Proceedings of the 2023 International Conference on Mechatronics and Smart Systems
DOI: 10.54254/2755-2721/9/20230116

Performance comparison between LQR and PID controllers for two-wheeled self-balancing vehicle

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Abstract. The two-wheeled balancing vehicle is an energy-efficient and eco-friendly transportation tool used for short distances. Both motors of the vehicle are required both motors to be regulated to maintain stability, and modern control methods have been applied to the system. In this paper, the effectiveness of traditional PID control and modern LQR control in the context of a two-wheeled balancing vehicle is evaluated. The motion principle of the vehicle is analyzed, and a corresponding dynamic model is established. PID and LQR control schemes are designed according to the model and the performance of the angle and speed of the vehicle is compared in MATLAB simulation. The two kinds of control performance are evaluated, offering theoretical feasibility verification for the application of two-wheeled balancing vehicles. It has been found that the LQR controller has better performance than the PID controller. LQR has a shorter response time and smaller steady-state error in controlling speed, and there is still room for improvement for both controllers.

Keywords: two-wheeled self-balancing vehicle, PID controller, LQR controller.

1. Introduction

The two-wheeled balancing vehicle boasts its simple structure, flexible movement, easy operation, energy efficiency, and eco-friendliness, as well as high portability. This kind of vehicle has gained considerable attention as a new, modern mode of transportation because of these features [1]. As a short distance transportation tool for people, it can be used in a wide range of scenarios, such as providing security personnel patrol in airports or stations, allowing citizens to enjoy themselves in parks or shopping malls.

The two-wheeled balancing vehicle has two wheels driven independently by motors. To maintain system stability, the controller must regulate the output torque of both motors. The controller ensures that the vehicle's center of gravity is positioned above the wheel shaft, allowing the balancing vehicle to remain in a balanced position [2]. The vehicle is a standard underactuated system. The only control input variables are the torques of the left and right motors, meaning just two control quantities. Additionally, the vehicle is a multi-input multi-output (MIMO) nonlinear system. With only two wheels, it is naturally unstable without any control exerted over it. Different from four-wheel mobile vehicle, two-wheeled balancing vehicle is more sensitive to disturbance in the process of motion control [3]. With advancements in control theory, modern control methods such as neural network control [4,5], fuzzy control, active disturbance rejection control (ADRC) [6], adaptive control [7], and

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other approaches have been applied to the control system of two-wheeled balancing vehicles, achieving positive results.

In order to evaluate and compare the effectiveness of traditional PID control and modern LQR control, the motion principle of the two-wheeled balancing vehicle is analyzed, and the corresponding dynamic model is established. According to the model, the PID and LQR control scheme is designed. The performance of the angle and speed of the vehicle is compared in MATLAB simulation, and the two kinds of control performance are compared and evaluated, offering theoretical feasibility verification for the application of two-wheeled balancing vehicles.

2. System modeling

The mechanical structure of the two-wheeled balancing vehicle is mainly composed of a body and two coaxial wheels, which is similar to the mobile inverted pendulum model [1]. In this paper, the Newton method is employed to analyze the wheel and body of a vehicle, establish the mathematical model, and ultimately derive the state equation of the entire system.

To simplify the analysis, it is assumed that the vehicle's positive x-axis represents the forward direction, and the positive y-axis represents the vertical upward direction. Since actual driving conditions are more complex, reasonable assumptions are made to simplify the analysis. It is considered that the wheels will not slide when the balancing car is running, and air resistance and friction are ignored [8].

Table 1 gives the symbols and their physical meanings for analysis and modeling.

Table 1. Two-wheeled balancing vehicle parameters

Variable	Definition	Variable	Definition
m_b	The mass of the body	θ	Pitch angle of the body
m_w	The mass of the wheel	$\dot{\theta}$	Pitch angular velocity of the body
l	Distance between the wheel axle and centroid of the body	J_b	Moment of inertia of the body with respect to wheel axle
R	Radius of the wheel	J_w	Moment of inertia of the wheel rotating around the wheel axle
x	Displacement of the wheel	T	The driven torque of wheel
$\dot{x}$	Velocity of the wheel		

The two-dimensional model diagram of the two-wheeled balancing vehicle is shown in the Figure 1. First of all, the mechanical analysis of the wheel is carried out. Assuming that the parameters of the wheels (mass and radius) are the same, the force analysis of the right wheel is shown below:

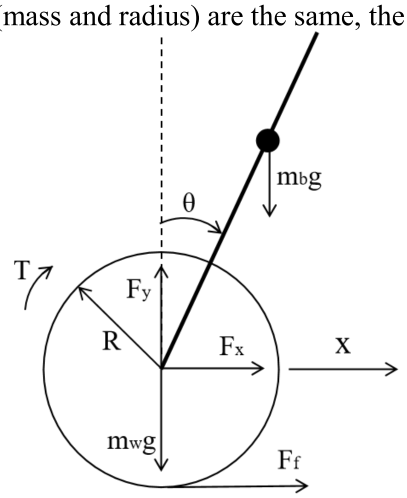

Figure 1. Two-dimension model of the vehicle.

$$F_f - F_x = m_w \ddot{x} \tag{1}$$

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$$J_w \frac{\ddot{x}}{R} = T - F_f R \tag{2}$$

where F_x is the horizontal component of the body's force on the wheel, F_f is the friction force between the floor and the wheel.

By applying Newton's force analysis to the body shown in Figure 1, the following equations can be obtained:

$$F_x = m_b \frac{\partial^2}{\partial t^2}(x + \sin\theta) \tag{3}$$

$$F_y - m_b g = m_b x \frac{\partial^2}{\partial t^2}(l \cos\theta) \tag{4}$$

$$J_p \theta =- F_x l \cos\theta + F_y l \sin\theta - T \tag{5}$$

where F_y is the vertical component of the body's force on the wheel.

By manipulating the dynamic equations from (1) to (5), the state variable for x and θ can be obtained:

$$\ddot{x} = \frac{TR + m_b l R^2 \sin\theta}{m_w R^2 + m_b R^2 + J_w} \tag{6}$$

$$\theta = \frac{T - l^2 m_b \cos^2\theta + l^2 m_b \cos\theta \sin\theta + \frac{l m_b (l m_b R^2 \sin\theta + TR)}{J_w + R^2 m_b + R^2 m_w}}{J_b} \tag{7}$$

To maintain balance of the two-wheeled self-balancing vehicle, the inclination angle of the body is near 0°. Based on this, the following assumptions can be made:

$$\begin{cases} \sin\theta \approx \theta \\ \cos\theta \approx 1 \end{cases} \tag{8}$$

By selecting state variable $Z = [\theta \quad \dot{\theta} \quad x \quad \dot{x}]^T$, the equation of state of the system can be written as:

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \\ \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ A_{21} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ A_{41} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ B_{11} \\ 0 \\ B_{41} \end{bmatrix} [T] \tag{9}$$

where

$$\begin{cases} A_{21} =- \frac{l^2 m_b + 2 l^2 m_b \theta - g l m_b - l^2 m_b \theta^2 + \frac{R^2 l^2 m_b{}^2}{\alpha} - \frac{l m_b \theta (l m_b \theta R^2 + TR)}{\alpha}}{J_b} \\ A_{41} = \frac{R^2 l m_b}{\alpha} \\ B_{21} =- \frac{\frac{R l m_b}{\alpha} + 1}{J_b} \\ B_{41} = \frac{R}{\alpha} \\ \alpha = J_w + R^2 m_b + R^2 m_w \end{cases} \tag{10}$$

The value of the vehicle is related variable designed in the paper are given in Table 2:

Table 2. The corresponding value of parameters.

Variable	Value	Variable	Value
m_b	0.527 kg	J_w	0.001384 kg · m^2
m_w	15.071 kg	R	0.07 m
J_b	0.474886 kg · m^2	l	0.024162 m

Substituting the parameters into the state equation yields the following formula:

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \\ \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 7.4863 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0.0229 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ -2.7956 \\ 0 \\ 0.8996 \end{bmatrix} [T] \tag{11}$$

3. Design of PID controller

The proportional-integral-differential (PID) control strategy is commonly utilized in industrial control processes, such as brushless DC motor closed-loop speed control and quadcopter unmanned aerial vehicle PID controller, due to its straightforward algorithm, dependable performance, and high reliability [9-11].

In computer-control system, the input and output of the controller are related through a PID relationship. Specifically:

$$u(k) = K_p e(k) + K_i \sum_{i=0}^{k} e(i) + K_d[e(k) - e(k-1)] \tag{12}$$

where K_p, K_i, and K_d are coefficients that respectively control the proportional, integral, and derivative behavior. k is the index of the sample, with k = 0,1, 2, $u(k)$ is the output value of the k-th sample, and $e(k)$ is the input deviation of the k-th sample.

K_p plays a key role in controlling feedback to achieve the target value but may cause oscillation. K_i reduces steady state error but increases overshoot. K_d provides damping to inhibit oscillation and overshoot but reduces response speed.

In this paper, the motion of a two-wheeled balancing vehicle can be divided into two parts: balance and speed change. Each part is controlled by a separate PID controller. The balance controller keeps the vehicle balanced by controlling the output of the wheels. The speed controller adjusts the inclination angle to achieve speed control, which has evolved into changing the motor speed to control wheel speed. The speed controller and balance controller are connected in series, and the overall control structure is depicted in Figure 2.

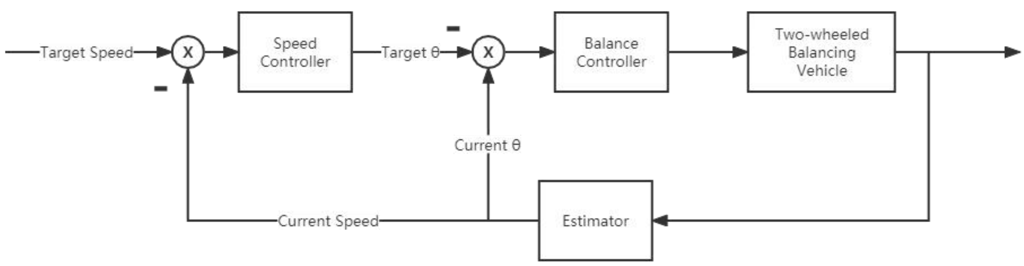

Figure 2. Diagram of PID controller.

The speed controller parameters are set as K_p = *0.14*, K_i = *0.012*, and K_d = *0.001*. The balance controller parameters are set as K_p = *7*, K_i = *0.5*, and K_d = *1.5*. These values have been found to be effective in achieving optimal performance.

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4. Design of LQR controller

Linear Quadratic Regulator (LQR) is a control design method used for linear systems represented in state-space form in modern control theory. The LQR optimal algorithm is significant in mathematics because it helps identify a state feedback control matrix K. This matrix improves the system's performance by optimizing tracking and minimizing the quadratic performance index function J [12].

$$J = \frac{1}{2}\int_0^{\infty} (x^T Q x + u^T R u) dt \tag{13}$$

where Q and R are the positive definite weight matrices for the state variables and control inputs, respectively.

The LQR algorithm is used to introduce state feedback for the relevant state variables and their differentials in the two-wheeled self-balancing vehicle. This approach achieves control of the inclination angle and position of the vehicle. The overall control structure is depicted in Figure 3.

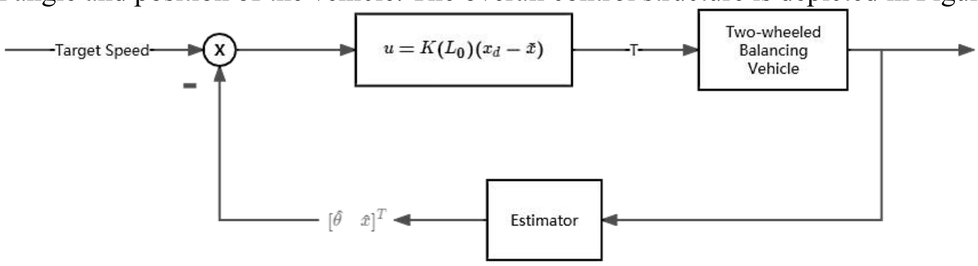

Figure 3. Diagram of LQR controller.

The state variable feedback matrix K for the system can be obtained using the MATLAB command function K = lqr(sys,Q, R). By comparing the simulation results with different weights in Matlab, it has been found that the system has better control performance when $Q = \begin{bmatrix} 100 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 50 \end{bmatrix}$ and $R = 0.1$, and the corresponding state variable feedback matrix is $K = [-78.0881 \quad -25.9818 \quad -31.6228 \quad -49.6101]^T$.

5. Analysis and results

The simulation results for the PID and LQR control methods of a two-wheeled self-balancing vehicle are presented, and their performances are compared. Figures 4 show the control responses of the system from the initial condition $Z = [0 \quad 0.698 \quad 0 \quad 0]^T$ to the equilibrium point under the PID and LQR controllers, respectively. Table 3 displays the corresponding performance characteristics of the inclination angle θ.

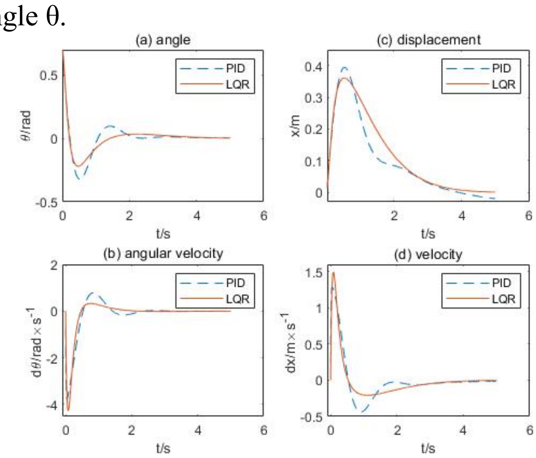

Figure 4. Control effectiveness between PID and LQR controller from initial condition to equilibrium point.

Table 3. Summary of performance characteristics for vehicle's angle.

Time Response	PID	LQR
Settling Time(T_s)	2.04 s	1.32 s
Rise Time(T_r)	0.23 s	0.21 s
Steady State Error (e_{ss})	4.75×10^{-3} rad	2.46×10^{-3} rad

When comparing the two controllers in Figure 4, it can be found that the PID controller exhibits a larger overshoot in controlling the angle, angular velocity, displacement, and velocity as compared to LQR. Additionally, the settling time for the PID controller is 2.04s, which is longer than LQR's settling time of 1.32s. Both controllers achieve close to zero steady-state error when controlling the angle and displacement, thereby allowing the vehicle to achieve good stationary performance.

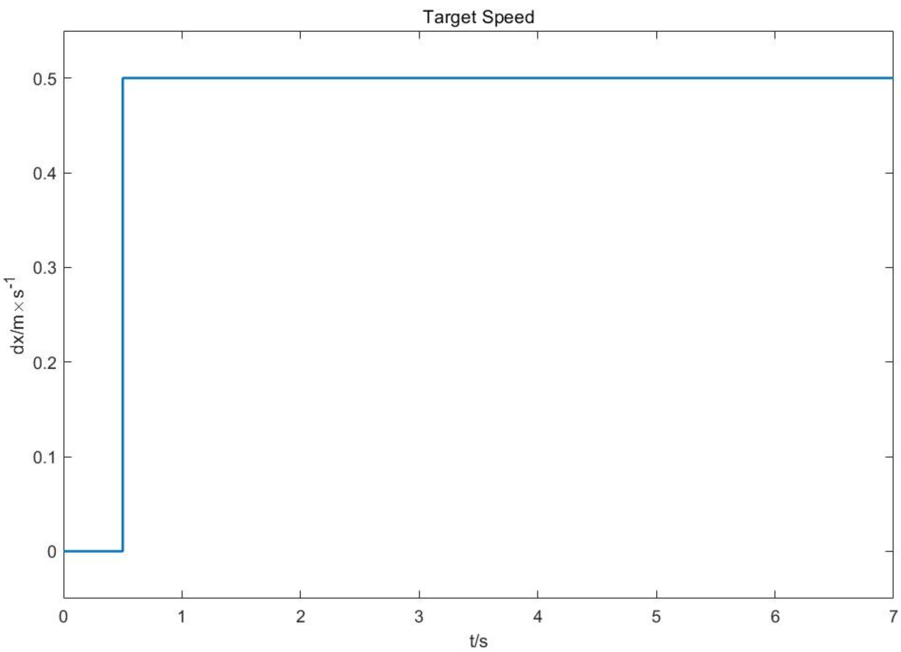

Figure 5. Step input signal with a target speed of 0.5 m/s.

Once the vehicle has reached its stable equilibrium point, a step signal is applied to it with a target speed of 0.5 m/s, as shown in Figure 5. Figure 6 displays the system's response under PID control and LQR control. Table 4 provides the velocity performance characteristics for both the PID and LQR controllers.

Table 4. Summary of acceleration performance characteristics for vehicle's velocity.

Time Response	PID	LQR
Settling Time (T_s)	5.01 s	3.50 s
Rise Time (T_r)	3.83 s	1.08 s
Steady State Error (e_{ss})	5.19×10^{-1} m/s	2.04×10^{-4} m/s

Comparing the two controllers in Figure 6, the speed stabilization time under the PID controller is 5.01 seconds, which is slower than the 3.50 seconds of the LQR controller. Additionally, the speed under PID control has a larger steady-state error. In contrast, LQR has a faster speed response, and the steady-state error of the speed response is close to zero. The only downside is that the system overshoot has increased, but the overall control effect is better than PID.

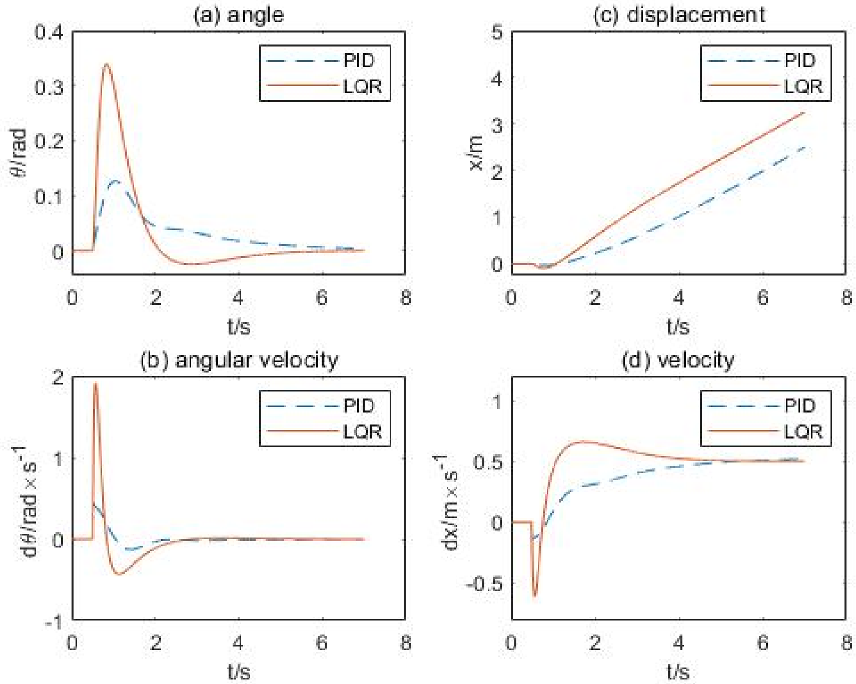

Figure 6. Control effectiveness between PID and LQR controller under applied acceleration step signal.

When the vehicle reaches the target speed, apply a step signal with a target speed of 0 to bring the vehicle to a stop, as shown in Figure 7. Figure 8 shows the braking response of the system under PID control and LQR control. Table 5 provides the speed performance characteristics of PID and LQR controllers.

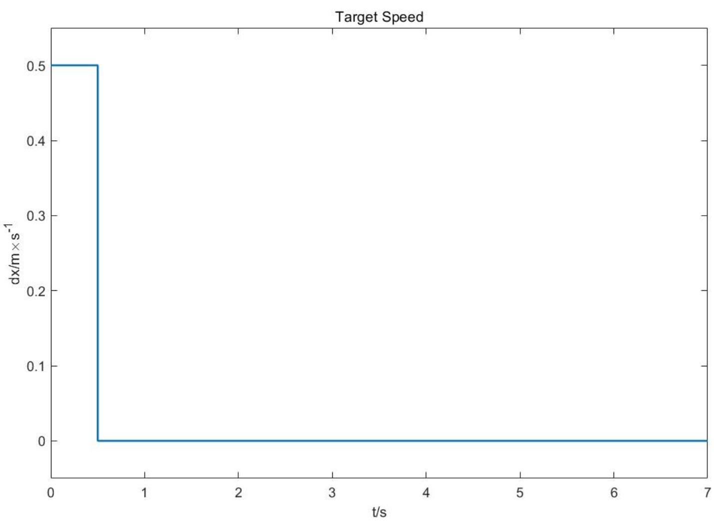

Figure 7. Step input signal with a target speed of 0 m/s.

Table 5. Summary of braking performance characteristics for vehicle's velocity.

Time Response	PID	LQR
Settling Time (T_s)	5.00 s	4.61 s
Rise Time (T_r)	3.83 s	1.08 s
Steady State Error (e_{ss})	-1.87×10^{-2} m/s	-1.73×10^{-4} m/s

When comparing the two controllers in Figure 8, it can be observed that the velocity rise time under the PID controller is slower than that achieved under the LQR controller, which is 1.08 seconds. Additionally, the displacement caused by braking under the PID control is almost four times greater than that under LQR control. On the other hand, faster braking over a shorter distance is achieved by the LQR controller, and the steady-state error of the velocity response is close to zero. Therefore, the LQR control effect is significantly better than that of PID.

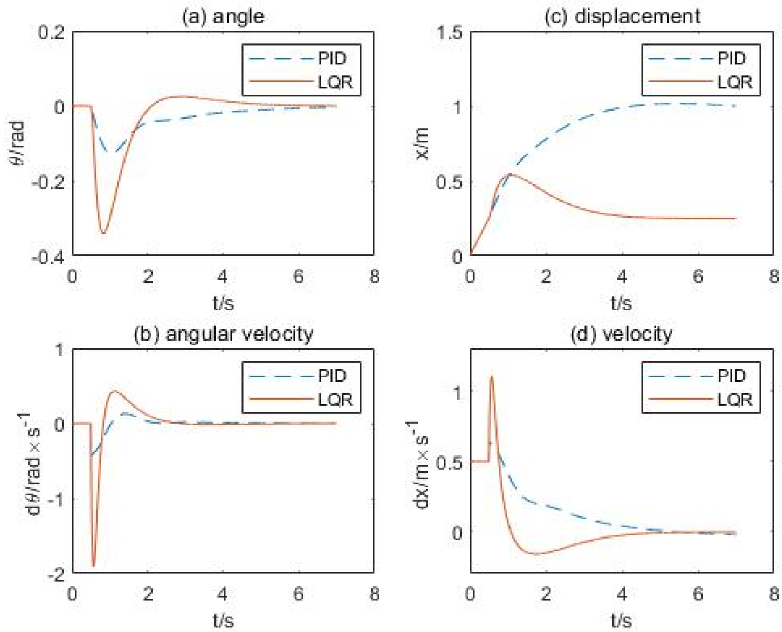

Figure 8. Control effectiveness between PID and LQR controller under applied brake signal.

Based on the analysis above, it can be found that the PID and LQR controllers designed in this article both achieve effective control of two-wheeled balancing vehicle. When comparing the performance from the initial state to the balancing point, the LQR parameters exhibit a shorter response time and smaller steady-state error. With regards to tracking the target speed, the LQR controller demonstrates better speed response than the PID controller, but it also has larger overshoot in both speed and angular velocity. Overall, the LQR controller exhibits superior control performance compared to the PID controller.

6. Conclusion

In this article, PID and LQR controllers are designed to control a two-wheeled balancing vehicle after establishing its dynamics model, and their control effects were compared. Based on simulation results, it is apparent that both controllers have the capability to control the angle and speed of the linearized system's vehicle. While the simulation results indicate that the LQR controller has better performance than the PID controller, with shorter response time and smaller steady-state error in controlling speed, there is still room for improvement for both controllers. The LQR controller should reduce overshoot to meet control requirements. Similarly, the PID controller can improve the parameters of the speed loop to shorten the response time to the target speed and achieve performance close to that of the LQR controller.

References

[1] Mao L W 2017 Design of control system for the two-wheeled self-balancing vehicle Zhejiang University
[2] Wei X 2014 Balance and motion tracking control method for a two-wheeled self-balancing electric vehicle Dalian University of Technology

[3] Yang S G 2020 Research on adaptive tracking control of two-wheel balanced mobile robot under various loads Chongqing University
[4] Sun C, Lu T and Yuan K 2013 Balance control of two-wheeled self-balancing robot based on Linear Quadratic Regulator and Neural Network *Intelligent Control and Information Processing (ICICIP), 2013 Fourth International Conference on IEEE* 862-867
[5] Ren T J, Chen T C and Chen C J 2007 Motion control for a two-wheeled vehicle using a self-tuning PID controller *Control Engineering Practice* 16 365-75
[6] Jian H U and Yan G F 2014 Analysis of two-wheeled self-balancing mobile robots based on ADRC *Journal of Mechanical & Electrical Engineering* 2 159-64
[7] Lin S C, Tsai C C and Huang H C 2011 Adaptive robust self-balancing and steering of a two-wheeled human transportation vehicle *Journal of Intelligent and Robotic Systems* 62 103-23
[8] Tsai C C, Huang H C and Lin S C 2010 Adaptive Neural Network Control of a self-balancing two-wheeled scooter *IEEE Transactions on Industrial Electronics* 57 1420-8
[9] Cao G 2004 Tuning of PID controller and its application Zhejiang University
[10] Xie S J, Chen S T and Lou S T 2004 Application of digital PID algorithms in brushless DC motor Modern *Electronics Technology* 2 59-61.
[11] Li C, Qi B, Miao H, Zou Y, Jiang B and Guo Z 2023 Cascade self-coupling PID attitude controller for quadrotor aircraft *Automation and Instrumentation* 280 103-6
[12] Gao Z W and Dai X W 2020 Design of LQR-PID optimal equilibrium and path tracking controller based on two-wheeled self-balancing vehicle *Control Engineer of China* 27 708-14