Literature review on the application of numerical model in improving face recognition performance

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Abstract. As an important biometric technology, the applications of face recognition technology have been applied in many fields. With the development of related technology, the application of different algorithms has improved the face recognition technology in different situations. This paper mainly reviews three mathematical models of face recognition based on neural networks, namely genetic algorithm and artificial neural network (GA-ANN), principal component analysis method and feed-forward neural networks (PCA-FNN) and Hidden Markov Models (HMMs) and State-Action-Reward-State-Action (SARSA). In this paper, three models are introduced and analysed in detail. Through analysis, it has been concluded that the GA-ANN improves classification accuracy, the PCA-FNN is used in face recognition with constant posture and the use of HMM-SARSA improves the accuracy of face recognition.

Keywords: Numerical models, Neural network, Genetic algorithm (GA), Principal component analysis method, Numerical models, Hidden Markov Models (HMMs).

1. Introduction

As a biological aspect of the recognition technology, face recognition technology uses a portrait of a person's face and combines the algorithm with a database of images to match [1]. Meanwhile, facial recognition technology is a computer algorithm. It includes face location, face detection, as well as identity recognition and image processing [2]. This technology can be used in all aspects of human life, such as smartphones, access control, and even when capturing fugitives. These algorithms are based on numerical models. Based on different numerical models, face recognition technology has also been improved in various aspects. Using a lot of mathematical formulas to solve essential problems and reveal potential processes.

With the continuous progress of modern technology, face recognition technology is also gradually optimized and improved. Face recognition technology is toward a higher precision; identify the direction of faster development. All these three numerical models can effectively improve face recognition technology. Many methods are proposed for face recognition. Neural network models are widely used in face recognition algorithms. As one of the achievements of face recognition, a genetic algorithm is combined with a neural network to select and classify facial features. Taraneh Kamyab et al. used this method to improve the accuracy of classification significantly. The combination of genetic algorithm and neural network can not only be easily executed in parallel, but also do not require any restrictions on the optimized functions [3]. In addition, Neural network face recognition and principal component

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analysis (PCA) is a common way used by people nowadays. To avoid the impact of changes in certain conditions like blinking position, left posture, right posture, etc., Dr. Kazi Kutubuddin Sayyad Liyakat et al. mainly studied the Attitude invariant face recognition system. In a result They used a face recognition system that uses PCA and a feed-forward neural (multi-layer) network to recognize faces regardless of their posture in the image. The use of PCA and neural networks increases the difficulty of identifying special situations [4]. However, Anil Kumar Yadav et al. proposed a fresh face recognition way based on Hidden Markov Models (HMMs) and State-Action-Reward-State-Action (SARSA). Compared with the PCA method, the HMMs-SARSA algorithm discovered that it reduces the number of iterations and plots, but it also promotes learning recognition. It provides a method that can be used to solve complex feature selection and other solutions capable of predicting further development of images [5].

This paper summarizes the literature on face recognition based on different models and algorithms and analyses the application of numerical models in different algorithms. At the same time, the application of the numerical model in the research is analyzed, including its characteristics and research results. The realization of face recognition technology is based on these numerical models. The application of different numerical models can also improve face recognition technology in different aspects.

2. Genetic Algorithm and Neural Network (GA-ANN)

Using genetic algorithms for network optimization in neural networks can make full use of the advantages of both, and realize global high-precision optimization of network weight. This section provides an overview of the methods used in the GA-ANN model.

2.1. Basic Principle - Genetic Algorithm (GA)

Influenced by Darwin's theory of biological evolution, GA was proposed as an optimization algorithm [6, 7]. In the theory of biological evolution, Renowned biologist Darwin has come up with a basic theory of Survival of the fittest. Organisms with favourable mutations are more likely to survive, and these high-quality genes are more likely to be passed on to the next generation. Through constant iteration, natural selection is the survival of the fittest. The offspring left behind eventually have the genetic structure of the highest fitness. J. H. Holland took inspiration from this theory and proposed GA [6, 7]. This algorithm is similar to the process of gene replication, selection, crossover and mutation in biological evolution [8].

In the GA algorithm, the parameter set of the actual problem is encoded and initialized. These individuals are then evaluated; In other words, different individuals in a population have different fitness. Then, the basic operation method was selected to process the data set, and the optimal solution was obtained for output.

- 2.1.1. Coding Process. GA searches by the population of points and uses the encoding of the parameter set, as well as the objective function information without any gradient information. It encodes the variables that describe the problem. Variables can be converted to binary strings or vectors [9]. In GA, the solution to the problem is represented as a chromosome. First, the solution of the problem is encoded as a finite-length string, so that the feasible solution of the problem can be mapped into the search space of the algorithm. This string is the genetic equivalent of a chromosome. The sum of the string of individuals produced by each generation is called the population.
- 2.1.2. Fitness Function. In the course of biological evolution, fitness refers to the individual's ability to adapt to the environment and the assessment of the individual's reproductive ability [10]. The evaluation function has another name, which is the fitness function in a genetic algorithm, which is used to judge the degree of the pros and cons of individuals in the population. It is evaluated according to the objective function of the problem [10]. At the end of each generation, it gives each chromosome a fitness value.

It is well known that the fitness result is calculated by the fitness function. Each chromosome will be awarded a score indicating how close the chromosome is to the best solution [11].

- 2.1.3. Basic Operation Procedure. In a genetic algorithm, it can be divided into selection operation, cross operation, and mutation operation. Selection arithmetic is like survival of the fittest in nature. Selection is made based on the assessment of individual fitness in the population. Crossover arithmetic is like genetic recombination in biology. The searchability of the genetic algorithm can be improved significantly by cross-operation. Mutation operation makes the genetic algorithm have a small area search function, but also ensures the diversity of the population [10].
- 2.1.3.1. Selection Operation. The main purpose of selection is to be able to pass on the best individuals directly to the next generation or to the next generation through the generation of new individuals by pairing and crossing. Selection operations are based on fitness assessments of individuals in the population [10]. Selection, as an important step in GA, determines the strings involved in the reproductive process [7]. Roulette wheel selection, rankings, tournaments, etc., are all relevant selection techniques [7].

Roulette wheel selection is completely dependent on random numbers, and the probability of an individual being selected is proportional to the result of the fitness function [13,14]. Suppose the population size is n and the fitness of individual i is F_i , then the probability of individual i being selected to inherit to the next generation population is:

$$p_i = \frac{F_i}{\sum_{i=1}^n F_i} \tag{1}$$

Modifying roulette choices is rank selection, which uses rank to select [7].

Tournament selection involves running several "tournaments" among several individuals randomly selected from the population, and individuals with higher fitness values are added to the talent pool for the next generation [7]. Table 1 enumerates the advantages and disadvantages of the different selection methods, and the differences between them can be seen

Selection method	Advantage	Disadvantage
Roulette wheel	Simple and easy	Risk of Premature convergence; Depends
selection		upon variance present in the fitness function
Rankings,	Preserve diversity; Free from bias	Slow convergence Sorting required; Computationally Expensive
Tournaments	Preserve diversity; Parallel Implementation; No sorting required	Loss of diversity when the tournament; Size is large

Table 1. Comparison of three selection methods

2.1.3.2. Cross Operation. Pairs were randomly matched from the newly selected chromosome individuals, and then the chromosome strings were exchanged for characters at different positions. Common crossover operators include single-point crossover, two-point crossover, uniform crossover, etc. [7].

Crossover points are randomly selected by single-point crossover, and genetic information after the crossover points is exchanged, as shown in Figure 1 [7]. Exchange two data sets after selecting one point.

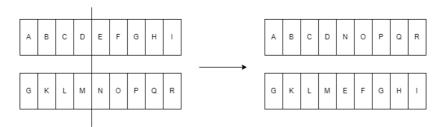


Figure 1. The theory of single-point crossover

Two random crossing points are selected for interchange, as shown in Figure 2 [7]. Exchange of data between two points in a data set.

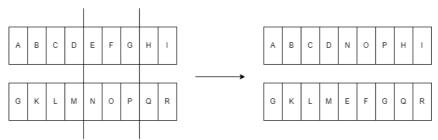


Figure 2. The theory of two random crossing points

Uniform crossing is the exchange of random corresponding positions on two chromosomes, as shown in Figure 3 [8]. Exchange randomly selected data.

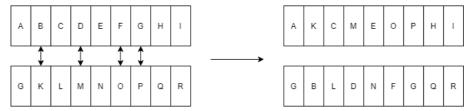


Figure 3. The theory of uniform crossing

2.1.3.3. Mutation Operation. Variation is to change some characters in chromosome individuals with a certain probability of obtaining new individuals [11], as shown in Figure 4.

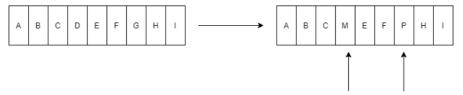


Figure 4. The theory of mutation operation

2.2. Basic principle -Artificial Neural Network (ANN)

ANN is an algorithmic mathematical model which imitates the behaviour characteristics of animal neural networks and carries out distributed and parallel information processing. In this system, the relationship between information is very complex. ANN processes information by adjusting the interconnecting relationships between many internal nodes [11].

ANN can realize pattern recognition efficiently. Compared with other pattern recognition methods, ANN can model complex tasks more easily [12]. No matter what kind of neural network, they have the characteristics of large-scale parallel processing, distributed storage, elastic topology, high redundancy,

and nonlinear operation. ANN has very high computing speed, strong association ability, strong adaptability, strong fault tolerance, and self-organization ability. At the same time, it is a system capable of learning. Its learning style can be divided into supervised learning and unsupervised learning. After computing our input learning set, it will summarize its thoughts based on these. The test examples in the test set are tested against the neural network, and if the tests pass (say 80% or 90% accuracy), then the neural network is built successfully [13]. In the network environment, there are three different types of processing units. The first is the input unit. The second is the output unit, and the third is an implicit unit. And the function of the input unit is used to receive data and signals from outside; The output unit realizes the output of system processing results; Hidden cells are cells that lie between input and output cells and cannot be viewed from outside the system, as shown in Figure 5.

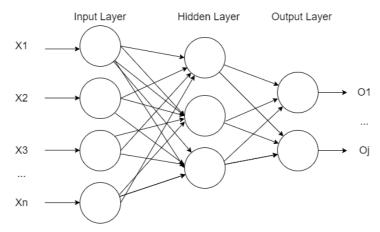


Figure 5. The process of ANN

2.3. The Use of GA-ANN Technique in Face Recognition

This method is based on a genetic algorithm and artificial neural network and combines principal component analysis techniques. In the application of this numerical model, we mainly recognize the important areas of the human face, integrate and select the features in this area, and then carry out face recognition. It can better recognize important areas in the image by removing the noise information in the unimportant areas, in order to achieve better face recognition technology. The facial area of a human face can include the eye area, the nose area and the mouse area. Each face area is provided with coordinate center information, and the threshold distance method is used to determine the size of the face area. After the location of the facial areas was determined, rectangular areas of the same size were used to divide each facial area and the mean value of the feature grey level was extracted from it. The formula is as follows:

$$gi = \frac{\sum p(x,y)}{w \times h \times v} \tag{2}$$

where gi is used to represent the average grey value of the small rectangular area i. p(x, y) represents the grey value of the pixel. w represents the width of the small rectangular area. h is the height. v represents the maximum grey level of the image.

GA was used to encode important features of the human face in binary form using chromosomes. 0 indicates the selected feature, and 1 indicates the unselected feature. This allows you to find this important feature further that are likely to generate more recognition. Each chromosome has its specific meaning of existence, and each chromosome is multiplied by the input feature set, and the input feature vector F is generated into ANN. In the following formula, C is a single chromosome and C_i is a gene in the chromosome. L is the length of the chromosome, which is the same as the input set P.

$$F = CP \tag{3}$$

$$C = (C_1, C_2, \dots, C_i C_i \in \{0, 1\}$$
(4)

$$P = L + R + N + M \tag{5}$$

Each generation of chromosomes in GA represents which traits are selected and which traits are not selected. In this technology, GA optimizes the selection of facial features and uses ANN for classification, which effectively improves the classification accuracy of face recognition system. To some extent, this technology is also a major advancement in science, and it has played a crucial role in people's lives, making them faster and more convenient.

2.4. Relevant Application

GA-ANN is widely used in the field of energy conservation. GA-ANN can predict and optimize building energy consumption. ANN can predict accurately and is very convenient for people to use [14]. ANN is also self-learning and fault-tolerant [14]. Since the energy consumption of buildings requires 24-hour testing, ANN can be used to quickly collect data sets [15]. GA optimization can deal with the nonlinear characteristics of building control, and we get the best global solution instead of the best local solution [15]. GA-ANN can also be used to optimize the operation of dual chillier systems in buildings [16]. It can also predict the strength of concrete in the presence of slag and fly ash [17].

GAA-ANN is also used in biomedicine. It can be used as a technique to diagnose coronary artery disease. Using GA on top of ANN can improve performance by 10% [18].

3. Principal Component Analysis-feed forward Neural (Multilayer) Networks (PCA-FNN)

PCA combined with neural networks is widely used in face recognition. The use of PCA for feature extraction of images and the use of feed-forward neural networks for classification are the principles of this technique used in face recognition [4].

3.1. Basic Principle -Principal Component Analysis (PCA)

PCA is a mathematical analysis method that can be used in many fields. We often use statistical analysis to solve problems. In the process of statistical analysis, the number of variables is often large. In this case, reducing the number of variables can help us solve the problem better. PCA technology is to use the method of dimension reduction, reflecting the main characteristics of things.

The data in the high-dimensional space is projected into the low-dimensional space by means of linear transformation. At the same time, it is also very important to preserve the most original data features as much as possible [19]. In the process of facial recognition, the data involved is very much. PCA was used to retain the component with large amount of information and remove the component with a relatively small amount of information to reduce the size of the original image data.

Suppose there are n vector data set X (). Each x^i is an instance in the data set.

PCA converts p-dimensional data set X () into L-dimensional data set Y (L < p), Y is the principal component of X [20]. As Eq. 6 shows:

$$Y = PC(X) \tag{6}$$

Calculate the average of the X data set as Eq. 7 shows.

$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n} \tag{7}$$

Calculate the variance of the X data set using Eq. 8..

$$s^{2} = \frac{\sum_{i=1}^{n} (X_{i} - \bar{X})^{2}}{(n-1)}$$
 (8)

Calculate the covariance of the X data set using Eq. 9..

$$X^{n*n} = (x_{i,j}, x_{i,j} = cov(Dim_i, Dim_j))$$
(9)

Where X^{n*n} is a data matrix that contains columns and rows, and Dim_i is the i th dimension [20].

If A is an n * n matrix, then the nonzero vector x in \mathbb{R}^n is an eigenvector of A. If the following equation is satisfied [20]:

$$Ax = \lambda x \tag{10}$$

 λ as A scalar is called the eigenvalue of A. x is the eigenvector corresponding to λ and the eigenvalue corresponding to the eigenvector is non-zero, as described in Eq. 11.:

$$(\lambda I - A)x = 0 \tag{11}$$

Define E as the feature space. The eigenspace satisfies all of the vectors $\mathbf{x}(20)$.

After finding the feature space, the feature vectors are sorted from highest to lowest according to the eigenvalues [20]. In this way, we retain the approximate principal component of the original data while eliminating the obscure component [20].

3.2. Basic Principle - Feed Forward Neural (Multilayer) Networks (FNN)

FNN is a kind of ANN, as shown in Figure 6. In FNN, information moves in one direction. The information in the FNN will only move forward from the input to the output. Each neuron and other neurons have a fixed connection mode, and each neuron can only connect to a specific neuron in the previous layer. It can only accept the output information of this layer and transmit this received information to the neurons in the lower layer. The most important thing is that there is no feedback between each layer and the other layers [21]. There are no loops or loops in the network.

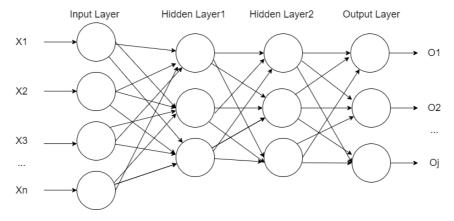


Figure 6. The process of FNN

3.3. The Use of PCA-FNN Technique in Face Recognition

PCA represents the input image data set as shown in Eq. 12.:

$$f = \{f_0, f_1, f_2, \dots, f_{p-1}\}^T$$
(12)

The correlation matrix is shown in Eq. 13:

$$R_f = E[FF^T] = R^T{}_f \tag{13}$$

These sets of coefficients are represented by the vector k^i .

$$k^{i} = U^{T}iF = F^{T}Ui, I = 0, 1, 2, ..., p - 1$$
 (14)

Ui = $[u_{i0}, u_{i1}, u_{i2}..., u_{ip}]^T$ Formed as unit vectors and T which forms matrix formed as $u = [u_0, u_1, u_2, ..., u_{p-1}]$, then have the Eigen vectors of

The Matrix R_f respectively corresponds to the Eigen values λ 0 , λ 1 , λ 2 , ... λ p – 1 where as λ 0 > λ 1 > ... > ··· > λ p – 1 which having λ 0 = λ max

Use an eigenvector equivalent to the highest eigenvalue of Rf.

Then, the principal components of PCA analysis are input into FNN for classification. The perceptron in FNN is used to add the weighted values of all the inputs values with threshold weights, and then the activation function as shown in Eq. 15

$$V_i = S_i + \sum_{i=1}^{\infty} Wij. fj \tag{15}$$

$$Yj = \varphi i(Vi) \tag{16}$$

Where the linear grouping of inputs fl, f2,..., fp is Vi of neurons = i and wjo = Sj is threshold value which is of the weights linked to input fo = -1. And Yj is output of j^{th} Neuron. Meanwhile $\varphi i(.)$ are the forms the activation function.

4. Hidden Markov Models (HMMs-SARSA)

4.1. Basic Principle - Hidden Markov Models & State-Action-Reward-State-Action (HMMs-SARSA) Statistical models designed using Markov processes with hidden states are called HMMs [5]. Markov models have no memory, and transitions between states depend on the current state [22]. At the same time, its state processes are hidden variables and cannot be directly observed [23]. It is a special state space model class with a finite number [23]. HMM In the process of face recognition, you can find the best model of data without thinking about it [5].

HMM is labeled based on Markov chain, as shown in Figure 7. We label the observed data sequence 0, and the labeled sequence I is equivalent to the unobservable sequence (hidden variable). How to solve the labeled sequence I with the highest probability is the core problem of HMM. Known observation sequence $O = \{O_1, O_2, O_3, ..., O_N\}$ and labeled sequence $I = \{I_1, I_2, I_3, ..., I_N\}$, in which the observed sequence is independent and related to the corresponding labeled sequence, which conforms to the Markov chain model, as shown in the figure below:

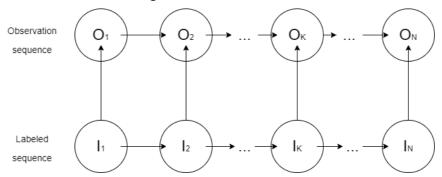


Figure 7. The process of HMM

4.2. Basic Principle - State-Action-Reward-State-Action (SARSA)

SARSA can be understood as a very important way of machine learning in a certain sense, and it is also continuously reinforcing learning [24]. SARSA algorithm is a kind of strategy, used in reinforcement learning strategy study Markov decision process [25]. SARSA needs to know the current state, the current action, the reward, the next state, and the next action when updating the value function, and the process is repeated until it converges [25].

4.3. The use of HMMs-SARSA Technique in Face Recognition

Face recognition based on HMM and SARSA is a new method that can accurately recognize and measure specific images [6]. It selects the overlap from each image separately. Then estimate the hidden attribute for each image through the above methods to improve the recognition rate [6].

- 1. Initialize each state and action in the image and define formula Q(s, a).
- 2. Each episode cycle has its corresponding iteration.
- 3. Assume that current state is (si).

- 4. Iteration process for each episode.
- 5. Make decisions based on greedy policy (π) .
- 6. Move on(a)
- 7. Updated state action pairs (look up table) and greedy policy-

$$Q\pi(s,a)\gamma(x1-i)\sum \pi a'Qt(st+1,at+1)$$
(17)

8. End when the target state is reached

5. Conclusion

In this paper, three kinds of numerical models used in face recognition are analyzed and summarized in order to better understand and apply face recognition technology. GA-ANN technology can be used in face recognition process to find more important features for analysis, improve the accuracy of recognition. This method focuses on improving the accuracy of face recognition technology. PCA technology is widely used in face recognition. The application of PCA-FNN in face recognition studies the face recognition under the condition of constant face posture. This method solves the problem of face recognition difficulty under the condition of changing posture. The application of HMMs-SARSA technology improves the recognition rate in face recognition. All these three models improve the accuracy of the recognition process to some extent. In the future development, we can further explore the improvement and application of these models, and constantly improve the face recognition technology.

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