DOI: 10.54254/2755-2721/6/20230809

An M/M/2 heterogeneous service encouraged arrival with feedback, reverse balking, reverse reneging and keeping of reneged customers

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Abstract: An infinite capacity This study develops a Markovian 2-server heterogeneous service encouraged arrival queuing model with the present challenges of customer feedback, customer impatience, customer keeping, and reverse balking. The model is then iteratively solved to produce several encouraged arrival probabilistic estimates and system performance measures. The results of the prescribed model , with regard to the modern scenarios described above are presented in this work that are having great value for developing strategies for the smooth operation of the system.

Keywords: encouraged arrival - reverse balking – reneging - heterogeneous service - impatient customers - keeping - feedback .

1. Introduction

The theoretical performance measures of the system can be theoretically determined in advance by organisations using the mathematical queuing model established in this research while taking into account a variety of behaviour of the costomers. Iteratively solving the model developed in this paper yields several encouraged arrival probabilistic measures and system performance measurements..

Reverse balking was first used by [1] to describe how, in many industries, such as restaurants and the healthcare industry, a large client base tends to draw in new customers more readily because they believe they will receive better or more reasonably priced service. [2] examined the single-channel queuing model and offered results for broad scenarios. Contrary to the traditional balking behaviour in the queuing theory investigated by [3, 4 and 5]. [6] examined numerical methods, analytical solutions, and simulation models while presenting a thorough examination of impatient behaviour in queuing models. Increased wait times frequently put strain on service providers, which leads to some customers

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experiencing discontent with the service they received and possibly starting over in the queue to receive satisfactory service. These clients are referred to as feedback customers. [7] examined the feedback queue to ascertain the length of the line and investigated a customer's average wait time distribution function. It was also investigated by [8] how long people wait on average in such queue systems. who also looked at line-ups where customers might either leave the queue permanently or re-join it with certain probability. [9] took into account single server lines with Bernoulli feedback and the impatient behaviour of the clients. Companies employ a number of retention techniques to keep these impatient customers because losing one means losing business. Reverse balking and feedback consumers were included in the finite capacity queuing model that [10] explored. They also looked at the idea of reverse reneging and retention. Using the concepts of reneging and retention of impatient consumers, [11] extended the work in [12] by examining an unlimited capacity queuing model with reverse balking and feedback. M/M/1/N queuing systems with encouraged arrival were examined by [13].

Reverse balking, reneging, and retention of reneged customers were explored in an M/M/2 heterogeneous service Markovian feedback queuing model .This paper along adding with encouraged arrival queuing model in[14]. an infinite capacity This work develops a Markovian two-server heterogeneous service encouraged arrival queuing model that takes into account the present challenges of customer feedback, customer irritation, customer retention, and reverse balking. The M/M/2 model is then theoretically solved to produce several encouraged arrival probabilistic estimates and system performance measurements.

The paper follows the following flow: In Section 2, a mathematical model is described. In Section 3, Model-probabilistic measures for the encouraged arrival solution is promoted. Encouraged arrival performance measures are covered in Section 4. Section 5 discusses the conclusion

2. Description of M/M/2 mathematical model

With the following presumptions, the conventional approach to developing an encouraged arrival queuing model that lists all mutually exclusive possibilities is used:

1 Arrivals subsequently encouraged the arrival process with a parameter $\lambda(1+\delta)$. δ represent discount values

2. Two servers, with varying service rates, μ_1 and μ_2, respectively, provide customer service. Exponentially spread service times are observed.

3. The system's capacity is not limited.

4. With an exponentially distributed parameter γ, a consumer in line for service may grow impatient after a certain amount of time 'T' and decide to leave the line.

5. By implementing a retention strategy, it is possible to keep a departing customer with probability q = (1-p).

6. With a chance a customer dissatisfied with the service may re-enter the line l= (1-k).

7. First in First served is the queue discipline.

8. ${}_pG_q$ Represents generalized H.G.F(Hyper Geometric Function)

9. Pochhammer symbol which is described as $(\alpha)_k$

10. For, $=0$, the probability of reverse balking is $q'=(1-p')$. For >0 , an arriving customer may reverse balking with the prob $\left(\frac{1}{m+1}\right)$ and may not reverse balking with the probability $\left(\frac{m}{m+1}\right)$.

Equations for the M/M/2 encouraged arrival queuing model in steady state are developed using the following premises:

$$0=-\lambda(1+\delta)p'P_0+\mu_1kP_{10}+\mu_2kP_{01}; m=0 \tag{1}$$

$$0=\lambda(1+\delta)p'\pi_1P_0-\left(\frac{\lambda(1+\delta)}{2}+\mu_1k\right)P_{10}+\mu_2kP_{11}; m=1 \text{ at server 1} \tag{2a}$$

$$0=\lambda(1+\delta)\ p^\wedge{}'\ \pi\_2\ P\_0-(\lambda(1+\delta)/2+\mu\_2\ k)\ P\_01+\mu\_1\ kP\_11; m=1\text{" } at\ server \text{ " 2(2b)}$$

$$0=\lambda(1+\delta)\left(\frac{m-1}{m}\right)P_{n-1}-\left\{\frac{\lambda(1+\delta)m}{m+1}+\mu_1k+\mu_2k+(n-2)\gamma p\right\}P_m+\{\mu_1k+\mu_2k+(m-1)\gamma p\}P_{m+1}; m\geq 2 \tag{3}$$

3. Model-probabilistic measures for the encouraged arrival solution

The following probabilistic metrics of the model were obtained through an iterative process:

$$P_1 = \frac{\left[\frac{\lambda(1+\delta)}{2}+\mu_1 k\pi_2+\mu_2 k\pi_1\right]}{[\lambda(1+\delta)+\mu_1 k+\mu_2 k]}\cdot\frac{\lambda(1+\delta)}{\mu_1\cdot\mu_2 k}\cdot(\mu_1+\mu_2)\cdot p' P_0 \tag{4}$$

$$P_2 = \frac{\left[\frac{\lambda(1+\delta)}{2}+\mu_1 k\pi_2+\mu_2 k\pi_1\right]}{[\lambda(1+\delta)+\mu_1 k+\mu_2 k]}\cdot\frac{1}{2}\cdot\frac{\lambda(1+\delta)}{\mu_1 k}\cdot\frac{\lambda(1+\delta)}{\mu_2 k}\cdot p' P_0 \tag{5}$$

$$P_m = \frac{\left[\frac{\lambda(1+\delta)}{2}+\mu_1 k\pi_2+\mu_2 k\pi_1\right]}{[\lambda(1+\delta)+\mu_1 k+\mu_2 k]}\cdot\frac{1}{n}\cdot\frac{\lambda(1+\delta)}{\mu_1 k}\cdot\frac{\lambda(1+\delta)}{\mu_2 k}\cdot\left\{\prod_{s=3}^{m}\frac{\lambda(1+\delta)}{[\mu_1 k+\mu_2 k+(s-2)\gamma p]}\right\} p' P_0, m\geq 3 \tag{6}$$

using the standard of normality$\sum_{m=0}^{\infty} P_m = 1$, we get

$$\begin{aligned} P_0 = \quad & P_l\{\text{empty }\} \\ & = \left[1+o\left(\frac{\lambda(1+\delta)}{2}+\mu_1 k+\mu_2 k\right)-\frac{o}{2}\{\lambda(1+\delta)+2(\mu_1 k+\mu_2 k) \right. \\ & \left.\left[1-{}_{\mathrm{p}}G_{\mathrm{q}}\left((1,1);\left(2,\frac{(\mu_1 k+\mu_2 k)}{\gamma p}\right);\left(2,\frac{\lambda(1+\delta)}{\gamma p}\right)\right)\right]\right\}\Bigg]^{-1} \end{aligned} \tag{7}$$

where $o = \frac{\left[\frac{\lambda(1+\delta)}{2}+\mu_1 k\pi_2+\mu_2 k\pi_1\right]}{[\lambda(1+\delta)+\mu_1 k+\mu_2 k]}\cdot\frac{\lambda(1+\delta)p'}{\mu_1\mu_2 k^2}$ and ${}_pG_q$ represents generalized H.G.F and this computation of expression through Wolfram Alpha. $pG_q(\alpha;\beta;o)$ has series expansion. $\sum_{s=0}^{\infty}(\alpha_1)_s \dots (\alpha_p)_s / (\beta_1)_s \dots (\beta_q)_s o^s/s!$, where $(\alpha)_s$ symbol representing increasing factorial and is defined asymbol for rising factorial, which is described as $(\alpha)_k = \alpha(\alpha+1)(\alpha+2)\dots(\alpha+k-1)$.

4. Encouraged arrival performance measures

Probabilistic measurements have been calculated, now we calculate the following performance measures :

1 L_k\{ system size anticipated) $= \sum_{m=0}^{\infty} mP_m$
2 L_{q}\{ Expected length of the queue) $= \sum_{m=3}^{\infty}(m-2)P_m$
3 R_{l}\{ Estimated Reneging Rate) $= \sum_{m=3}^{\infty}(m-2)\gamma q P_m$
4 R_{L}\{ Rate of Keeping Anticipated) $= \sum_{m=3}^{\infty=3}(m-2)\gamma p P_m$
5 $R_{\mathrm{L}\beta}$\{ Reverse Balking Expected Rate) $= \sum_{m=0}^{\infty}\left(\frac{1}{m+1}\right)\lambda(1+\delta)p' P_m$.

5. Conclusion

We developed a reverse balking, reneging, and keeping of reneged client's infinite capacity Markovian two server heterogeneous feedback encouraged arrival queuing model. This model addressed the actual, pressing contemporary issues. The mathematical findings will be extremely helpful for corporate organisations that are facing the aforementioned difficulties. The encouraged arrival model is obtained to measure the system's overall performance theoretically. By implementing the encouraged arrival approach, organization's measures will be increased and will enable better customer service and hence the organisation will stay ahead in the situation of fierce competition.

References

[1] Jain, N. K., Kumar, R., & Som, B. K. (2014). An M/M/1/N queuing system with reverse balking. American Journal of Operational Research, 4(2), 17-20.
[2] Robert, E. (1979). Reneging phenomenon of single channel queues. Mathematics of Operations Research, 4(2), 162-178.
[3] Ancker, C. J., Jr., & Gafarian, A. V. (1963). Some queuing problems with balking and reneging I. Operations Research, 11, 88-100.

DOI: 10.54254/2755-2721/6/20230809

[4] Ancker, C. J., Jr., & Gafarian, A. V. (1963). Some queuing problems with balking and reneging II. Operations Research, 11, 928-937.

[5] Haight, F. A. (1957). Queuing with balking I. Biometriika, 44, 360-369.

[6] Wang, K., Li, N., & Jiang, Z. (2010). Queuing system with impatient customers: A review. In 2010 IEEE International Conference on Service Operations and Logistics and Informatics (pp. 82-87), July 15-17, 2010, Shandong.

[7] Takács, L. (1963). A single-server queue with feedback. Bell System Technical Journal, 42(2), $505-519$.

[8] Davignon, G. R., & Disney, R. L. (1976). Single server queue with state dependent feedback. INFOR, 14, 71-85.

[9] Santhakumaran, A., & Thangaraj, V. (2000). A single server queue with impatient and feedback customers. Information and Management Science, 11, 71-79.

[10] Kumar, R., & Som, B. K. (2015). An M/M/1/N feedback queuing system with reverse balking, reverse reneging and retention of reneged customers. Indian Journal of Industrial and Applied Mathematics, 6(2), 173-183.

[11] Som, B. K., & Seth, S. (2017). A multi-server infinite capacity markovian feedback queuing system with reverse balking. In Fifth International Conference on Business Analytics and Intelligence, December 11-13, 2017, IIM Bangalore

[12] Som, B. K., & Seth, S. (2018). A stochastic multi-server infinite capacity feedback queuing system with reverse balking, customer impatience and retention of impatient customers. In The 13th International Conference on Queuing Theory and Network Applications (QTNA2018), July 25-27, 2018, Tsukuba, Japan.

[13] Bhubendra kumar som,sunny seth, An M/M/1/N queuing system with encouraged arrivals,Global journal and pure applied mathematics vol 13,no7 (2017).

[14] Bhubendra kumar som Vivek Kumar Sharma and Sunny Seth, An M/M/2 Heterogeneous Service Markovian Feedback Queuing Model with Reverse Balking, Reneging and Retention of Reneged Customers, Advanced in computing and intelligent systems pp291-296,(2020)