

Optimizing traffic light signal timing using a yellow interval model: Methodology and empirical evaluation

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Abstract. All over the world, especially in urban areas, the growing population and prosperity of the car industry have resulted in a steady increment in vehicles on the road, leading that already-built car-transportation infrastructure can not be commensurate with the traffic demand. Consequently, this will cause traffic congestion and thus a higher rate of traffic accident. In such circumstances, the ability of traffic management becomes indispensable and crucial to ameliorate or even solve this problem. Although various advanced approaches can provide effective traffic management, the cost can be prohibitive to execute them pervasively. Many algorithms, especially machine learning related, require a mass of input data and a high hash rate. As a result, massive sensor systems, advanced computers, and elaborate programming will be unavoidable, which are costly. Two models in this paper, respectively, yellow light interval and traffic-light signalization, are based on summarizing and improving on previous models and are easy to understand and apply. The model focuses on timing the traffic light to maximize the number of vehicles passing through a traffic intersection in a given time. Many authors in their paper on traffic-light signalization hardly consider the effect of yellow light intervals. This paper provides a relatively comprehensive model of traffic-light signalization. Although the model in this paper is inferior to those advanced models utilized in effectiveness, this model can be applied simpler than others.

Keywords: Signal Light Timing, Best Yellow Light Time, Traffic Flow.

1. Introduction

This study introduces two models related to traffic-light signalization and yellow light intervals. These models have been developed by summarizing and enhancing prior models in the field, leading to a formulation that is easy to comprehend and implement. It is noteworthy that the impact of yellow light intervals is frequently overlooked in many scholarly papers on traffic-light signalization [1]. To address this gap, the current paper presents a more comprehensive model of traffic-light signalization that includes the role of yellow light intervals. Although the proposed model might not outperform some of the advanced models in terms of effectiveness, it provides an advantage in its relative simplicity and ease of application, making it a practical tool for real-world scenarios [2].

2. Methodology

2.1. Model Assumptions

(1) When permissible, all vehicles traverse the parking line at a steady speed [3]. (2) Vehicles maintain a constant speed before initiating braking and after starting [4]. (3) The terrain is considered to be completely horizontal. (4) The dynamic friction coefficient between the tire and the road surface remains constant [5]. (5) The length of pedestrians and bicycles is not taken into account. (6) During the green light phase, traffic reaches and maintains saturation [6]. (7) Lane occupancy scenarios are not considered in this study.

2.2. Yellow Light Interval Model

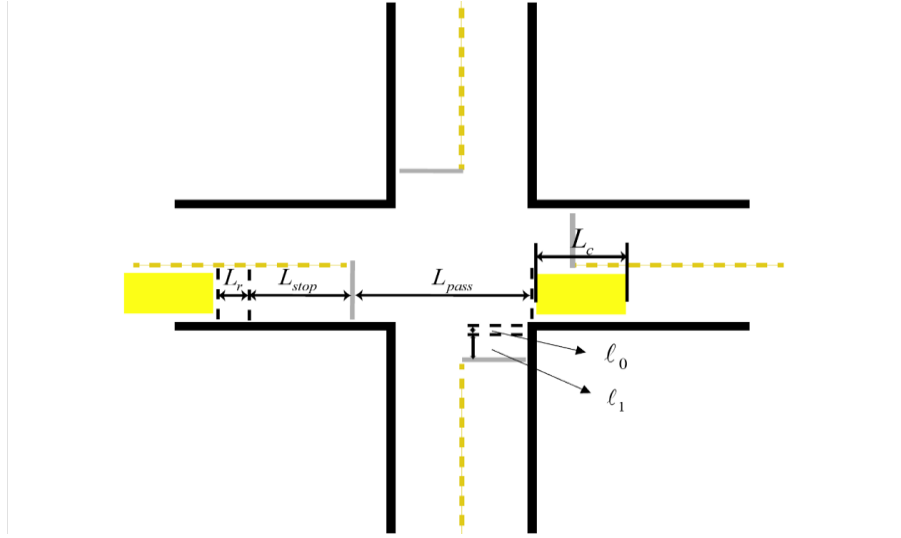


Figure 1. Yellow light interval model diagram (Photo/Picture credit: Original).

Setting the reasonable timing of the yellow light T_Y to the Yellow light dilemma zone assumes that the driver can make a perfect decision on whether to brake or not. Then the total time is equal to the sum of reaction time, braking time, and time to pass the intersection minus the exit time of the vertical vehicle T_Y (to improve the efficiency of intersection traffic, under reasonable calculations, the vertical vehicle can exit before the vehicle reaches the other side of the intersection) [7]. Therefore, it can be concluded that to calculate the total time, the distance of each part should be calculated first, $T_Y = T_r + T_{stop} + T_{pass} - T_a$ so the optimization model can be derived as:

$$T_Y = \frac{L_r + L_{stop} + L + L_{car}}{v_0} - T_a \quad (1)$$

2.2.1. Calculation of braking distance. Because it is Newton's second theorem $\sum F = a$ and $F = \mu mg$, $\frac{d^2p}{dt^2}m = \mu mg$. And because the second derivative function of the distance function is acceleration, it can be obtained $p'(0) = v_0$. Because the primary derivative function of the distance function is the velocity, and when the time is equal to 0 and the distance is equal to 0, it can be obtained $p(0) = 0$ [8]. Collated, differential equations for the braking process model.

$$\begin{cases} \frac{d^2p}{dt^2}m = \mu mg \\ p'(0) = v_0 \\ p(0) = 0 \end{cases} \quad (2)$$

For (1.1) points, get:

$$\frac{dp}{dt} = \mu g t + v_0 \quad (3)$$

Therefore, $v(t) = \frac{dp}{dx} = \mu g t + v_0 = 0$ the braking time is:

$$T_{stop} = \frac{v_0}{\mu g} \quad (4)$$

Points are awarded for (5).

$$p(t) = \frac{1}{2} \mu g t^2 + v_0 t \quad (5)$$

Bring (4) into (5) to solve its distance, therefore:

$$L_{stop} = p(T_{stop}) = \frac{v_0^2}{2\mu g} \quad (6)$$

2.2.2. Calculation of the distance driven out of the vertical vehicle. As shown in Figure 1, the distance from the stop line to the horizontal road is, and the vertical vehicle accelerates from rest ($v = 0$). To reduce time waste, vertical vehicles can leave early, and the duration of the yellow light can be reasonably shortened [9]. So should subtract T_a .

And because the two workshops need a certain insured distance to reduce the risk of accidents, this distance needs to be subtracted ℓ_0 (total duration plus insurance time).

The ℓ_0 time at which the next phase of traffic flows from the stop line on the entrance road to a safe distance before the conflict point.

$$T_a = \sqrt{\frac{2(\ell_l - \ell_0)}{\alpha}} \quad (7)$$

Optimization model of total time when the yellow light is reasonably allocated. Combine the above formula (1), (6), and (7).

The optimization model of the yellow light time of sorting is shown below:

$$T_Y = T_r + \frac{v_0}{2\mu g} + \frac{L+L_c}{v_0} - \sqrt{\frac{2(\ell_l - \ell_0)}{\alpha}} \quad (8)$$

2.3. Traffic-Light Signalization Model

2.3.1. Definition of equivalent traffic. Passenger Car Unit refers to the maximum flow of vehicles passing through the intersection stop line per unit time, that is, the stable traffic flow through the stop line per unit time when the queuing vehicles accelerate to normal driving speed, in PCU/H [10].

PCU (Passenger Car Unit) is also called equivalent traffic, which is the actual various motor vehicles and non-motor vehicle traffic volume according to a certain conversion coefficient into a standard model of equivalent traffic. In a fixed-timing traffic light system, traffic lights are periodic, so only one of the cycles needs to be analyzed. There are 3 cases. The green light times of Figure 2, 3 and 4 are denoted separately T_1, T_2, T_3, T_4 .

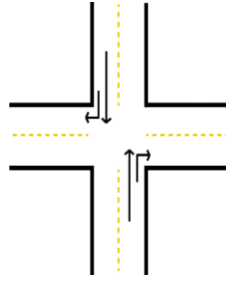


Figure 2. Example 1 of Traffic Light Signal Model (Photo/Picture credit: Original)

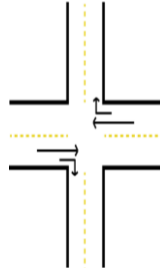


Figure 3. Example 2 of Traffic Light Signal Model (Photo/Picture credit: Original)

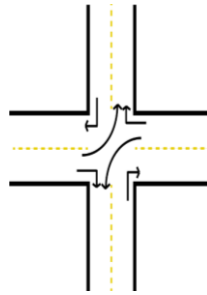


Figure 4. Example 3 of Traffic Light Signal Model (Photo/Picture credit: Original).

2.3.2. Modeling. The goal is to maximize the PCU/h in the intersection. In other words, the maximum number of vehicles passing through the intersection per unit of time.

Vehicles accumulate on each road at its corresponding normal traffic flow velocity. Consequently, during yellow and red lights interval, the queue of vehicles will enlarge. As a result, the number (more specifically PCU) of vehicles in the queue accumulating on the road is $q_i = (T_{cycle} - T_i) \cdot Q_i$, T_{cycle} is defined as the total time of a traffic cycle.

Assume that each road situation needs to ensure that the queue does not accumulate excessively. In other words, the number of vehicles accumulating during the yellow and red lights interval should be lower or at least be commensurate to the number of vehicles passing through the traffic intersection while the green light. Therefore, after all the vehicles in the waiting queue pass through the intersection, the traffic flow of vehicles passing through the intersection will not be the saturated traffic flow but the normal traffic flow on each road.

$$Flow_i = q_i + (T_i - \frac{q_i}{S_i}) \cdot Q_i \quad (9)$$

Therefore, the total Passenger Car Unit number is:

$$Flow_{intersection} = \sum_{i=1}^8 (q_i + (T_i - \frac{q_i}{S_i}) \cdot Q_i) \quad (10)$$

So the total traffic flow of the intersection is:

$$PCU_{intersection} = \frac{\sum_{i=1}^8 (q_i + (T_i - \frac{q_i}{S_i}) \cdot Q_i)}{T_{cycle}} \quad (11)$$

The model is subject to the following conditions: when the motor vehicle traffic light is green, this green light should be long enough for pedestrians and bicycles to pass—assuming that pedestrians are going straight and the bicycle turning route is a quarter circle with a radius of three-quarters L (the length of the intersection).

$$s. t. \begin{cases} T_1, T_3 \geq \frac{L_{pass}}{v_{pd}} \\ T_2, T_4 \geq \frac{3 \cdot \pi \cdot L_{pass}}{8 \cdot v_b} \end{cases} \quad (12)$$

The waiting time at each intersection should be reasonable and not too long to cause vehicles or pedestrians to violate traffic rules (such as running a red light).

$$T_{cycle} - T_i \leq T_{max R} \quad (13)$$

Each road situation needs to ensure that the queue does not accumulate excessively. In other words, the number of vehicles accumulating during the yellow and red lights interval should be lower or at least be commensurate to the number of vehicles passing through the traffic intersection while the green light.

$$T_i \cdot S_i - T_{cycle} \cdot Q_i \geq 0 \quad (14)$$

Each road situation needs to guarantee that the length of the maximum vehicle queue should not be greater than the distance to the adjacent traffic intersection.

$$(T_{cycle} - T_i) \cdot Q_i \cdot L_{vehicle} < L_{intersection_i} \quad (15)$$

Based on the above formula, the optimization model is deduced as:

$$\begin{aligned} & \max \frac{\sum_{i=1}^8 (q_i + (T_i - \frac{q_i}{S_i}) \cdot Q_i)}{T_{cycle}} \\ & T_i \cdot S_i - T_{cycle} \cdot Q_i \geq 0 \\ & T_{cycle} - T_i \leq T_{max R} \\ & s. t. \begin{cases} T_1, T_3 \geq \frac{L_{pass}}{v_{pd}} \\ T_2, T_4 \geq \frac{3 \cdot \pi \cdot L_{pass}}{8 \cdot v_b} \end{cases} \\ & (T_{cycle} - T_i) \cdot Q_i \cdot L_{vehicle} < L_{intersection_i} \end{aligned} \quad (16)$$

3. Experiment

The intersection of Qian Xuesen Road and Chengdong Road is selected for analysis.

3.1. Data Set

As shown in the image in Figure 5. This image is drawn using a scale bar based on the graphic of this intersection on a satellite map. The data is accurate. There is a case as shown in Figure 6.

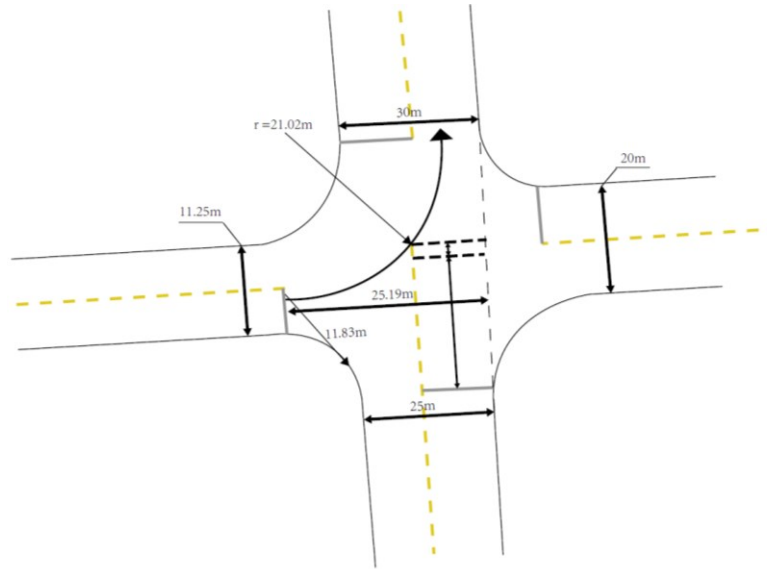


Figure 5. Detailed Intersection Map with Notations for Two Instances of Yellow Lights (Photo/Picture credit: Original).

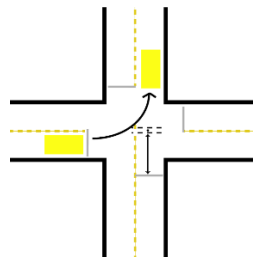


Figure 6. Traffic Cases (Photo/Picture credit: Original)

3.2. Analysis

Based on reliable data, an average vehicle acceleration of 2.778 m/s^2 is assumed. Human reaction time typically falls within the 0.1 to 0.3-second range, with a median value of 0.2 seconds considered for this analysis. Standard car length ranges between 3,800mm and 4,300mm, whereas truck length varies between 5,200mm and 9,200mm. Due to the absence of truck traffic observed on-site at this intersection, the possibility of trucks has been dismissed, setting the average car length at 4,050mm. Considering pavement friction, it is typically 0.6 for dry asphalt, around 0.4 for rainy conditions, about 0.28 for snow-covered roads, and it decreases to 0.18 for icy conditions. For safety considerations and given that Hangzhou experiences mostly rainy conditions with rare instances of snow, ice, or other extreme weather, the pavement friction coefficient is set to 0.4. A safety distance of 4 meters is maintained, and a speed limit of 40 km/h is observed, as demonstrated in Table 1.

Table 1. The dynamic friction coefficient of vehicle tire rubber and asphalt road ground.

Dynamic friction coefficient	\
Normal drying of asphalt pavement	0.6
Rainy pavement	0.4
Snowy pavement	0.28
Icy pavement	0.1

3.3. Result

Best time to yellow light. The yellow light time to go straight after the optimal turn corresponds to Figure 5 is 2.1. The yellow light time for the optimal turn after going straight corresponds to Figure 6 is 2.4. Best time to green light. The survey data was processed using the exhaustive method programmed in C++, following the previously mentioned model. $T1=46.5$, $T2=46.4$, $T3=46.5$, $T4=54.4$.

4. Discussion

The model outlined here takes safety into consideration. In the first scenario, the yellow light time is reserved to enhance safety, while in the second scenario, it accounts for the commencement of the green light phase, along with other factors. Assumptions made about non-motorized vehicles provide a more nuanced approach that aligns with real-world circumstances, and hence, offers valuable insights. The utilization of field investigations for data collection captures more realistic data, addressing issues related to source ambiguity and enhancing credibility.

However, the exhaustive method used makes the algorithm slow, suggesting there could be more efficient alternatives. Data collection from field investigations is not exempt from potential human errors, which could lead to inaccuracies.

Room for model improvements exists. The model currently does not cater specifically to intersections with high vehicle throughput or consider variations in acceleration and average speed between different vehicle types like trucks and cars. The model also assumes standard intersections, without accounting for special surrounding areas such as schools or shopping malls that can cause significant traffic increases in certain directions. Modifications to accommodate these unique situations would enhance the model's utility. Additionally, alternative algorithms could be explored for solving the model. The paper provides a method for analyzing and calculating traffic light timings at intersections, which can be implemented at specific locations. Inserting actual data allows for the determination of optimal traffic light timings that can serve as a reference point. Adjusting these timings according to different time periods can help alleviate traffic congestion and ensure safety, underlining the practical significance of this work.

5. Conclusion

The two models presented in this paper - the Yellow Light Interval and the Traffic Light Signalization - derive from enhancements made to previous models, offering ease of understanding and application. Notably, a significant number of authors in their work on traffic-light signalization seldom consider the influence of yellow light intervals. This paper addresses this gap by proposing a relatively comprehensive model of traffic-light signalization. While it's acknowledged that this model may not outperform some advanced models in terms of effectiveness, its comparative simplicity renders it more accessible for application.

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