Proceedings of the 3rd International Conference on Signal Processing and Machine Learning
DOI: 10.54254/2755-2721/6/20230811

Diffusion-limited aggregation

Junze Huang[1,3,†] , Binyang Xu[2,†]

[1]Department of Mathematics, Pennsylvania State University, State college, The United States
[2]Department of Applied Mathematics, Ocean University of China, Qingdao, China

[3]jfh5948@psu.edu
†These authors contributed equally

Abstract. Models of physical processes from the fields of physics and chemistry may be used to create a variety of beautiful visuals and structures that resemble living things. Diffusion limited aggregation, also known as DLA, is an illustration of this. It is a term used to explain, among other things, how zinc ions diffuse and assemble onto electrodes in an electrolytic solution. "Diffusion" refers to the random movement of the particles that make up the structure before they adhere ("Aggregate") to it. "Diffusion-limited" refers to the fact that the structure grows one particle at a time rather than in chunks of particles because the particles are thought to be in low concentrations, preventing them from coming into touch with one another. Other instances include the development of coral, the route of lightning, the coalescence of dust or smoke particles, and the development of certain crystals.Nature uses a variety of interesting and often simple processes to produce amazing shapes, patterns, and forms that span all scales and never cease to surprise and enlighten the keen observer. From the From the micro to the macro, matter uses a variety of logical, observable processes to arrange, enhance, and transform, often stacking on top of each other in complex ways. In this paper, Mathematicians will discuss a process called diffusion-restricted aggregation or DLA, which uses randomly moving and "sticky" particles to produce fractal branching structures.

Keywords: diffusion-limited aggregation, random walk, non-linear aggregation.

1. Introduction

Witten and Sander proposed the diffusion limited aggregation (DLA) model in 1981. Numerous pattern-forming processes, including as dielectric breakdown, two-fluid flow, and electrochemical deposition, have been proven to be explained by the model. The model starts with one particle fixed at the origin of coordinates in d dimensions. Next, a cluster is created by releasing a random walker from infinity, which is then allowed to roam until it collides with any particle in the cluster. Once there, the new particle is let loose from infinity and joined to the expanding cluster. The model was investigated both on and off the lattice in multiple dimensions d 2; DLA has garnered a lot of attention throughout the years as a stunning illustration of fractal objects [1-20].This is a research report on diffusion-limited Aggregation. Diffusion-limited aggregation is a process in which particles of matter stick together (aggregate) as they chaotically move (diffuse) through a medium that provides some sort of resistive (limiting) force. HoMathematiciansver, in this study, Mathematicians added another TRAP. As the name implies, when random points move to TRAP, they will stop moving. Trap will invalidate the lattice for which trap (i, j) == true. And pTr is associated with trap. When the pTr goes lager the greater the probability that the lattice trap (i, j) == true. When particles are moving around freely, they are called walkers. When they get stuck together, they are collectively called clusters. Mathematicians will also complete the study based on this process. Models of physical processes from the fields of physics and chemistry may be used to create a variety of beautiful visuals and structures that resemble living things. Diffusion limited aggregation, also known as DLA, is an illustration of this. It is a term used to explain, among other things, how zinc ions diffuse and assemble onto electrodes in an electrolytic solution. "Diffusion" refers to the random movement of the particles that make up the structure before they adhere ("Aggregate") to it. "Diffusion-limited" refers to the fact that the structure grows one particle at a time rather than in chunks of particles because the particles are thought to be in low concentrations, preventing them from coming into touch with one another [1-20]. Other instances include the development of coral, the route of lightning, the coalescence of dust or smoke particles, and the development of certain crystals. The DLA process over the real axis, which is quite visible, illustrates the usage of iterated conformal maps to represent cluster expansion. The relative simplicity is due to the fact that the size of the strike is independent of the order of iteration, unlike the usual off-lattice DLA in two dimensions, where the strike size changes with each iteration to allow the insertion of a fixed size particle in the physical domain. This allows one to precisely estimate the growth rate and fractal dimension of the entire cluster or of certain trees.

2. Organization of the text

2.1. Aggregation model with no trap

The first model Mathematicians'll look at is aggregation with no Traps. So, the model grows from a random point in the center of the circle, and they can move left or right on the lattice, and gradually it will fill up the whole circle. In this model, Mathematicians have 200 lattices, and Mathematicians have 10,000 particles. The smaller the number of lattices, the faster it will grow, and the faster it will go through the circle, but it actually determines the size of the model, so it seems like slow or fast, but they have the same speed. The aggregation model with no trap is shown as in Figure 1.

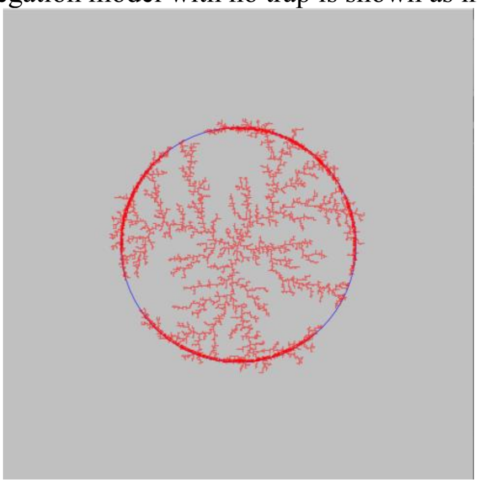


Figure 1. Aggregation model with no trap.

2.2. Aggregate dynamic model

The next one is aggregation dynamic. Aggregation dynamic is the simultaneous expansion of the radius of the circle as aggregate grows, in which case it will not grow through the circle. And Mathematicians can see that this is what happens when it doesn't go through the circle. But it's important to note that when the lattice is small, it might go through the boundary. The aggregation dynamic model is shown as below in Figure 2.

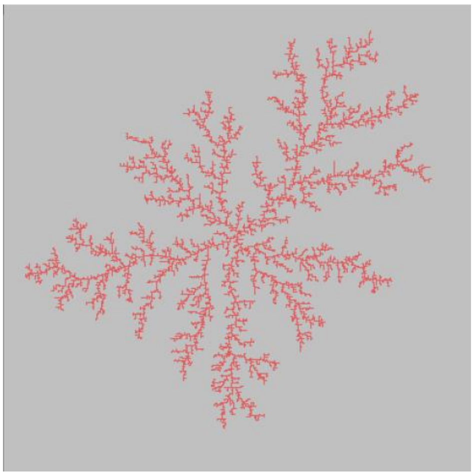
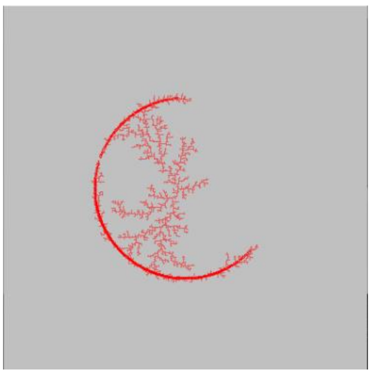
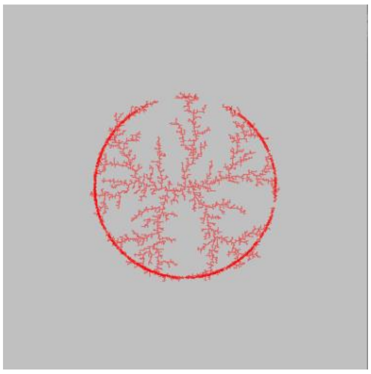


Figure 2. Aggregate dynamic model.

2.3. Aggregation model with traps

The last one is aggregation with traps. Here is the difference in the image it produces in the case of different pTr. Trap will invalidate the lattice for which trap (i, j) == true. And pTr is associated with trap. When the pTr goes lager the greater the probability that the lattice trap (i, j) == true. In other words, when the random walker goes into trap it would disappear. When the pTr is large, they will fill part of the circle, and the development of the whole model will be more concentrated. When the pTr is small, it develops more evenly. Three different types are shown respectively in Figure 3, Figure 4, and Figure 5.





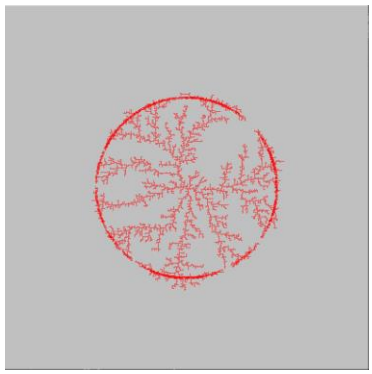
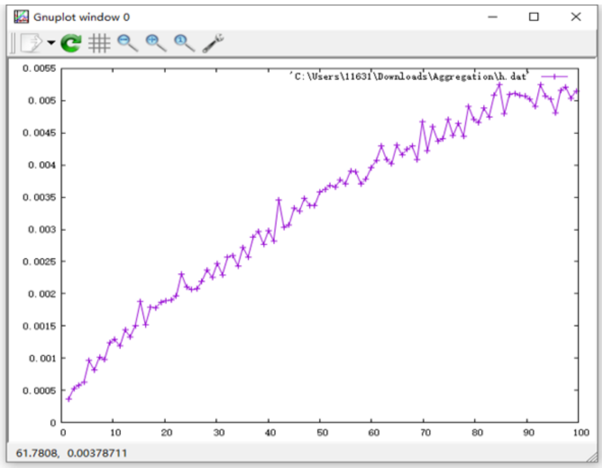


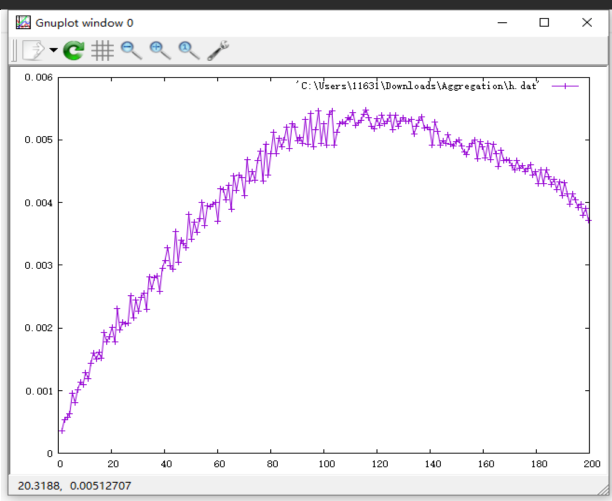
Figure 3. pTr = 0.001. **Figure 4.** pTr = 0.0001. **Figure 5.** pTr = 0.00001.

And then after that Mathematicians can get the distance betMathematiciansen the points and so on and figure out its fractal dimension and the 2-point correlation function. When the distance is small, the dependence is poMathematiciansrful. When the distance becomes larger, the 2-point correlation function will go to zero. Images of 2 point correlation function are shown in Figure 6.

DOI: 10.54254/2755-2721/6/20230811




```
Give Number of particles in the cluster, N:9994
Give Number of histogram bins, M:100
Give Rmin for the histogram:1
Give Rmax for the histogram:100
SumX = 3.652077
SumY = -5.8402944
SumXX= 14.118452
SumXY= -20.818743
beta=0.65377975 ;  b=2.670838E-4
   D=1.6537797451019287
```




```
Give Number of particles in the cluster, N:9994
Give Number of histogram bins, M:200
Give Rmin for the histogram:1
Give Rmax for the histogram:200
SumX = 4.3251395
SumY = -5.5816107
SumXX= 19.563368
SumXY= -23.72572
beta=0.48512265 ;  b=4.6205407E-4
   D=1.4851226508617401
```

Figure 6. Image of 2 point correlation function.

DOI: 10.54254/2755-2721/6/20230811

2.4. The fractal dimension

2.4.1. Introduction for fractal dimension. Fractal dimension is known as the Fractal theory of nature geometry. It is a new branch of modern mathematics, but its essence is a new world outlook and methodology. Fractal dimension reflects the efficiency of space occupied by complex forms and is a measure of the irregularity of complex forms. It is cross-combined with the chaos theory of dynamical systems and complement each other. It acknowledges that parts of the world may, under certain conditions or in certain processes, show similarities with the whole in a certain aspect (form, structure, information, function, time, energy, etc.), and it acknowledges that the change of spatial dimension can be either discrete or continuous, thus expanding the horizon.

2.4.2. The method of obtaining fractal dimension. Use this formula to get a series of different values of N by changing the value of R, and then build a linear model in turn. Finally, the slope of the line that comes out of this is going to be the D that needed to be figured out.

$$D = -\log N / \log R \quad (1)$$

In this way, use the related r min and r max that figured out before, then input all the relevant variables and it will be computed by a compiled Java program. The result looks like picture 1. The corresponding numreical result is given in Figure 7.

```
Give Number of particles in the cluster, N:18000
Give Number of histogram bins, M:30
Give Rmin for the histogram:2
Give Rmax for the histogram:180
SumX = 4.246721
SumY = -4.1798415
SumXX= 18.783379
SumXY= -17.263876
beta=0.65008307 ;  b=9.6772396E-4
   D=1.650083065032959
```

Figure 7. Solution for D.

2.5. The nonlinear regression

2.5.1. Confirm the model. Express rho in R and put it into the code according to the derivation formula in lecture which is shown below:

$$\rho_2(R) = \pi \frac{2R}{N_{pairs} a^2} \propto R^{D-1} \quad (2)$$

Analyze a set of data that came out and make an image out of these points which is shown in figure 2. Look at this image and ensure the model like below in Fig.8:

DOI: 10.54254/2755-2721/6/20230811

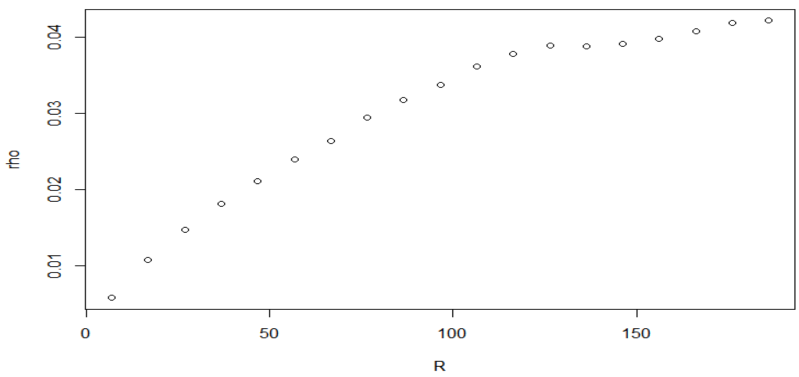


Figure 8. Solution for D.

Look at this image and ensure the model like below:

$$rho = aR^{D-1} + b \quad (3)$$

2.5.2. Complete the model. Therefore, construct a new variable based on D obtained before, and make this variable directly proportional to the dependent variable in order to complete this nonlinear regression. Finally, this project by using RStudio and got the nonlinear fitted formula shown in formula 4.

$$rho = 0.0038595R^{D-1} + 0.0014048 \quad (4)$$

2.6. The discussion of D(pTr)

2.6.1. Introduction. Make it clear that D is the exponent of the typing dimension and pTr is the probability that each unit becomes a TRAP. And here are a couple of pictures showing different D's for different probabilities which are created by Java. The correspoingding results are given in Figure 9, Figure 10 and Figure 11.

```
Give Number of particles in the cluster, N:15000
Give Number of histogram bins, M:30
Give Rmin for the histogram:2
Give Rmax for the histogram:180
SumX = 4.246721
SumY = -4.042356
SumXX= 18.783379
SumXY= -16.732367
beta=0.5801618 ;  b=0.0014942375
   D=1.5801618099212646
```

Figure 9. D for pTr = 1e-4.

DOI: 10.54254/2755-2721/6/20230811

```
Give Number of particles in the cluster, N:18000
Give Number of histogram bins, M:30
Give Rmin for the histogram:2
Give Rmax for the histogram:180
SumX = 4.246721
SumY = -4.210805
SumXX= 18.783379
SumXY= -17.415373
beta=0.6233658 ;  b=0.0010509525
   D=1.6233658194541931
```

Figure 10. D for pTr = 1e-5.

```
Give Number of particles in the cluster, N:18000
Give Number of histogram bins, M:30
Give Rmin for the histogram:2
Give Rmax for the histogram:180
SumX = 4.246721
SumY = -4.155113
SumXX= 18.783379
SumXY= -17.15652
beta=0.65320873 ;  b=9.788711E-4
   D=1.6532087326049805
```

Figure 11. D for pTr = 1e-6.

2.6.2. The analysis reason. In terms of function expression, when the cardinal number is >0, the larger D is, the faster mass grows when x expands; on the contrary, it is obvious that the cardinal number is greater than zero in this model, that is, the larger D is, the more the number of Particles increases with R. My understanding of pTr is that the larger the pTr is, the higher the probability of each unit becoming trap and the less Particles can walk, that is, the less Particles in a circle with radius R. So at first Mathematicians guessed that as pTr gets bigger, D gets loMathematiciansr. HoMathematiciansver, after adjusting parameters, calculation and testing data for many times, Mathematicians obtained the following results. It can be seen from the data in the figure that the size of D does change with the size of pTr, but the trend of change is different from our prediction. When pTr <= 1e-5, D decreases with the increase of pTr. HoMathematiciansver, for individual values, D fluctuates greatly around pTr = 1e-5, and the peak value also appears around it. When pTr > 1e-5, D is similar to that at pTr = 0. D is very close and stable. Based on the above data results and conclusions, Mathematicians believe that when pTr increases (greater than a certain value), the probability of each unit becoming TRAP is higher, and the part of Particles that can be moved is indeed less, and D is smaller. HoMathematiciansver, when pTr is too small, Particles are not obstructed, which may be too easy to randomly gather, and D tends to be stable. HoMathematiciansver, an appropriate pTr can sometimes cause Particles to gather and limit the development of Particles to a distance, thus easily causing D to peak in a certain range of R. The corresponding results are given in Figure 12 and Figure 13.

DOI: 10.54254/2755-2721/6/20230811

```
Give Number of particles in the cluster, N:18000
Give Number of histogram bins, M:30
Give Rmin for the histogram:2
Give Rmax for the histogram:180
SumX = 4.246721
SumY = -4.2030168
SumXX= 18.783379
SumXY= -17.37041
beta=0.63924384 ;  b=9.901061E-4
   D=1.6392438411712646
```

Figure 12. D for pTr = 1e-7.

```
Give Number of particles in the cluster, N:18000
Give Number of histogram bins, M:30
Give Rmin for the histogram:2
Give Rmax for the histogram:180
SumX = 4.246721
SumY = -4.202597
SumXX= 18.783379
SumXY= -17.362791
beta=0.6470389 ;  b=9.582546E-4
   D=1.6470388770103455
```

Figure 13. D for smaller pTr.

3. Summary

The diffusion limited aggregation (DLA) model was presented by Witten and Sander in 1981. The model has been shown to describe a wide range of pattern-forming processes, such as dielectric breakdown, two-fluid flow, and electrochemical deposition. At the beginning of the model, one particle is fixed at the d-dimensional coordinate system's origin. A random walker is then let loose from infinity to build a cluster, after which it is permitted to roam until it collides with any particle within the cluster. The new particle is then released from infinity and added to the growing cluster once it has arrived there. The model was examined in various dimensions d 2 both on and off the lattice; DLA has attracted a lot of interest over the years as a magnificent example of fractal objects. First of all, Mathematicians may draw the conclusion that modeling a (off-lattice) DLA using a square lattice approach is a severe simplification, changing both the DLA's size and structure. Mathematicians have also observed that when Mathematicians increase the impact of the square lattice using the minValue principle, the dimension doesn't change considerably but the structure changes. Therefore, using the minValue noise reduction approach to increase the visibility of some attributes is a smart idea. Implementing an off-lattice approach is an option if Mathematicians don't want these negative impacts. But this is less effective and more difficult. Mathematicians may infer from the results that the DLA is not significantly altered by the simplifications used for the off-lattice DLA. In conclusion, the use of iterated conformal maps to describe cluster expansion is demonstrated by the DLA process over the real axis in a manner that is comparatively transparent. The reason for the relative simplicity is because, unlike the traditional off-lattice DLA in two dimensions, where the strike size varies with each iteration to accommodate the addition of a fixed size particle in the physical domain, the size of the strike is independent of the order of iteration. As a result, in this instance, one may accurately determine the growth rate and fractal dimension of the entire cluster or of specific trees.

References

[1] Mais Khaskia, Dina Shpasser, Roy Cohen, Omer Yehezkeli, Ofer Manor, Oz M. Gazit. First-Principle Colloidal Gate for Controlling Liquid and Molecule Flow Using 2D Claylike Nanoparticles. ACS Applied Materials & Interfaces 2022, 14 (28) , 32657-32664. https://doi.org/10.1021/acsami.2c05077

[2] Rong Sheng Li, Jiahui Liu, Yuan Yan, Chuanyang Su, Tiancheng Lai, Yunwen Liao, Yuan Fang Li, Na Li, Cheng Zhi Huang. Aggregation-Enhanced Energy Transfer for Mitochondria-Targeted ATP Ratiometric Imaging in Living Cells. Analytical Chemistry 2021, 93 (34) , 11878-11886. https://doi.org/10.1021/acs.analchem.1c02833

[3] Marc Mueller, Samaneh Rasoulinejad, Sukant Garg, Seraphine V. Wegner. The Importance of Cell–Cell Interaction Dynamics in Bottom-Up Tissue Engineering: Concepts of Colloidal Self-Assembly in the Fabrication of Multicellular Architectures. Nano Letters 2020, 20 (4) , 2257-2263. https://doi.org/10.1021/acs.nanolett.9b04160

[4] Kristian Schneider, Inga Melnyk, Karl Hiekel, Andreas Fery, Günter K. Auernhammer, Alexander Eychmüller. Mechanical Characterization of Self-Supported Noble Metal Gel Monoliths. The Journal of Physical Chemistry C 2019, 123 (45) , 27651-27658. https://doi.org/10.1021/acs.jpcc.9b08607

[5] Kasturee Nayak, Subhasish Sahoo, Priyadarsi De. Chirality and solvent assisted gelation modulation with stearoyl appended macromolecules. European Polymer Journal 2022, 177 , 111439. https://doi.org/10.1016/j.eurpolymj.2022.111439

[6] Soumyajit Roy. Time-Gel. Journal of the Indian Chemical Society 2022, 157 , 100431. https://doi.org/10.1016/j.jics.2022.100431

[7] Yu Xiong, Xinmin Liu, Hailing Xiong. Aggregation modeling of the influence of pH on the aggregation of variably charged nanoparticles. Scientific Reports 2021, 11 (1) https://doi.org/10.1038/s41598-021-96798-3

[8] Zaoming Wang, Christian Villa Santos, Alexandre Legrand, Frederik Haase, Yosuke Hara, Kazuyoshi Kanamori, Takuma Aoyama, Kenji Urayama, Cara M. Doherty, Glen J. Smales, Brian R. Pauw, Yamil J. Colón, Shuhei Furukawa. Multiscale structural control of linked metal–organic polyhedra gel by aging-induced linkage-reorganization. Chemical Science 2021, 12 (38) , 12556-12563. https://doi.org/10.1039/D1SC02883A

[9] Emily Olson, Fei Liu, Jonathan Blisko, Yifan Li, Ayuna Tsyrenova, Rebecca Mort, Keith Vorst, Greg Curtzwiler, Xin Yong, Shan Jiang. Self-assembly in biobased nanocomposites for multifunctionality and improved performance. Nanoscale Advances 2021, 3 (15) , 4321-4348. https://doi.org/10.1039/D1NA00391G

[10] Prithwish Biswas, Pankaj Ghildiyal, George W. Mulholland, Michael R. Zachariah. Modelling and simulation of field directed linear assembly of aerosol particles. Journal of Colloid and Interface Science 2021, 592 , 195-204. https://doi.org/10.1016/j.jcis.2021.02.050

[11] Elisha Martin, Martin Prostredny, Ashleigh Fletcher, Paul Mulheran. Modelling Organic Gel Growth in Three Dimensions: Textural and Fractal Properties of Resorcinol–Formaldehyde Gels. Gels 2020, 6 (3) , 23. https://doi.org/10.3390/gels6030023

[12] Fabian Matter, Ana Laura Luna, Markus Niederberger. From colloidal dispersions to aerogels: How to master nanoparticle gelation. Nano Today 2020, 30 , 100827. https://doi.org/10.1016/j.nantod.2019.100827

[13] Anna Gialitaki, Alexandra Tsekeri, Vassilis Amiridis, Romain Ceolato, Lucas Paulien, Emmanouil Proestakis, Eleni Marinou, Moritz Haarig, Holger Baars, Dimitris Balis, , , , , . Is Near-Spherical Shape "the New Black" for Smoke ?. EPJ Web of Conferences 2020, 237 , 02017. https://doi.org/10.1051/epjconf/202023702017

[14] Karunamuni L. Silva, Leenah Silmi, Stephanie L. Brock. Effect of metal ion solubility on the

oxidative assembly of metal sulfide quantum dots. The Journal of Chemical Physics 2019, 151 (23) , 234715. https://doi.org/10.1063/1.5128932
[15] Michael Kanetidis, Maria Tsouchnika, Panos Argyrakis, Raoul Kopelman. A simple model for charged particle aggregation and polarization. Physica A: Statistical Mechanics and its Applications 2019, 528 , 121433. https://doi.org/10.1016/j.physa.2019.121433
[16] Swetlana Jungblut, Jan-Ole Joswig, Alexander Eychmüller. Diffusion- and reaction-limited cluster aggregation revisited. Physical Chemistry Chemical Physics 2019, 21 (10) , 5723-5729. https://doi.org/10.1039/C9CP00549H
[17] Matheus Brendon Francisco, Diego Morais Junqueira, Guilherme Antônio Oliver, João Luiz Junho Pereira, Sebastião Simões da Cunha Jr Jr, Guilherme Ferreira Gomes. (2021) Design optimizations of carbon fibre reinforced polymer isogrid loMathematiciansr limb prosthesis using particle swarm optimization and Lichtenberg algorithm. Engineering Optimization 53:11, pages 1922-1945.
[18] Urol Makhmanov, Abdulmutallib Kokhkharov, Sagdilla Bakhramov. (2019) Organic fractal nano-dimensional structures based on fullerene C60. Fullerenes, Nanotubes and Carbon Nanostructures 27:3, pages 273-278.
[19] Röder, Holger, Karsten Bromann, Harald Brune, and Klaus Kern. "Diffusion-limited aggregation with active edge diffusion." Physical review letters 74, no. 16 (1995): 3217.
[20] Meakin, Paul. "Diffusion-limited aggregation in three dimensions: results from a new cluster-cluster aggregation model." Journal of colloid and interface science 102, no. 2 (1984): 491-504.