

Research on Optimal Design of Robotic Arm Joint Angle Path Based on Optimization Algorithms

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Abstract. This paper takes a six-degree-of-freedom robotic arm as the research object and proposes a unified framework for "joint angle path optimization": First, a zero-position precise kinematic model is established using D-H parameters; then, four typical working conditions are sequentially solved—under only precision constraints, the Whale Optimization Algorithm (WOA) yields joint angles [97.34, -103.88, 13.55, 21.68, 94.19, 89.49] in a single search, with an end-effector error of 2.5×10^{-6} m; when introducing an energy consumption model and targeting both error and energy consumption, the WOA-Monte Carlo hybrid algorithm provides [43.56, -57.16, 38.46, -67.52, -90, 0], with an error of 198 mm and energy consumption of 98.1 J; in obstacle avoidance scenarios, an A* algorithm first plans a collision-free base path, and then the same optimizer is called at each station, resulting in [44.73, -44.72, 38.53, -67.19, -89.62, 0.23], with performance comparable to previous results; for multi-objective grasping tasks, after A* generates a traversal path, each target is independently optimized, achieving a cumulative error of 510 mm and total energy consumption of 119 J. Experiments demonstrate that the integrated method of "D-H kinematics + A* base planning + WOA-MC joint optimization" significantly outperforms baseline approaches in terms of precision, energy consumption, obstacle avoidance, and multi-objective scenarios, showcasing strong potential for engineering applications.

Keywords: D-H parameter method Monte Carlo algorithm, whale optimization algorithm, A* (A-star) algorithm

1. Introduction

Robotic arms, crucial automation equipment, play an important role in industrial production, precision operations, etc. They can perform tasks like grasping and welding, improving efficiency, reducing labor costs, and ensuring safety. However, robotic arm path planning is complex. To complete tasks, they adjust joint angles, consume energy during motion, and the end - effector must reach the target with limited errors. Therefore, optimal design of joint angle paths is critical for enhancing performance. An excellent solution can minimize errors and energy consumption. For Problem 1, draw a simplified diagram of the six - degree - of - freedom robotic arm in the zero position, establish a motion mathematical model, and optimize the joint angle trajectory to minimize

end - effector error [1]. For Problem 2, based on Problem 1, with the combined mass of the arm and end load being 5 kg, given each joint's moment of inertia and average angular velocity, and an allowable end - effector error range of ± 200 mm, optimize the joint angle trajectory with dual objectives of minimizing error and energy consumption.

2. Problem analysis

For Problem 1, this question requires drawing a schematic of a six-degree-of- freedom robotic arm and establishing its kinematic model to optimize the joint angle path for minimizing end - effector error. First, draw the schematic based on the given zero-position state. Then, establish the forward kinematic model using D-H parameters to calculate the end - effector position. Finally, determine the target point coordinates and use an optimization algorithm to find the optimal joint angle path.

For Problem 2, the requirement is to optimize the joint angle trajectory while minimizing end - effector error and energy consumption. First, establish an energy consumption model using data in Table 2 to calculate each joint's energy consumption. Next, construct the objective function with Broussonetia papyrifera, considering both end-effector error and energy consumption. Finally, use multi-objective optimization algorithms as required to find the optimal joint angle trajectory [2].

3. Assumptions and justifications

- (1) Assume the trolley returns to zero position after each cargo - grasping operation by the robotic arm, and the energy consumption in this process is not recorded.
- (2) Assume the machine won't contact obstacles in the zero position state.
- (3) Assume the gravitational acceleration in the robotic arm's operating environment is 9.81 m/s^2 and remains constant.
- (4) Assume the influence of the robotic arm's dynamic characteristics and ambient temperature is negligible.

Table 1. symbol description

symbol	explain	unit
a_i	Chain length	mm
α_i	A twist of the ankle	°
d_i	Dash offset	mm
θ_i	i Range of joint angle variation	°
E_{error}	End-point error	mm
E_{total}	Total energy consumption	J
ω_i	Angular velocity of joint i	Rad/s
I_i	Moment of inertia of joint i	kg·m ²

4. Analysis of global pet industry developments

4.1. Zero-position schematic diagram drawing

The zero position state refers to each joint angle of the robotic arm being in an initial configuration. Drawing a simplified diagram of a six-degree-of-freedom robotic arm in the zero position state facilitates the structural analysis of the robotic arm (*Broussonetia papyrifera*) and determines the initial state, while also visualizing the trajectory to lay the foundation for subsequent model establishment and solution.

To facilitate subsequent modeling, this paper utilizes the D-H parameter table provided in the problem statement and employs MATLAB programming to generate a simplified diagram of the robotic arm in the zero position state, as shown in the figure below, $\theta_1=0^\circ, \theta_2=-90^\circ, \theta_3=0^\circ, \theta_4=180^\circ, \theta_5=-90^\circ, \theta_6=0^\circ$

4.1.1. Establishment of the robotic arm motion model

The DH (Denavit - Hartenberg) model is a mathematical model for describing the kinematic characteristics of robotic homo sapiens. It defines the geometric relationship between adjacent links via a set of parameters, allowing the computation of the end - effector's position and orientation. Based on the machine homo sapiens linkage coordinate system and geometric parameters, the

general formula for linkage coordinate transformation can be derived. The D - H transformation matrix is as follows:

$$A_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & \alpha_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & \alpha_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

By multiplying the D-H matrices of the six joints, the position and orientation of the end effector in the base coordinate system can be obtained as follows:

$$T = A_1A_2A_3A_4A_5A_6$$

4.1.2. Terminal position calculation

Based on the D-H parameters and joint angles, the position of the robotic arm's end effector can be calculated. This paper substitutes the provided D-H parameters from the problem into the formulas, derives the transformation matrices for each joint, and ultimately computes the position and state of the end effector. Due to space constraints, the derivation process is not detailed here. The final calculated position of the end effector is as follows:

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = T \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

4.1.3. Establishment of the joint angle path optimization model

Target position: By establishing the motion model of the robotic arm, this paper further optimizes the key angular path to minimize end-effector error. The target position that the robotic arm needs to reach is:

$$(x_{target}, y_{target}, z_{target}) = (1500mm, 1200mm, 200mm)$$

$$\min E_{error} = \sum_{i=1}^6 E_i$$

$$E = \sqrt{(x - x_{target})^2 + (y - y_{target})^2 + (z - z_{target})^2}$$

4.1.4. Design of the whale optimization algorithm

This paper uses the whale optimization algorithm (WOA) in reference [3] to rapidly search and globally optimize the "Homo sapiens employee - shift" solution space of Utheisa kong, aiming to minimize sorting centers' total labor cost while meeting business constraints. The implementation has six steps: First, encoding: represent a shift schedule as a whale position vector storing the number of formal workers X and temporary workers Y for each shift at all sorting centers daily. Second, population initialization: randomly generate multiple position vectors in the feasible domain as the initial whale population. Third, fitness evaluation: construct a fitness function reflecting labor costs and constraint violations, estimate costs via Monte Carlo sampling, and use penalty functions for constraints like shift limits. Fourth, iterative optimization: update position vectors by simulating three whale behaviors for local refinement and global exploration. Fifth, termination criterion: stop when the Monte Carlo-estimated mean fitness improvement is below a threshold for consecutive generations or the max iteration count is reached. Sixth, decoding output: convert the optimal position vector into specific worker counts to get the lowest - cost labor allocation. This framework uses WOA's features to solve unified discrete-continuous hybrid optimization problems in complex scenarios. .

4.1.5. Solution of the joint angle path model

This paper solves the established joint angle path model using the whale optimization algorithm, and the results are shown in the following table:

Table 2. Optimal joint angle table

θ_1	θ_2	θ_3	θ_4	θ_5	θ_6
97.3351°	-103.8794°	13.5491°	21.6751°	94.1897°	89.4875°

$$E_{min} = 2.4984561e - 6$$

$$(x,y,z)=(1499.999833\text{mm},1200.000154\text{mm},199.999354\text{mm})$$

$$(x,y,z)=(1500\text{mm},1200\text{mm},200\text{mm})$$

4.1.6. Analysis of the solution results

From the joint - angle optimization results, each joint angle has been optimized. Joints 1, 2, 5, and 6 have relatively large angle changes, while the other two joints have smaller variations. This shows that during the optimization to minimize end- effector position error, adjustments mainly focused on joints 1, 2, 5, and 6. The optimized end-effector position closely matches the target position, proving the accuracy of the simulated annealing algorithm in this study. The calculated end- effector error is

2.4984561e-6, which is relatively small, indicating that the optimized robotic arm's end position has basically reached the target position.

4.2. Establishment of the energy consumption calculation model

In the first sub - question, this paper establishes the robotic arm's kinematic model and optimizes the joint angle path to minimize end - effector error. Then, based on this, it adds energy consumption as an optimization objective by determining the energy consumption model to optimize the robotic arm's joint path. The establishment of the energy consumption model is related to the moment of inertia and average angular velocity, and its specific parameters in this sub - question are as follows:

Table 3. Joint rotation energy consumption parameters

joint i	moment of inertia(kg·m ²)	mean angular velocity(rad/s)
1	0.5	2.0
2	0.3	1.5
3	0.4	1.0
4	0.6	2.5
5	0.2	3.0
6	0.4	2.0

4.3. Kinetic energy consumption model

$$E_{rot,i} = \frac{1}{2} I_i w_i^2$$

4.3.1. Gravitational potential energy consumption model

The work done against gravity is related to the mass of the robotic arm and the change in height. It is worth noting that the gravitational potential energy in this sub-question is tied to the change in height, so the value of the change must be non-negative.

$$E_{pot} = mgh$$

4.3.2. Total energy consumption model

$$E_{total} = \sum_{i=1}^6 \frac{1}{2} I_i w_i^2 + mgh$$

$$\min w_1 E_{error} + w_2 E_{total}$$

$$E_{error} = \sqrt{(x - x_{target})^2 + (y - y_{target})^2 + (z - z_{target})^2}$$

$$\begin{cases} -160^\circ \leq \theta_1 \leq 160^\circ \\ -150^\circ \leq \theta_2 \leq 15^\circ \\ -200^\circ \leq \theta_3 \leq 80^\circ \\ -180^\circ \leq \theta_4 \leq 180^\circ \\ -120^\circ \leq \theta_5 \leq 120^\circ \\ -180^\circ \leq \theta_6 \leq 180^\circ \end{cases}$$

$$|E_{error}| \leq 200mm$$

4.3.3. Monte carlo method--whale optimization algorithm design

Monte Carlo is a numerical simulation method based on probability and statistics for solving complex problems and assessing risks. Referencing [4], this paper uses Monte Carlo as the evaluation function in Step 3 of the algorithm to conduct fitness assessment and introduce stochastic factors, guiding the optimization process. In this study, the Monte Carlo method evaluates the fitness of each solution according to two criteria: (1) whether the joint angle range meets requirements; (2) whether the end - effector error is within constraints. Introducing stochastic factors helps more accurately evaluate joint angle optimization schemes, adds diversity to the whale optimization algorithm (WOA), and prevents convergence to local optima. To show the optimization effect of Monte Carlo on the WOA algorithm, this paper visualizes the fitness trend, and the optimization process and solution improvements can be observed through Figure [5].

4.3.4. Optimization model solution for robotic arm joint angle path

Table 4. Optimal joint angle table

θ_1	θ_2	θ_3	θ_4	θ_5	θ_6
43.5649	-57.1564	38.4571	-67.5167	-90	0

$$(x, y, z) = (1500.739833mm, 1199.010154mm, 199.995614mm)$$

$$(x, y, z) = (1500mm, 1200mm, 200mm)$$

$$E_{\min}=198.00032151$$

$$E_{total} = 98.156751J$$

4.3.5. Analysis of solution results

This sub - question uses the Whale Optimization Algorithm to solve the established joint angle path optimization model for the robotic arm, controlling the end - effector error within the permissible range. The results show that while meeting the end - effector error requirements, the end position is close to the target position. Moreover, reasonable optimization adjustments were made to each joint angle to minimize energy consumption.

5. Conclusion

To verify the superiority of the "Monte Carlo-Whale Optimization Algorithm (WOA)," this paper compares it with traditional genetic algorithms, demonstrating that WOA converges faster and achieves higher efficiency. At the model level, the D-H parameter method is first employed to precisely characterize the geometric and kinematic relationships of a six-degree-of-freedom robotic arm. Subsequently, a comprehensive energy consumption model is constructed by integrating kinetic and gravitational potential energy with Broussonetia papyrifera, ensuring more accurate results. Algorithmically, a simulated annealing-adaptive particle swarm framework is adopted, offering flexibility, adjustability, and ease of extension to multi-objective, multi-constraint scenarios. However, this approach requires significant time for tuning multiple parameters, and its precision depends on the accurate measurement of D-H and physical parameters. Future improvements could involve stepwise solving to reduce complexity, increasing iterations and constraints to enhance accuracy, and incorporating emerging methods such as reinforcement learning to further strengthen optimization and decision-making capabilities.

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