

Gait analysis of biped robot based on inverted pendulum model

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Abstract. The spring inverted pendulum model was used to analyze the motion of the biped robots under different motion states and the effective model parameters were obtained. In this paper, the biped robot was simplified to an inverted pendulum model and the kinematics equations of the robot under different motion states were obtained. The kinematics equations were solved by MATLAB, and the analysis was used to get the right sports condition during motion process. The corresponding simulation of the inverted pendulum model was carried out based on MATLAB, a series of data such as the trajectory of mass point of the robot and the variation of the ground reaction force during motion process were obtained. By analyzing the results, the model parameters and conditions of the biped robots to keep stable in different motion states were obtained.

Keywords: biped robots, inverted pendulum model, gait analysis.

1. Introduction

With the development of science and technology, robot technology has been widely used in many fields in human society. Robots can not only be used in industrial fields like manufacturing, logistics, transportation and geological exploration, but also be used in the daily life for entertainment, catering, medical treatment, or education. So robotics has been a hot research area of many domestic or foreign experts and scholars' research.

1.1. Domestic and foreign research

In the field of robot research, biped robot is a research hotspot. Compared with other types of robots, like quadruped robot or wheeled robots, bipedal robots have their own unique advantages: Bipedal robots usually cover a smaller area, have strong adaptability to different terrain and can work in restenosis environment, so they have small moving blind area; biped robots generally have higher degrees of motion freedom, better movement performance and strong flexibility than wheeled robots or quadruped robots; biped robots have human-like structures, which can be used in fields like medical and rehabilitation, replace prosthetics for disabled people, biped robots can also be used in catering, logistics and other industries. Therefore, the analysis and research of biped robot is of great value. This paper mainly studies the stability and gaits of biped robot. Since the 1990s, many domestic and international robotics scholars began to study biped robots, then in recent years, with the development of artificial intelligence technology and robot-related industries, the robot industry has received more investment

and development, and there have been great advances in robotics related technologies, such as sensors or motion control [1]. The United States, France, Japan and some other countries currently leads the industry in robotics, the robot industry in these countries has experienced years of development and accumulated lots of theoretical knowledge and technology. The Chinese robot industry has started relatively late, but with the efforts of robotics scholars in recent years, domestic robot industry also had significant development.

In 1986, American scholars developed SD-1 and SD-2 biped robot, these two bipedal robots can walk on flat ground [2]. Then in 1998, the MIT (Massachusetts Institute of Technology) developed a biped robot called M2 which has 12 degrees of motion freedom and can walk in different gaits. In the 2010s, Boston dynamics developed a biped robot called Atlas, the early version of this kind of robots need to connect the cable, it can walk on complicated terrain, then in 2015 and 2018, Boston dynamics updated the design of Atlas robot [3], new Atlas robots are no longer need to connect the cable, they can stand up after falling, achieve the long jump and high jump, back flip and other movements [4]. The speed of new Atlas robots has also been improved [5, 6, 7].

China began the robotics research in the 1980s. Robotics scholars at Harbin Institute of Technology started their research on biped robots in 1985 and developed HIT-series robot which could walk on grounds and slopes [8], go up and down stairs. In 2015, the robotics team of Harbin Institute of Technology developed FDUBR robot with flexible units that can absorb vibrations while walking, reducing the impact and inertia of the robot's motion. In addition to universities, many Chinese companies are also investing in robotics. In 2019, UBTECH Robotics Corp Ltd developed Walker biped robot that can not only walk on different terrains, go up and down stairs, but also dance, play soccer and deliver things [9, 10].

1.2. Theoretical research method

The motion pattern of biped robot is similar to human, biped robot move by alternating feet swing. This motion pattern has high flexibility and can adapt to different terrains and complex surroundings. Besides, biped robots have many leg joints and high motion freedom, so more effective gait planning methods are needed to ensure the stability of biped robots.

There are many methods for gait planning of biped robots, one of the most common methods is gait planning based on motion model. This method needs to establish a simplified model of the structure of the biped robot and analyze the motion model to plan an effective biped walking model [11].

When building the simplified motion model of the biped robot, a common method is the inverted pendulum model analysis. In the inverted pendulum model analysis, the biped robot structure is simplified as the combination of a mass point which has the whole weight of the robot and the lightweight springs [12, 13]. One end of the lightweight spring is connected with the mass point, and the other end is contacted with the ground. The point of the spring which is connected with the mass point can rotate, and the support point of the spring which is in contact with the ground can also rotate, and the torque of the ground end is zero. In the motion process, the robot can simulate the swing of legs and the rotation of the joints of biped robots or human beings in the motion process by stretching out or drawing back the springs, the motion of legs and joints also means the motion of the biped robots [14]. Besides, the inverted pendulum model is a fast and effective model analysis method, so it is usually more suitable for robots with small mass and motion inertia because it can only simulate few motion states.

So this paper will first analysis kinematics principles and formulas of the biped robot, and then this paper will use the inverted pendulum model to simulate the structure of the biped robot. Based on the simulation, this paper will analyse the motion process of the robots and complete the gait planning to keep the motion stability of the biped robots. The simulation and analysis of this paper will be realized via MATLAB platform.

2. Kinematic analysis of inverted pendulum model

2.1. Inverted pendulum model

The inverted pendulum model was proposed by Blickha in 1989. As shown in figure 1, the original model consisted of a mass point and a lightweight spring. The parameters of the spring and the mass point could be adjusted to simulate the running and jumping motion of animals such as humans and kangaroos. Because the common inverted pendulum model can only simulate few motion states, so it can not effectively analyze the motion of the biped robots in different gaits, so this experiment introduced the double spring inverted pendulum model.

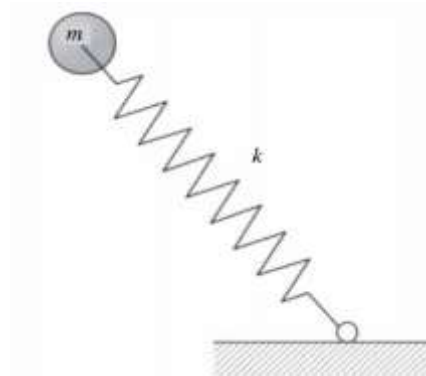


Figure 1. Structure of the single-spring inverted pendulum model.

The double-spring inverted pendulum model is a combination of a mass point with the whole quality of the robot and two lightweight springs. Because the model increases a spring, the model is closer to the motion characteristics of normal human beings, so it can simulate more motion states and is more suitable for gait planning of biped robots. Thus, to analyze the gait of biped robots, the double-spring loaded inverted pendulum model which was more consistent with the characteristics of human walking and running in the movement can be used [15, 16].

2.2. Kinematic analysis

As shown in the figure 2, the double-spring loaded inverted pendulum model consisted of a mass point and two lightweight springs, the stiffness coefficient of the springs was K and the initial length of the springs was l_0 . In the figure 2, the motion states were respectively single-leg phase, double-leg phase and off-ground phase from left to right. When analyzing the forward movement of the model, the range of movement of the mass point was limited in the vertical plane, and there were three motion states: single-leg phase, off-ground phase and double-leg phase [17].

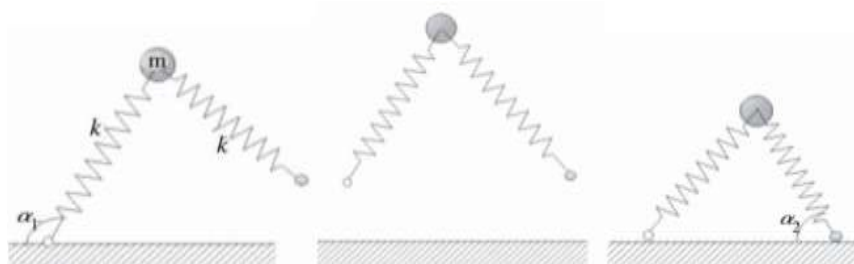


Figure 2. Different motion states of the double-spring inverted pendulum model.

In single-leg phase only one of the two springs touches the ground and the other moves in the air; in double-leg phase the two springs both touch the ground; in off-ground phase both two springs are off the ground, the robot is moving faster and the mass point is only affected by gravity [18]. We can

establish a coordinate system in the vertical plane, the forward direction of the inverted pendulum model is x direction and the vertical direction is y direction. Then we can get the kinematics formulas of the double spring inverted pendulum model in different motion phases.

In single-leg phase, the inverted pendulum system is affected by the gravity and the reactive force of the only spring touching the ground:

$$\begin{cases} m\ddot{x} = -kx(1 - \frac{l_0}{\sqrt{x^2+y^2}}) \\ m\ddot{y} = -ky(1 - \frac{l_0}{\sqrt{x^2+y^2}}) - mg \end{cases} \quad (1)$$

In double-leg phase, the system is affected by gravity and the reaction force of the two springs touching the ground:

$$\begin{cases} m\ddot{x} = -kx(1 - \frac{l_0}{\sqrt{x^2+y^2}}) - k(x - x_0)(1 - \frac{l_0}{\sqrt{(x^2-x_0^2)+y^2}}) \\ m\ddot{y} = -ky(1 - \frac{l_0}{\sqrt{x^2+y^2}}) - mg - ky(1 - \frac{l_0}{\sqrt{(x^2-x_0^2)+y^2}}) \end{cases} \quad (2)$$

In off-ground phase, the system is only affected by gravity:

$$\begin{cases} m\ddot{x} = 0 \\ m\ddot{y} = -g \end{cases} \quad (3)$$

Under different moving states, the inverted pendulum model will switch between three phases. When the model is in the walking-gait, it circularly switches between single-leg phase and double-leg phase. When the model is in the running-gait, it circularly switches between single-leg phase and off-ground phase [19].

From the above dynamic equation, when the inverted pendulum model switches between different phases, the parameters of its mass point and two lightweight springs need to meet certain conditions. Suppose that the initial coordinates of the contact points between the two springs and the ground are (0,0) and (x_0, y_0) in the vertical plane, and the coordinate of the mass point is (x_1, y_1) . The angle between the spring moving in the first double-leg phase and the ground is α , and the angle between the spring which doesn't lift and the ground is β .

When the model switches from the single-leg phase to the double-leg phase, the position of the mass point needs to be such that the other spring can just touch the ground:

$$y_1 = l_0 \sin \alpha \quad (4)$$

When the model switches from the double-leg phase to the single-leg phase, the spring which last touched the ground needs to be restored to its original length:

$$l_0 = \max(\sqrt{x_1^2 + y_1^2}, \sqrt{(x_1 - x_0)^2 + y_1^2}) \quad (5)$$

When the model is in the running gait, it switches between the single-leg phase and the off-ground phase circularly. When the model switches from the single-leg phase to the off-ground phase, the spring touching the ground need to just leave the ground and the force between the spring and the ground is zero, the spring length is the original length:

$$\sqrt{x_1^2 + y_1^2} = l_0 \quad (6)$$

When the model switches from the off-ground phase to the single-leg phase, the position of the mass point should be such that the other spring can touch the ground:

$$y_1 = l_0 \sin \beta \quad (7)$$

According to the above kinematics formulas, we can use matlab platform to simulate the walking gait and running gait of the double spring inverted pendulum model.

3. Simulation of inverted pendulum model

The kinematic equations of inverted pendulum model in different states above most are nonlinear ordinary differential equations. To solve the equations, we can use the forth and fifth order Runge-Kutta equation function 'ode45' which is provided by matlab to solve them. In this paper, the author will use matlab to establish the double spring inverted pendulum model, also simulate and analyze the inverted pendulum model in different gaits or states [20].

When establishing the double spring inverted pendulum model, it can be found that the motion stability of the double spring model depends on the parameters of each part of it. When the motion parameters of the model are improper, the model can not maintain stability in the process of movement and may fall down. Therefore, we imitate the movement characteristics of human beings to design the initial parameters, then we can adjust the parameters to make the spring inverted pendulum model can keep stable in both walking gait and running gait through the process of movement.

3.1. Model parameter design

In this experiment, the parameters of the double-spring inverted pendulum model were based on the movement characteristics of normal adults. The original length of the two springs which represent the robot's legs was 1 meter, the stiffness coefficient of the springs was $30k\text{ N}\cdot\text{m}^{-1}$, the acceleration of gravity was $g=9.81\text{ m}\cdot\text{s}^{-2}$, and the weight of the mass point which was also the weight of whole model was 80 kg. The starting position of the mass point of the inverted pendulum model was (x,y), x was 0 and y was 1. The angles between two springs and the ground were respectively α_1 and α_2 , the reaction force between the inverted pendulum system and the ground was f, and the component forces of the reaction force in the horizontal and vertical directions were respectively f_x and f_y , the step frequency of the model was set to be one step pre second. The simulative movement progress of the model in this experiment was demonstrated by matlab animation function, as shown in the figure below.

In the above analysis of inverted pendulum model, the parameters have great influence on the motion stability of the inverted pendulum model. Thus, the author input the initial parameters of the model into the matlab simulation program in the experiment first, and then the author adjusted the parameters according to the motion process of the model. Finally, the proper parameters which can make the inverted pendulum model keep stable during movement process in different states or gaits was obtained.

3.2. Walking gait analysis

The initial conditions of the double-spring inverted pendulum model obtained after debugging were as follow. The initial position of the mass point was (0,1), the angles between two springs and the ground were respectively 74 degree and 106 degree. At this moment, according to the solution of the kinematics equation, it can be found that the robot can maintain the walking gait. There were only two motion states in this process, which were single-leg phase and double-leg phase, and these two states continuously switched during the whole movement process.

As shown in the figure 3, the inverted pendulum model completed the gait adjustment process within about 0.5 seconds. Then the movement trajectory of the mass point of the inverted pendulum model tended to be stable, the model began to walk steadily. At this time, the average walking speed of this model was $\bar{v}=0.449\text{ m}\cdot\text{s}^{-1}$.

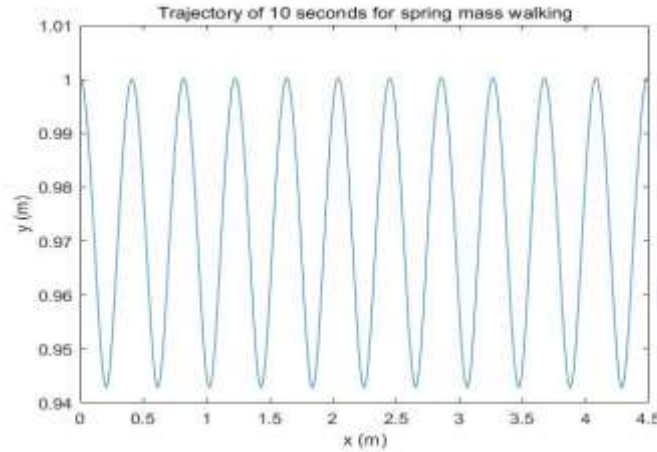


Figure 3. Trajectory of mass point of the walking gait model in 10 seconds.

The reaction force variation of the inverted pendulum model during movement process was shown in the figure 4. It can be seen that, during one second movement process, the component force f_y of the reaction force of the inverted pendulum model in the vertical direction fluctuated twice and the component force f_x in the horizontal direction showed an overall trend of a slight decrease, then a continuous increase and then a decrease again in the walking gait.

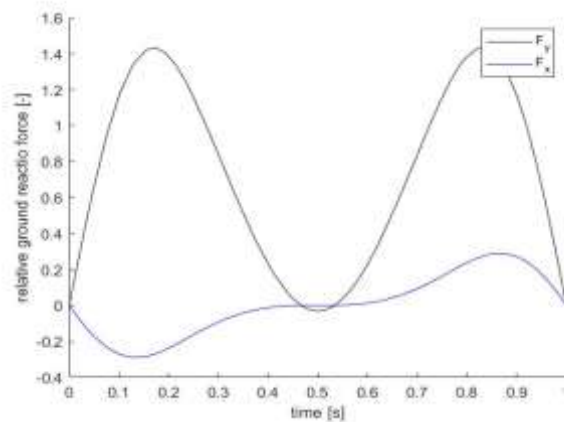


Figure 4. Variation of the reaction force between the walking gait model and the ground in 1 second.

The figure 5 showed the return map of the motion apexes of the mass point of the double-spring inverted pendulum model during 20 seconds of motion. The return map curve almost coincided with the ideal straight line with slope 1. It can be confirmed that the motion state of the inverted pendulum model is quite stable after a short time of gait adjustment, so every step of the model's walking was almost the same.

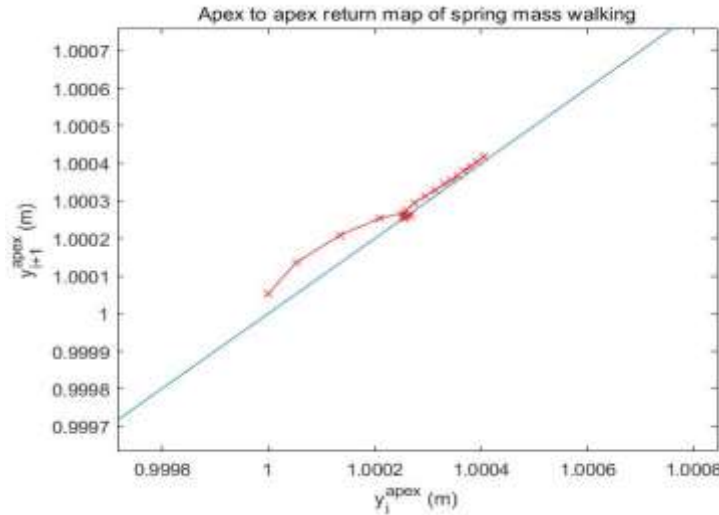


Figure 5. Return map of the motion apexes of the walking gait model in 20 seconds.

3.3. Running gait analysis

Then, we analyze the simulation results of the running gait. The initial conditions of the double-spring inverted pendulum model were shown below, the initial position of the mass point was (0,1), and the angles between two springs and the ground were respectively 72 degree and 108 degree. At this moment, according to the solution of the kinematics equation, the biped robot can keep stable in the walking gait. There were two motion states in this process, which were single-leg phase and off-ground phase, the inverted pendulum model can run forward by cycling between these two motion states.

As shown in the figure 6, the inverted pendulum model completed the gait adjustment process within about 3 seconds. Then the movement trajectory of the mass point of the inverted pendulum model was almost stable, the model began to run forward steadily. At this time, the average running speed of the biped robot was $\bar{v} = 1.28m \cdot s^{-1}$.

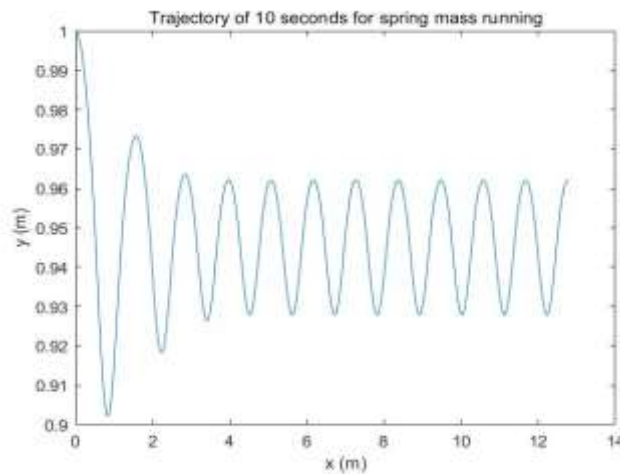


Figure 6. Trajectory of mass point of the running gait model in 10 seconds.

The variation of the reaction force during motion process in one step was shown in the figure 7. It can be seen that, during the one step process, the component force f_y of the reaction force of the inverted pendulum model in the vertical direction rose first and then went down, the component force f_x in the

horizontal direction also showed an overall trend of a slight decrease, then a continuous increase and then a decrease again in the running gait, which was similar to the walking gait.

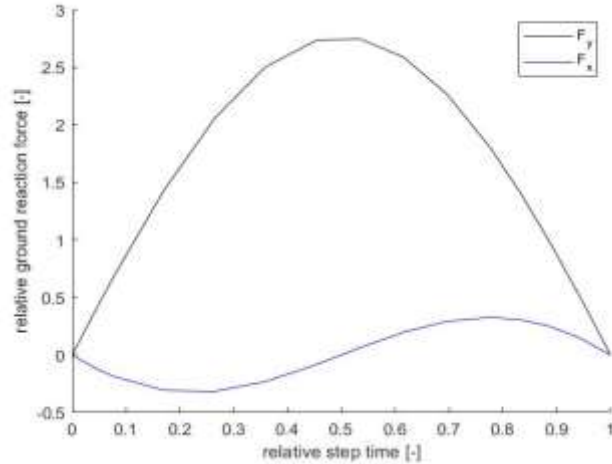


Figure 7. Variation of the reaction force between the running gait model and the ground in 1 second.

The figure 8 showed the return map of the motion apexes of the mass point of the double-spring inverted pendulum model during 20 seconds of motion. The return map curve was basically similar to the ideal straight line with slope 1. It can be confirmed that the motion state of the inverted pendulum model is quite stable after a short time of gait adjustment, every step of the model was almost the same in running gait.

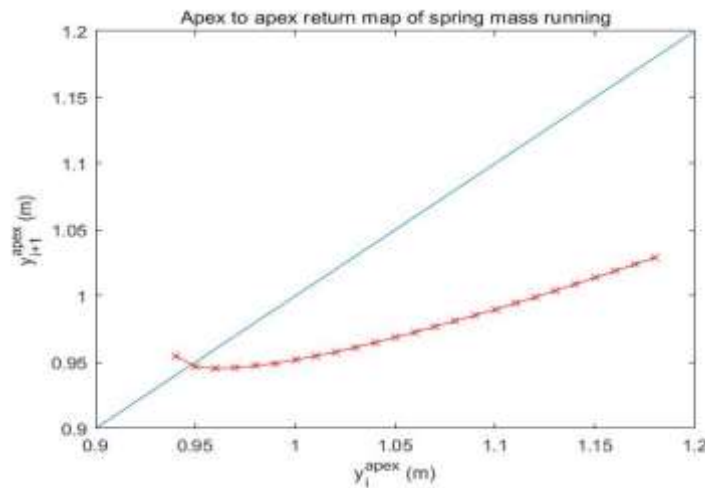


Figure 8. Return map of the motion apexes of the running gait model in 20 seconds.

4. Conclusion

The conclusions are shown as follow:

(1) Double inverted pendulum model can complete the simulation of biped robot's movement. The model can get the proper parameters and motion conditions of the biped robots to keep stable during movement process by analyzing the simulation results of the walking gait and running gait.

(2) The stability of the inverted pendulum model depends on the initial design of the parameters and motion conditions of the inverted pendulum model. Sometimes when the angle between the lightweight spring and the ground has a small range of adjustment, the model cannot remain stable and will fall

down in the movement process after a little period of time, so to get the proper parameters which can keep the stability of the biped robot, designers need to consider carefully in the initial design.

(3) With different design parameters, the biped robot can keep stable in different states, like walking or running gait. However, it is still a problem to design effective simulation algorithm to find proper parameters which can get the motion stability of biped robots in two different gaits at the same time, and this will be the focus of future work.

References

- [1] Christensen, Henrik I. et al. "A Roadmap for US Robotics - From Internet to Robotics 2020 Edition." *Found. Trends Robotics* 8 (2021): 307-424.
- [2] Gong, Daoxiong et al. "Study of human-like locomotion for humanoid robot based on human motion capture data." 2016 IEEE International Conference on Robotics and Biomimetics (ROBIO) (2016): 933-938.
- [3] Li, Tianyu et al. "Using Deep Reinforcement Learning to Learn High-Level Policies on the ATRIAS Biped." 2019 International Conference on Robotics and Automation (ICRA) (2019): 263-269.
- [4] Wang, Xiang et al. "Locomotion Control for Quadruped Robot Combining Central Pattern Generators with Virtual Model Control." 2019 IEEE 15th International Conference on Control and Automation (ICCA) (2019): 399-404.
- [5] Kuindersma, Scott et al. "Optimization-based locomotion planning, estimation, and control design for the atlas humanoid robot." *Autonomous Robots* 40 (2016): 429-455.
- [6] Yoshiike, Takahide et al. "The Experimental Humanoid Robot E2-DR: A Design for Inspection and Disaster Response in Industrial Environments." *IEEE Robotics & Automation Magazine* 26 (2019): 46-58.
- [7] Fujita, Masahiro et al. "Autonomous behavior control architecture of entertainment humanoid robot SDR-4X." *Proceedings 2003 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS 2003)* (Cat. No.03CH37453) 1 (2003): 960-967 vol.1.
- [8] Ma, Hongxu et al. "Humanoid walking pattern modification based on foot-ground equivalent contact control." 2009 IEEE International Conference on Robotics and Biomimetics (ROBIO) (2009): 457-462. David Harel. 1979. *First-Order Dynamic Logic*. Lecture Notes in Computer Science, Vol. 68. Springer-Verlag, New York, NY. <https://doi.org/10.1007/3-540-09237-4>
- [9] Wang, Fei et al. "Nao humanoid robot gait planning based on the linear inverted pendulum." 2012 24th Chinese Control and Decision Conference (CCDC) (2012): 986-990.
- [10] Kasaei, Mohammadreza Mohades et al. "A Fast and Stable Omnidirectional Walking Engine for the Nao Humanoid Robot." *RoboCup* (2019).
- [11] Yu, Zhangguo et al. "Gait Planning of Omnidirectional Walk on Inclined Ground for Biped Robots." *IEEE Transactions on Systems, Man, and Cybernetics: Systems* 46 (2016): 888-897.
- [12] Lei, Yu et al. "Mechanical design and gait plan of a hydraulic-actuated biped robot." 2015 IEEE International Conference on Mechatronics and Automation (ICMA) (2015): 1132-1137.
- [13] Liu, Tao et al. "Motion Simulation of Bionic Hexapod Robot Based on ADAMS/MATLAB Co-simulation." *Journal of Physics: Conference Series* 1601 (2020): n. pag.
- [14] Yu, Haitao et al. "Toward a Unified Approximate Analytical Representation for Spatially Running Spring-Loaded Inverted Pendulum Model." *IEEE Transactions on Robotics* 37 (2021): 691-698. Hou, Xiang-lin et al. "Study on swing-up control of linear spring-connected double inverted pendulum." 2010 Chinese Control and Decision Conference (2010): 2971-2974.
- [15] Xiang-lin, Hou and Zhang Ning. "Modeling of Linear Spring-Connected Double Inverted Pendulum System and Simulating of Free Motion." *Journal of Shenyang Jianzhu University* 25 (2009): 934-937.
- [16] Yang, H.W. et al. "A spring-loaded inverted pendulum model for analysis of human-structure interaction on vibrating surfaces." *Journal of Sound and Vibration* (2021): n. pag.

- [17] Hao, Wu. "Controller Design of Linear Spring-connected Double Inverted Pendulum System." 2003.
- [18] Pelit, Mustafa Melih et al. "Bipedal Walking Based on Improved Spring Loaded Inverted Pendulum Model with Swing Leg (SLIP-SL)." 2020 IEEE/ASME International Conference on Advanced Intelligent Mechatronics (AIM) (2020): 72-77.
- [19] Kim, Bumjoon J. et al. "Spring-loaded inverted pendulum modeling improves neural network estimation of ground reaction forces." Journal of biomechanics 113 (2020): 110069 .
- [20] Xie, Sicheng et al. "Compliant Bipedal Walking Based on Variable Spring-Loaded Inverted Pendulum Model with Finite-sized Foot*." 2021 6th IEEE International Conference on Advanced Robotics and Mechatronics (ICARM) (2021): 667-672.