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Model and algorithm analysis of MPC control in aerospace engineering

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Abstract. With the continuous development of time and technology, the control system has thus evolved in the direction of great complexity. In the field of path control, once control algorithms cannot absolutely meet the requirements of all people, so this paper introduces an alternative algorithm-MPC. This article focuses on the development of MPC from when it is invented and its application in aerospace today with specified examples, and then lists and analyses the mainstream MPC algorithms, namely LTI, LTV, LPV, PWA and nonlinear systems. Article below also illustrates the derivation of sample of non-parametric model-based algorithms (Dynamic matrix control) and parametric model-based algorithms (Generalized Predictive Control) and the elaboration of mainstream systems and their applications in the form of an overview. Eventually gives a conclusion to the current situation of MPC and relationships among various algorithms. Note that this article is mainly aimed at scholars who are new to MPC algorithms.

Keywords: aerospace, model predictive control, control system.

1. Introduction

The origin of MPC can be traced back to "Self-tuning regulators for multivariable systems: the Quadratic Regulator Case" published in 1966, which is a computer control algorithm developed in the early 1980s. It has the advantages of low accuracy to the model, robustness and good control due to its use of forward feedback such as multi-part prediction, roll optimization and feedback correction. And because of the large number of scenarios in which the processes are characterized by non-linearity and uncertainty, it makes the classical control algorithms such as PID, i.e., the modern control theory, achieve unsatisfactory results. PID algorithms are suitable for control loops and cannot couple variables, making them slightly inferior to control systems. The advantages of MPC in controlling multiple loops and constraints have led to its development in aerospace and chemical production systems. MPC has proved itself to be a state-of-the-art control framework, which has penetrated deeply into the industrial and academic control domains and has had a great impact on them. Its ability to handle multiple constraints at the same time and its stepwise optimization approach have made it popular in the aerospace industry.

The aerospace control system is limited by such conditions as special environment, long working time, high precision requirement and energy consumption limitation, which is a system with multiple state variables and there is also a relationship between the states. Since the 1950s, a set of gradually improved modern control theory has been gradually established in the control field, which makes the aerospace control system have a theoretical basis. Afterwards, MPC was born in such an atmosphere.

The general MPC algorithm can be both suitable on linear and nonlinear models. Consequently, MPC is able to take into account complex nonlinearities and couplings, and it is easy to generalize the benefits to scenarios such as time-delay, non-minimum phase, and nonlinearity. Examples include control of structural interactions, rocket wobble modes and mission-specific modes of spacecraft, which are key features of aerospace system dynamics [1]. In addition, MPC is usually applicable to any type of SISO/SIMO/MISO/MIMO system. From the algorithm itself, the step-by-step optimization of MPC is able to compensate the uncertainties due to factors such as model fit and disturbances in a timely manner, possesses high dynamics, and has better overall robustness and stability.

Nowadays, MPC appears in many fields of aerospace. In the last five years, MPC has served to maximize the thermal durability of aircraft, by incorporating the guardian maps (GM) theory to establish a transient trajectory design methodology, reconstructing the trajectory on-line and correcting the tracking commands [2,3]. Furthermore, one paper examines the problem of coupled orbit-attitude control for the flight of solar sail formation. [4]. This paper is aimed at scholars who are new to MPC. The paper gives a general overview of MPC, then discusses the mainstream control algorithms, and then resolves the system modelling at the level of the aerospace domain.

2. Brief overview of MPC

2.1. MPC information

MPC is mainly used for autonomous navigation of spacecraft [5], i.e., it collects its own speed, angle, position and other remaining parameters without relying on ground equipment, and combines dynamics and kinematics to calculate, immediately predict and optimize the speed, angle, position and such kinds of variables of the spacecraft.

The mainstream predictive control algorithms that currently exist are MAC (Model Algorithm Control), DMC (Dynamic Matrix Control), GPC (Generalized Predictive Control), and GPP (Generalized Predictive Pole Placement Control). Among them, MAC uses the impulse response model, meanwhile, DMC applies the step response model of the object, while the other two (GPP, GPC) use the CARIMA model to neutralize the predictive thought control and adaptive control [6].

The MPC consists of three main components consisting of a predictive model, a controller and a corrector, where the reference trajectory and roll optimization are incorporated into the controller. The prediction of the future output values is determined by the model system, the controller determines the dynamic properties of the system output, and the corrector works only when there is an error in the prediction. A large closed-loop negative feedback system is formed by these three.

2.2. Predictive model

The core of the MPC algorithm is the predictive model it applies. The responsibility of the predictive model is to obtain the current and past deviation values from the ideal values to get the future state and response of the system. Models are categorized into parametric and nonparametric models. The former includes differential equations, difference equations, etc. The latter include impulse response, step response, etc.

$$y(k) = f(x, u) \tag{1}$$

Where $y(k) \in R$ is the output, i.e., the successor state; $x \in R$ and $u \in R$ symbolize current state and current control respectively.

2.3. Rolling optimization

The original thought of Moving Horizon Estimation (MHE) is to give a solution to a specific optimization problem during each time step using the information at the current moment t to obtain the optimal control policy from the current moment to sometime in the future and to execute the first control input of the control horizon, also known as the optimal input. At time t+1, the information at the current moment is reused to update the optimization problem, obtain the optimal control policy for the prediction horizon, and execute the first control input, and so on iteratively, to realize the real-time control and optimization of the system state.

$$J = \sum_{i=1}^{N} w_y (x_i - y_i)^2 + \sum_{i=1}^{N} w_{pi} {\Delta_{pi}}^2 \tag{2}$$

2.4. Error correction

Autonomous navigation measurement errors are usually one of the major factors affecting the state estimation accuracy. Measurement errors are generally categorized into random and systematic errors, which can generally be eliminated by means of weighted averaging or filters, whereas systematic errors, as a significant bottleneck to improve accuracy, are generally resolved by mounting errors and measurement biases. The errors are eliminated at the technical level by modeling-compensation. Among the modeling methods broadly include geocentric orientation systematic error modeling method [6], spatial geometric phase mapping model [7], and Fourier transform modeling [8]. And the main compensation methods are information-assisted correction, on-orbit correction of systematic errors based on state separation estimation, compensation methods for error distribution estimation, and self-correction methods for systematic errors.

3. MPC classification

The main focus of this chapter is centered around an overview of parametric model-based algorithms and nonparametric model-based algorithms, as well as their use in the aerospace field.

3.1. Model algorithm control

Model algorithm control is a class of predictive control algorithms proposed by Richalet and Mehra et al. in the late 1970s, based on impulse response and consisting of three components: an internal model, a reference trajectory and a control algorithm.

3.1.1. Predictive model. For a linear system, if the sampling values$x_1,\ x_2, \dots, x_N$ of the unit impulse response are known, the predictive model description of the system can be approximated by a finite term convolution using the discrete convolution formula:

$$y_M(k+i) = \sum_{j=1}^{N} x(j) * u(k+i-j) \tag{3}$$

3.1.2. Reference track. The desired output of the control system in MAC is specified by a reference trajectory starting from the actual output $y(k)$ at the present time and smoothly overshooting to the set value y_{sp}, which can improve the system robustness. At moment k, the future reference trajectory is represented by $y_r(k+i)$. and there are

$$y_r(k+i) = \left(1 - \alpha^i\right) y_{sp} + \alpha^i y(k) \tag{4}$$

Where α^i is the smoothness coefficient and $0 < \alpha^i < 1$,$y_{sp}$is the target constant.

3.1.3. Rolling optimization. The following optimization criterion is used:

$$minJ(k) = \sum_{i=1}^{P} q_i [y_p(k+i) - y_r(k+i)]^2 + \sum_{j=1}^{M} r_j u^2(k+j-1) \tag{5}$$

where P and M represent predictive horizon and control horizon respectively. Generally, $M \le P$; q_i,r_j are the output tracking weighting coefficients and input weighting coefficients, respectively, and $y_p(k+i)$ is the predicted output

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3.1.4. Feedback correction. A feedback correction is introduced for the $y_p(k+i)$ summation.

$$y_p(k) = y_M(k) + h * e(k) \tag{6}$$

Expressed as a matrix, then where

$$y_p(k) = \left[y_p(k+1), \dots, y_p(k+P)\right]^T \tag{7}$$

$$h = [h_1, \dots h_P]^T \tag{8}$$

$$e(k) = y(k) - y_M(k) = y(k) - \sum_{j=1}^{N} x_j * u(k-j) \tag{9}$$

h is the matrix of feedback coefficients, which is a constant.

3.2. Dynamic matrix control

Dynamic matrix control was also proposed by Cutler in the late 1970's based on the step response. DMC is suitable for stable linear systems, and its use is not affected by time delay or non-minimum phase characteristics in the dynamics of the system. Due to its use of discrete coefficients of the step response and the multi-step prediction technique, it can effectively solve the time delay problem and improve the system error. The specific operation is the same as MAC.

3.3. Generalized Predictive Control. Clarke researched and published the Generalized Predictive Control in the late 1980s based on time series modelling and online identification. In the previous decade or so, the resulting self-correctors have been successfully applied in practice. However, for the process of variable time delay, variable order and poop parameter, the effect is not satisfactory. The hotspot of the era thus becomes the development of self-correctors with robustness. It possesses the consideration of control increment weighting in the objective function; remote forecasting using the output; the introduction of the concept of control time domain length and the recursive solution of the Diophantine equation.

3.3.1. Predictive model. GPC uses CARIMA model as a forecasting model, i.e., Controlled Auto-Regressive Integrated Moving-Average model, which can be represented as:

$$A(z^{-1})y(k) = B(z^{-1})u(k-1) + \frac{C(z^{-1})\xi(k)}{\Delta} \tag{10}$$

Where $A(z^{-1})$,$B(z^{-1})$,$C(z^{-1})$ are n, m and z^{-1} polynomials of n respectively,$\Delta = 1 - z^{-1}$; and $y(k)$, $u(k)$ and $\xi(k)$ represent the outputs, inputs, and white noise sequences with zero mean, respectively. It is worth noting that the vast majority of scenarios will have $C(z^{-1})$ for simplicity.

3.3.2. Rolling optimization.

$$minJ(k) = \sum_{i=1}^{n} [y(k+i) - y_r(k+i)]^2 + \sum_{i=1}^{m} r_i u^2(k+i-1) \tag{11}$$

where m and n are referred to control horizon and predictive horizon ($m \leq n$); $r_i = \lambda$(constant), $y_{sp}$is the target constant, and

$$y_r(k+i) = \left(1 - \alpha^i\right)y_{sp} + \alpha^i y(k) \tag{12}$$

Then introduce Diophantine equation

$$1 = E_i(z^{-1})A(z^{-1})\Delta + z^{-i}F_i(z^{-1}) \tag{13}$$

With combining equation (2.8) and equation (2.11), then the equation will be

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$$y(k+i) = E_i(z^{-1})B(z^{-1})\Delta u(k+i-1) + F_i(z^{-1})y(k) + E(z^{-1})\xi(k+i) \quad (14)$$

Let

$$G_i(z^{-1}) = E_i(z^{-1})B(z^{-1}) \quad (15)$$

So that

$$G_i(z^{-1}) = x_0 + x_1 z^{-1} + \cdots + x_{i-1} z^{-i+1} + x_i z^{-i} \quad (16)$$

Eventually,

$$y(k+i) = G_i(z^{-1})\Delta u(k+i-1) + F_i(z^{-1})y(k) + E(z^{-1})\xi(k+i) \quad (17)$$

3.3.3. Feedback correction. In the derivation of GPC, the calculation of feedback is not explicitly given though. However, during the rolling optimization process, the system and the actual situation must agree as much as possible. This then means that at each step of the control, it is still necessary to detect the difference between the calculated value and the factual output. This is necessary to maximize the robustness of the model when there is a large uncertainty in the system.

4. System modelling

MPC in aerospace, unlike other industrial domains, the realizability of MPC depends heavily on the accuracy of the model, the numerical properties of the solver, the cost and performance of the spacecraft, and the flexibility of the control problem formulation. Therefore, the modeling of MPC systems needs to be subdivided accordingly in different refinements. The main cornerstones of today's aerospace modeling are dynamics and kinematics models. Among them, the kinematic model calculates the position, velocity, and acceleration of the system; while the dynamics model takes into account the input or output moments, i.e., factors such as inertia, mass, and friction, while considering the state of motion from the system. There are two basic models and five mainstream aerospace models, namely, linear model and nonlinear model, where the former one can be further divided into linear time-invariant (LTI), linear time-varying (LTV), linear parameter-variable (LPV) and Piecewise affine system (PWA) [1].

4.1. LTI

Linear time-invariant systems are first described separately here. In the field of autonomous spacecraft navigation, static systems are less considered, so only the relationship between dynamic and linear systems is discussed here. A dynamic system (memory system) that satisfies the requirements of a linear system must satisfy three conditions, namely: satisfy decomposability, zero-state linearity, and zero-input linearity.

$$y = x(t) + u(t) \quad (18)$$

That is, in this equation, if the equation is decomposable and remains linear when x(t), which represents the input, is zero, and remains linear when u(t), which represents the state, is zero, then the dynamical system can be regarded as linearity.

A time-invariant system, on the other hand, is one in which the output signal of the system is related only to the shape of the input signal and not to time. If the input delay time of the system is equal to the zero-state response delay time, the system can be recognized as a time-invariant system, i.e:

$$x_k \to u_k \quad (19)$$

$$x_{k+\Delta k} \to u_{k+\Delta k} \quad (20)$$

LTI systems possess three important characteristics: Homogeneity, Superposition and Time invariance. They are Since linear systems are solvable, it is achievable to build accurate algebraic models and thus predict the state of the system from the solution of the model. The properties of the system do not vary in time with time and there is a linear relationship between the output of the system and its inputs. This means that the output response of the system is stable and predictable for a given input signal and the properties of the system such as frequency response and transfer function are constant.

Among these five models, LTI is the simplest and most straightforward one. It appears in aerospace applications to simplify linear dynamics or approximate results about complex nonlinear structures. However, it is also characterized by the fact that it can only be locally accurate. Currently the system is widely used at the level of flight control systems, signal processing and inertial navigation systems.

Previously, the LTI system was applied to autonomous navigation of satellites stabilized in orbit [9]; it is worth noting that the paper [9] appropriately transforms the time-varying system into a time-invariant one and proves the effectiveness of the method;

4.2. LTV

An LTV system is one in which the properties of the system change over time, but the input-output relationship of the system remains linear. Such systems can describe time-varying physical systems whose properties and behavior can change over time but can still be analyzed and designed using linear control theory.

The model can be expressed as

$$x_{k+1} = A_k x_k + B_k u_k \tag{21}$$

$$y_k = C_k x_k + D_k u_k \tag{22}$$

Rocket dynamics, time-variable linear circuitry, satellite systems and pneumatic actuators are among the applications of time-variable linear systems. In adaptive and standard gain schedule systems, linear time-varying structures are commonly assumed.

Currently LTV systems are also widely used in autonomous navigation systems. In satellite attitude simulation systems, neural network predictive time-varying models based on minimum variance optimization algorithms exist [10]. Moreover, trajectory tracking for helicopters [11].

4.3. LPV

An LPV system is a system whose properties vary with time and parameters. Such systems can describe those systems that are affected by external environmental or operational conditions that cause the parameters to vary. In LPV systems, it is common to regard the system dynamic properties as one or more parameters that can vary with time and environment.

The model can be expressed as

$$x_{k+1} = A(p_k)x_k + B(p_k)u_k \tag{23}$$

$$y_k = C(p_k)x_k + D(p_k)u_k \tag{24}$$

In aerospace, polyhedral models suitable for expressing the behavior of LPV vehicle systems have been explored [12]; and flight attitude control [13].

4.4. PWA

Segmented affine systems are essentially non-smooth, discontinuous and complex nonlinear systems, and due to their broad engineering background, they are one of the current research hotspots and difficulties in the research region of systems engineering and automatic control.

Before talking about the segmented affine system, the switching system should be understood first. For some complex control systems, the system may be in multiple operating states within its operating range, and each operating state has a corresponding dynamics equation, and the dynamics equation corresponding to each operating state describes the dynamic behavior of the system in that operating state. In addition, the development of multi-controller theory is also an important reason that prompts the proposal of switching system.

On the basis of the switching system, the segmented affine system has been studied and popularized by previous researchers. The modeling idea of the segmented affine system is similar to that of the switching system, which is the simplest and the most important one in the hybrid system model. Due to the introduction of switching, the structural parameters and modes of the whole system are not fixed, and the system motion often corresponds to multiple motion modes, which cannot be described by a simple equation of motion, and thus the control of them becomes quite complicated. When the subsystems of the switching system are attached with affine constant terms, the switching system is extended to a segmented affine system.

The study of multi-model modeling and control methods based on the "decomposition-synthesis" strategy consists of two aspects: firstly, the decomposition of the problem; secondly, the synthesis of the solution [14]. The specific tasks can be summarized as follows: (1) divide the whole system into several condition intervals according to some decomposition criterion, e.g., based on the range of conditions, define the whole range of conditions and the variables used to characterize the condition intervals; (2) select the structure of the local models/controllers in each condition interval; (3) identify the parameters of the local models/controllers; (4) finally, combine the local models/controllers to derive a solution based on some scheduling mechanism or performance index. local models/controllers according to some scheduling mechanism or performance index to derive the solution of the original problem.

A method for predicting unpredictable segmental affine of LTI state-delay systems using saturated linear feedback controllers using a robust constrained model is proposed here [15].

4.5. Nonlinear model

As the most accurate but complex of all models, nonlinear models have a wider range of uses. For nonlinear models, MPC is usually expressed using discretization. Models of nonlinear systems can be expressed using nonlinear difference equations or nonlinear state space models. These models then need to be transformed into discretized models for use in MPC. However, due to their complex nature, one still cannot describe them precisely. For example, there are very many kinds of stability describing the zeros of nonlinear systems. Arbitrary singular equilibrium points lead to systems with more complex convergence situations. In addition, there are no good mathematical description tools for nonlinear control systems. Consequently, most of the theories describing nonlinear systems are still under development.

Here is a sample MPC for automobile autopilot with nonlinear style [16]. The paper examines the problem of avoiding obstacles for a fully non-linear vehicle model using model predictive control. To avoid collision with unknown obstacles, its controller consists the function of generating safe trajectories based on the MPC, as well as producing actual control inputs meanwhile. It should be noted that the simplification of the vehicle model and the formulation of reasonable assumptions remain a necessary step due to the large number of nonlinear calculations.

5. Formulation modelling

The problem formulation mainly includes state and control constraints, cost function, resource optimality and controller architecture [1]. In the following, only the cost function and state and control constraints are discussed. The optimization problems are always applied to solve for the sequence of inputs of control. In this process, the loss function needs to be involved, while resource efficiency maximization and control and state constraints need to be considered. When the control signal amplitude exceeds the constraints, the closed-loop performance of the system deteriorates rapidly, leading to severe output overshoot. So a realistic spacecraft would be physically constrained and would appear in the MPC in a constrained attitude. In one paper the implementation of control constraints addressed to the maneuvering design for complex underdriven systems modeled as multi-body problems is discussed [17].

6. Conclusion

This paper analyzes the main models and main systems of MPC. In aerospace, each model and algorithm has its strengths and weaknesses. Since the specific application may involve state secrets, this paper cannot conclude which algorithm or model is more advanced. However, at the system level, the more common measure is to transform most of the nonlinear systems into linear systems and then perform roll optimization and error compensation. It can be seen that there is no bottomless gap between most of the systems, through reasonable assumptions and summarization of previous research, the system can always choose a solution that best suits the current situation by choosing between the amount of computation and the accuracy of the system.

References

[1] Eren, Utku, et al. 2017 Model predictive control in aerospace systems: Current state and opportunities. Journal of Guidance, Control, and Dynamics 40.7: 1541-1566.
[2] Leister, Daniel D, and Justin P Koeln. 2022 Nonlinear Hierarchical MPC With Application to Aircraft Fuel Thermal Management Systems. IEEE Transactions on Control Systems Technology 31.3: 1165-1178.
[3] C Rui, Y B Liu, and Y P Lu. 2021 Robust multiple model predictive control for ascent trajectory tracking of aerospace vehicles. IEEE Transactions on Aerospace and Electronic Systems 58.2: 1333-1351.
[4] G Chen and Y K Zhao. 2023 Model predictive control-based coupled orbit-attitude control for solar sail formation flying in the Earth-Moon system. Astrophysics and Space Science 368.6: 47.
[5] D Y Wang, B W Hou, J Q Wang. 2021 State estimation method for spacecraft autonomous navJgation:Review. Acta Aeronamica et Astronautica sinica, 42(4).
[6] Li G Yand Yang L J 2018 Neuro-Fuzzy-Predictive Control and its MATLAB Implementation Electronic Industry Publishing House, Beijing.
[7] Bemporad, Alberto, and Carlo Filippi. 2001 Suboptimal explicit MPC via approximate multiparametric quadratic programming. Proceedings of the 40th IEEE conference on decision and control (Cat. No. 01CH37228). Vol. 5. IEEE.
[8] Li M D, W X Jing, and X Y Huang. 2012 Dual cone-scanning horizon sensor orbit and attitude corrections for Earth oblateness. Journal of guidance, control, and dynamics 35.1: 344-349.
[9] Morozov, V M, and V I Kalenova. 2020 Satellite control using magnetic moments: controllability and stabilization algorithms. Cosmic Research 58: 158-166.
[10] LI Y Q, HAN J G, LI K, et al. 2003 Neural network predictive control and its application in satellite attitude simulation system. Computer Measurement and Control,11(5):359-361,364.
[11] Dutka, Arkadiusz S, Andrzej W Ordys, and Michael J Grimble. 2003 Non-linear predictive control of 2 DOF helicopter model. 42nd IEEE International Conference on Decision and Control (IEEE Cat. No. 03CH37475). Vol. 4. IEEE.
[12] Fujimori, Atsushi, and Lennart Ljung. 2006 Model identification of linear parameter varying aircraft systems. Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering 220.4: 337-346.
[13] Alexis, Kostas, George Nikolakopoulos, and Anthony Tzes. 2011 Switching model predictive attitude control for a quadrotor helicopter subject to atmospheric disturbances. Control Engineering Practice 19.10: 1195-1207.
[14] Liu Z L 2007 Research on controller design and predictive control method of segmented affine shooting system. Heilongjiang:Harbin Institute of Technology.
[15] Wijaya, Santo, and Manop Wongsaisuwan. 2010 Robust MPC for PWA with time-delay systems using saturated linear feedback controller. 2010 The 2nd International Conference on Computer and Automation Engineering (ICCAE). Vol. 1. IEEE.
[16] Park, J M, et al. 2009 Obstacle avoidance of autonomous vehicles based on model predictive control. Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering 223.12: 1499-1516.
[17] Masarati, Pierangelo; Morandini, Marco; Fumagalli, Alessandro 2014. Control Constraint of Underactuated Aerospace Systems. Journal of Computational and Nonlinear Dynamics, 9(2), 021014.