q branches select  $s_i(t)$  and  $s_j(t)$  in equation (4) respectively according to index value  $i$  and  $j$ .

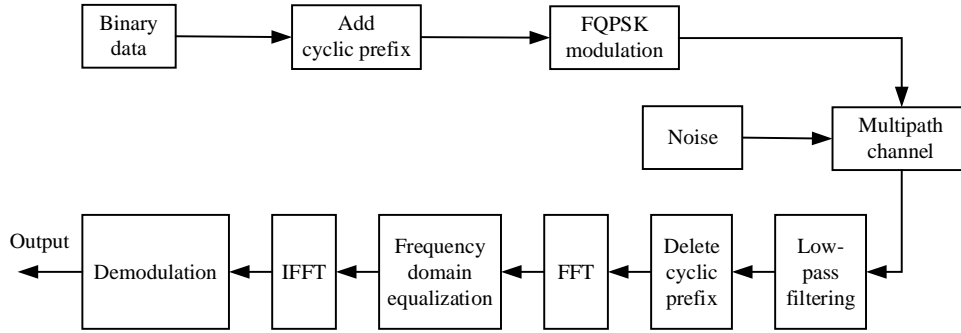
### 3. FS-FDE system based on FQPSK modulation

To reduce the influence of the ISI, an equalizer is usually implemented to compensate and correct the characteristics of the system. The time domain equalizer directly corrects the distorted waveform. In contrast, the frequency domain equalizer compensates the frequency parts of the channel or system to make the system approximately meet the condition of nearly no ISI.

### The structure of FS-FDE system

Current modulated symbol of FQPSK is associated with the preceding and following symbols while combining the conventional symbol spaced frequency domain equalization (SS-FDE) [9,10]. In this framework, the memory of the modulated symbol will be destroyed, and the subsequent demodulation will be affected. To solve this problem, this paper proposes the FS-FDE system based on FQPSK.

The schematic diagram of the FS-FDE system based on FQPSK is shown as Figure 2. SS-FDE conducts FDE on the symbol, while the fractionally spaced equalization system directly performs equalization based on the sampled information. In the FS-FDE system, the transmitter first inserts the cyclic prefix (CP) to the binary data and then performs FQPSK modulation. Meanwhile, the receiver firstly accomplishes the low-pass filtering on receiving the signal, then performs FFT by deleting the CP from the synchronized signal. After all, the channel estimation obtained based on the pilot is used to the FDE. The FDE results are converted to the time domain by IFFT and then performs demodulation. It should be concerned that, the FDE based on preserving the memory of modulated symbols could enhances the equalization system's bit error rate (BER) performance.



**Figure 2.** The schematic diagram of FS-FDE system based on FQPSK.

### Frequency domain equalization algorithm

If the transmitted symbol is represented as the vector  $\mathbf{x}_{N \times 1}$ , the channel's tap is  $L$ , the additive white Gaussian noise with the variance of  $\sigma_0^2$  is  $\mathbf{n}_{(N+L) \times 1}$ , the received symbol is represented as the vector  $\mathbf{y}_{(N+L) \times 1}$ , and the channel matrix with the taps  $h_l, l=0, \dots, L-1$  is expressed as  $\mathbf{H}_{(N+L) \times N}$ . The matrix of channel model is expressed as

$$\mathbf{y}_{(N+L) \times 1} = \mathbf{H}_{(N+L) \times N} \mathbf{x}_{N \times 1} + \mathbf{n}_{(N+L) \times 1} \quad (5)$$

In the FQPSK system, the baseband waveform of I and Q branches are selected in equation (4), and the number of samples in each symbol is expressed as the over-sampling rate  $M$ . Then, the channel model after over-sampling at the transmitter is shown by

$$\mathbf{y}_{fs} = \mathbf{H}_{fs} \mathbf{x}_{fs} + \mathbf{n}_{fs} \quad (6)$$

Where,  $\mathbf{x}_{fs}$  is  $[x_1, x_2, \dots, x_{M \times N}]^T$ ,  $\mathbf{y}_{fs}$  is  $[y_1, y_2, \dots, y_{M \times (N+L)}]^T$ ,  $\mathbf{n}_{fs}$  is  $[n_1, n_2, \dots, n_{M \times (N+L)}]^T$ , and  $\mathbf{H}_{fs}$  is a  $(M \times N + L) \times (M \times N)$  channel matrix. The FS-FDE algorithm adopts the minimum mean square error (MMSE) criterion [11,12]. The error  $\mathbf{e}_{(M \times N) \times 1} = \hat{\mathbf{x}}_{fs} - \mathbf{x}_{fs}$ , and  $\hat{\mathbf{x}}_{fs}$  is the expected response. When the mean square error shown in equation (7) is the minimum, the required equalizer can be obtained.

$$E[\|\mathbf{e}\|_2^2] = E\left[\sum_{i=1}^{M \times N} |e_i|^2\right] = E[\text{trace}(\mathbf{e}\mathbf{e}^H)] \quad (7)$$

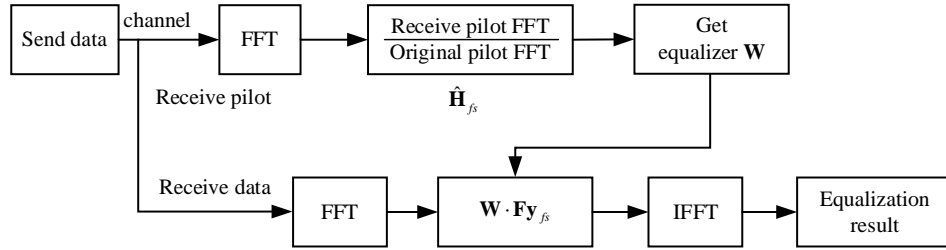
The coefficient of the MMSE equalizer is expressed as

$$\mathbf{W} = \text{diag}\left(\frac{\hat{H}_1^*}{|\hat{H}_1|^2 + \sigma^2}, \frac{\hat{H}_2^*}{|\hat{H}_2|^2 + \sigma^2}, \dots, \frac{\hat{H}_{M \times N}^*}{|\hat{H}_{M \times N}|^2 + \sigma^2}\right) \quad (8)$$

Approximate transmission data  $\hat{\mathbf{x}}_{fs}$  after equalizing can be obtained from

$$\hat{\mathbf{x}}_{fs} = \mathbf{F}^{-1}[\mathbf{W} \cdot \mathbf{F} \mathbf{y}_{fs((M \times N) \times 1)}] \quad (9)$$

where  $\mathbf{W}$  is the coefficient matrix of the equalizer,  $\hat{\mathbf{H}}_{fs} = [\hat{H}_1, \hat{H}_2, \dots, \hat{H}_{M \times N}]^T$  is the channel frequency response estimation, which can be obtained by dividing the FFT representation of the received pilot by the FFT representation of the original pilot [13]. Furthermore,  $P$  is the average power of the transmitted signal,  $\sigma^2 = \sigma_0^2/P$ ,  $\mathbf{F}$  is the Fourier transform matrix, and  $\mathbf{F}^{-1}$  is the inverse Fourier transform matrix. The schematic diagram of SC-FDE-MMSE algorithm is shown in Figure 3.

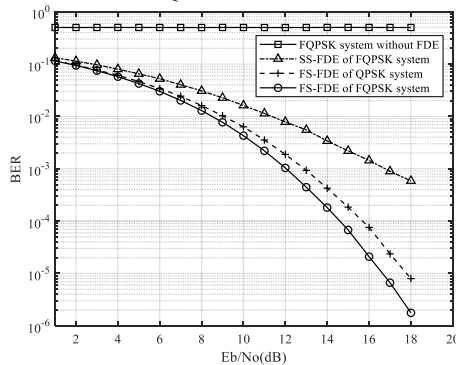


**Figure 3.** The schematic diagram of SC-FDE-MMSE algorithm.

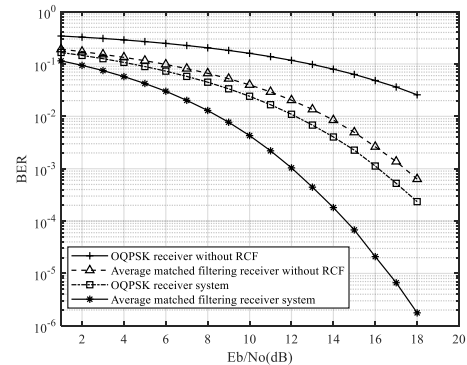
#### 4. Simulation results

Without loss of generality, the CP's length is assumed as 32 symbols, and the length of a single pilot as well as data segment is 128 symbols. The over-sampling rate is set to  $M = 8$ . Furthermore, the low-pass filter of the receiver uses a raised cosine filter (RCF) with the coefficient of 1, which is closer to the matched filter corresponding to the FQPSK modulation. Besides, the simulation channel uses the 10-path channels with the maximum multi-path delay of 2 microseconds, and the number of simulation channels is 10000.

Figure 4 shows the comparison of BER performance of QPSK and FQPSK systems added by FDE. It can be seen that the system with FDE algorithm has a certain ability to erase ISI, and FS-FDE system has better BER performance than SS-FDE system. When the BER is  $10^{-3}$ ,  $E_b/N_0$  increases about 4 dB. Furthermore, the BER performance of FS-FDE system based on FQPSK modulation is also better than that of FS-FDE QPSK system. In addition, if the BER reaches to  $10^{-5}$ ,  $E_b/N_0$  increases about 1 dB. Figure 5 shows that the system with RCF has the better BER performance, and the FS-FDE algorithm is suitable to the FQPSK modulation.



**Figure 4.** Performance of different modulation systems and equalization systems.



**Figure 5.** Performance of different FQPSK receiver systems.

## 5. Conclusions

In this paper, to deal with the problem of multi-path effect in the communication among unmanned aerial vehicles, under the background of increasingly limited frequency band resources, a FS-FDE algorithm based on FQPSK modulation is proposed, which takes advantage of the high spectrum efficiency and anti-multi-path characteristics of FQPSK modulation to enhance the performance of high-speed data transmission. Simulation results prove the better performance of the proposed FS-FDE system based on FQPSK modulation, on reducing the BER, improving spectrum efficiency.

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