DOI: 10.54254/2755-2721/33/20230283

A review of metaheuristic optimization methods

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Abstract. In this paper, the metaheuristic optimization methods are categorized. Each method is explained detailly in basic principle and working mechanism. The article is separated to four parts. The second part shows the principle and implementation for each method such as GA, PSO, ACO and TS. The algorithms are categorized according to their searching type. The third part shows the combination of different optimization methods used to solve complex, multi-objective, high-dimensional problems. The fourth part is the future and challenges for metaheuristic optimization. The paper includes recent experiments to 2022.

Keywords: PSO, metaheuristic optimization, GA, ACO.

1. Introduction

Metaheuristic optimization, is an advanced problem-solving algorithm used to find a near-optimal solution or select a heuristic function to optimize the solution [1]. The term metaheuristic is created by Fred Glover in 1977. 'Meta', meaning of 'beyond' and 'after in Greek. Metaheuristic, therefore, is an upgraded version or a higher level of heuristic optimization. Metaheuristic optimization is always used to lead the search algorithm. Unlike other optimization that aimed to produce an optimal solution for the problem, metaheuristic as mentioned is aimed to find a good solution in a relatively short time. And most of the time, it is used in complex environments such as multiple goals, non-linear, discontinuous, and high dimensional. So, usually, there is no perfect or best solution for metaheuristic optimization. Metaheuristic optimizations produce a flexible and steady good solution in a short time for different environments and problems. The flexibility and strong problem-solving skill make it valuable in many different areas and industries. The optimization methods are applied in timetable management, industry, and even finical data [2,3]. For example, the PSO (Particle Swarm Optimization) can find the best shape of an airfoil, its work starts by testing with a population of 'particles', in this case, particle means different shapes of airfoil [4]. The process can be done without adjusting the parameters on the airfoil which saves lots of time on calculation. This example shows simply how PSO works as a metaheuristic optimization to produce a range of good solutions rather than the best solution.

2. Categories

Metaheuristic optimization can be classified according to many different standards: the number of solutions, nature-inspired or not, trajectory or population-based, and whether the method involves randomness. The two main categories of metaheuristic optimization are whether it is a global search or local search and whether the solution is single, or population based. Local search only searches nearby regions rather than the whole region. It works by searching the region around the previous step and optimizing the next search region to make changes to the current solution. An optimal solution is produced by repeating these steps. Global search finds good solutions by searching the bigger region of the entire space. The examples for local search are SA (simulated annealing) and TS (tabu search). And global search algorithm includes GE (generic algorithm), SA (simulated annealing), PSO and ACO (ant colony optimization). The global search algorithms are also the ones that are population-based search methods and local search algorithms usually produce a single solution.

2.1. Global search algorithm

An algorithm searches the entire space to find optimal solutions. Random samples and other functions are used to explore the whole area.

2.1.1. GA (Genetic Algorithm).

2.1.1.1. Definition and mechanism. Genetic Algorithm is an advanced optimization method that belongs to the evolution algorithm and is inspired by Darwinian natural selection. The solution is chosen among the population to generate the best solution for the next population. By repeating this process, a genetic algorithm can search nearly all regions and produce an optimal outcome. Representation of solutions is always the first step for a genetic algorithm. In this step, representations are called 'chromosomes' or 'genomes. The first population is randomly chosen. However, in most cases, the population are the solution that is most likely to become the optimal solution. The size of the population depends on a specific scenario, it can be big like 1000 or only 50. After choosing the population, these samples are calculated by a fitness function. The fitness function also called the objective function, is used to calculate the fitness value for each solution. The fitness score indicates the behavior of each chromosome. The algorithm will select high-fitness score chromosomes to be the parent for the next generation. The selected parents will go through a genetic operator to generate their children. Crossover(recombination) and mutation are used in genetic operators. Crossover is a method combining parents to produce the same children, this method makes sure the solutions or new generation produced will be closer to the optimal solution compared to their parents. And mutation is used to change a chromosome to a randomly selected element. The probability of chromosome solution being changed is the mutation rate. The higher the mutation rate, the more random elements will replace the new generation of solutions. The mutation makes sure the diversity of the new generation so that genetic algorithm optimization can search all the regions of solutions. The process is repeated until the new generation of solutions are in the range of good solution. A detailed explanation of genetic algorithm and explains the future development of the genetic algorithm [5].

*2.1.1.2. Improvement and implementation.*In this article an improved genetic algorithm to overcome the limitations of a simple genetic algorithm is proposed [6]. Two problems of a single genetic algorithm are focused to be solved. One is the premature convergence and the other is the long convergence time. The premature convergence is caused by inappropriate mutation and crossover factors. The algorithm may lose the diversity of solutions if the value of mutation rate is too small. How to adjust these rates becomes the key to generate a good optimization. Zhu Si-rui has proposed an improved genetic algorithm that is self-adaptive according to the fitness function with a function that adjusts the fitness-value for avoiding repeating excellent solutions and increasing the convergence speed. Results show the comparison between IGA and SGA, the computing time is clearly shorter, and the result is closer to the optimal solution. GA has been proven to be one of the best algorithms for solving ARP (anycast routing problems). A combination of GA and PSO (particle swarm optimization) is proposed to solve ARP [7]. There is also an adaptive genetic algorithm used to solve ARP. The adaptive genetic algorithm gets stuck into local optimal point fewer times, however there is still a chance the algorithm will not be able to escape from local optimal [8]. However, PSO-GA algorithm can easily escape from local optimal, and the table shows a result better than GA or PSO.

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2.1.2. PSO (Particle Swarm Optimization).

2.1.2.1. Definition and mechanism. Particle Swarm optimization is also a natural inspired algorithm. It is inspired by the movement of groups of animals such as birds flocking and fish schooling. The concept of PSO was first introduced by James Kennedy and Russell Eberhart in 1995. PSO is also a stochastic algorithm which means the algorithm searches space randomly. Every particle in the particle swarm optimization is a solution. By movement of particles in the searching space, an optimal solution can be generated. Same to the genetic algorithm, the first step is to define the number of particles also known as population. With a random position and velocity of a particle, each particle has an optimal position, and the system also has an overall optimal position. The next position is produced by the previous position of particles, as well as the velocity. The best position for each particle so far and the best position overall keep updating after each step. PSO is divided into global searching and local searching. The general equation of local search particle swarm optimization is modified by Shi and Eberhart 1998.

$$vi, t + 1d = \omega * vi, td + c1 * rnad * pi, td - xi, td + c2 * rand * (pl, td - xi, td) \quad (1)$$

In equation 1 ω stands for inertia weight which factorized the initial velocity. ‘c1’ and ‘c2’ are acceleration coefficients, and their numerical values are usually constant. ‘c1’ controls the new position generated for each particle and ‘c2’ represents the learning factor which means the new position of each particle is compared and data is shared among the particles [9].

2.1.2.2. Improvement and implementation. In article, author gives an overview of PSO and conclude the recent studies [10]. The focus of article is on the basic rules of particle swarm optimization. It includes examples like how the particles interact with each other and how to produce a more stable optimization but keep the same convergence speed. The essays referenced are from 1998 to 2016 in the theoretical analysis. The focus has been on the constant value determined in the general equation, Clerc and Kennedy (2002) are the first researchers that simplify the standard PSO to a dynamic system. There is much research carried out to find stable and reasonable values for ω, 'c1' and 'c2'. There is also a clarification of the improvement of PSO algorithm structure in article 2. The methods have been divided into eight categories and each of them is studied to improve a part of PSO algorithm. Both articles have mentioned a combination of PSO, multi-objective PSO, improve learning strategy and selecting strategy. PSO can be applied to a variety of occasions which include Healthcare, Environment, Industry, Commercial and even in smart cities. Overall, these articles have demonstrated PSO is an algorithm that can be easily implemented and can be used in many different scenarios in combination with other methods or with little change in algorithm. However, how to adjust the parameters to increase the diversity and find a closer optimal solution is still the researchers need to work on. There has not been a mathematical proof of convergence rate, and binary and discontinuous problems which refer to more complex and high dimensional problems are still required modification and exploration.

2.1.3. ACO (Ant Colony Optimization).

2.1.3.1. Definition and mechanism. According to its name, ACO (Ant Colony Algorithm) is obviously inspired by the ant colony. ACO and PSO are both natural inspired and belong to the same algorithm called Swarm intelligence algorithm. Both algorithms show the inspiration from collective behaviour of animals. The basic mechanism of ant colony optimization is simple. To avoid the attack from predators, ants need to find the shortest path between their home and the location of food. In most cases, the ant sacrifices themselves and explore different paths. The information is gathered around so there can be an optimal path generated. In a mathematical view, the ants are different nodes in the path. Each node probabilistically constructs a solution, according to heuristic information and pheromone levels the solution is decided to either increase the probability or decrease. Pheromone levels are updated and compared, the higher the pheromone level the better the path. The pheromone is then brought back by ants and compared to find a better solution. The pheromone level is gradually reduced to avoid convergence to a suboptimal solution and the process is repeated until a satisfactory level of solution is reached.

2.1.3.2. Improvement and implementation. The simple model of the process is described in detailly in the article [11]. The applications for ACO are basically path-finding related problems. Examples like Traveling Salesman Problem, Vehicle Routing Problem and Job Scheduling. One of the articles uses FA to optimize the initial parameters of ACO to maximize the efficiency of ACO algorithm [12]. The initial parameters include the number of 'ants', pheromone initial value and evaporation rate. The research has also found the best parameters for ACO in TSP which is 31. Although the researchers conclude that when there are many paths if we make the pheromone concentration too big, it is easy to fall into local optimum. It is not possible to adjust all the parameters before getting into a problem. How to adjust the parameters for ACO to make it more efficient and accurate should be researched more in future. Article (Y. Xu and X. Yao ,2023) introduces many different combined optimization problems to solve NP-hard combinational optimization problems [13]. ACO hybrid algorithm provides a closer solution to desired one and with a shorter computing time, the more detailed combination of different algorithms is explained and justified in section 3 future and challenges.

2.2. Local search algorithm

An algorithm focuses on searching for an optimal solution by iteratively improving a candidate solution. The local search focuses on neighbour candidate solutions rather than the whole area.

2.2.1. Hill climbing.

2.2.1.1. Definition and mechanism. Hill climbing is a simple and basic local search algorithm. It explores neighbouring solutions by making small changes to the current solution. Solutions go through fitness function, and each has a fitness value. If the new solution has a better fitness value, the algorithm will select the new solution. The process stops when a desired fitness value is reached. It is very easy to understand and implanted. However, the simplicity also brings disadvantages like easily stacking in local optimal and lack of backtracking. The disadvantages are more obvious in ROSA problem, their inaccuracy and low convergence rate are the two main things researchers need to focus on and solve [14].

2.2.1.2. Improvement and implementation. Hill Climbing is a common method used in finding maximum power in an electric circuit. Hill climbing works by first measuring the power and voltage output of solar panels. The algorithm finds an initial operation point and starts to change the voltage value. The neighbourhood solution is produced and compared to the initial solution. However, as mentioned, HC is easily lost in local minimal or maxima. So, it is usually implemented with other techniques. In article (C. -Y. Tan, N. A. Rahim and J. Selvaraj, 2013), there has been an improvement in the hill-climbing method used in solar power systems [15]. Hill climbing combines with MPPT (maximum power point tracking) control technique when aimed to find a point of maximum power. Article (M. S. Bouakkaz, A. Boukadoum, O. Boudebbouz, I. Attoui, N. Boutasseta and A. Bouraiou, 2020) shows a combination of fuzzy logic and Adaptive step HC MPPT algorithm [16]. The algorithm improves the stability of output and the behaviour under sudden changes in temperature.

2.2.2. Tabu search.

2.2.2.1. Definition and mechanism. Tabu search is a local search optimization method with memory and diversification strategies. The solution is also generated randomly in a search space same as other optimization methods. Tabu starts to visit neighbored solutions and compare them using an objective function. The solution will be memorized as a tabu list to avoid repeat solutions. Tabu uses different memory to search further regions. The memory structure can be short-term, intermediate, and long-term.

Short-term memory includes recent searching solutions, this avoids repeat in solutions. Intermediate and long-term memory have intensification and diversification rules that lead the search to unexplored areas.

2.2.2.2. Improvement and implementation. Tabu search is usually combined with other simulations to solve complex problems such as TSP as mentioned in PSO and other planning or finical problems. There is an example of combining genetic algorithm with local tabu search, the algorithm is called HGALTBS (hybrid genetic algorithm with local search and tabu search) [17]. It has been demonstrated that for timetable arranging problems, combing of a population-based method with a local-based algorithm can always have a better result than a single meta-heuristic method. PECTP (post-enrolment-based course timetabling problem) is mainly focused. The tabu search works simultaneously with the genetic algorithm, tabu search is used to improve the equality of the initial population and make the solution jumps out of the local optimal. So, there will fitness value in comparison in a genetic algorithm is closer to the optimal value. After applying tabu search, the results show that the new algorithm can solve the problem with large data while simple GA is not able to come up with a result. And the hybrid algorithm always generates an optimal solution with the aid of tabu search.

2.3. SA(Simulated Annealing)

2.3.1. Definition and mechanism. SA is an optimization method that combines local search and global search. Initially, SA starts by finding a better neighbor solution, but the 'worse' mechanism makes it jump out of local search into searching a larger space. SA is inspired by the annealing process in metallurgy. Where metal is heated and cooled slowly. It is a strong optimization method that is usually used to search a big region and find the global optimal solution. The method allows an increase in probability to increase objective function to make the optimal value escape from local optima. SA starts by initializing the start state and objective function. The start state is generated randomly, and the objective function is used to compare different values for different states. Defining a temperature schedule is the key to this process, the schedule indicates the probability of accepting worse solutions. The next solution is generated by every iteration, small random changes are made to the current solution to produce a neighbored solution. The random changes could be adding, splitting and other mathematical calculations. These random changes are called 'perturbations'. If the neighbored solution has a better objective value, it will be kept. If the neighbored solution has a higher objective value, it still has a probability to be kept. The probability is generated by the previous step in temperature scheduling. Accepting a portion of a worse solution allows the algorithm to explore more regions and avoid local optimal values.

2.3.2. Improvement and implementation. Like ACO as a global search algorithm, SA is also used widely in complex combinatorial optimization problems such as job scheduling and TSP. In article (*X. Liang and Z. Du, 2020*), the authors proposed a combination of genetic algorithms and simulated annealing [18]. The results show that the ISAGA (improved simulated annealing genetic algorithm) not only improves the convergence of the whole system but also helps to ensure diversity in the population. This method uses the temperature cooling technique in SA to keep bad solutions in a certain probability to make sure the algorithm jumps out of local optima.

3. Combination of different methods

Much research has demonstrated the combination of different optimization methods can always lead to a good result. By combining these intelligent methods, life will be easier around us. In Article (T.O.Ting, Xin-Shen Yang, Shi Cheng, Kaizhu huang, 2015), the author has analyzed the past, present, and future of hybrid metaheuristic algorithms [19]. When the word 'combination' is mentioned, the hybrid algorithm should have the advantages of both algorithms and minimize their disadvantages. The author splits the hybrid algorithm into two different types according to the methods of combination. One is called collaborative hybrids and the other is Integrative hybrids. Collaborative hybrids are algorithms each put 50% into the total outcomes. In most cases, one algorithm is a local search to improve individual solutions for global search algorithms. There are cases that the algorithms work alternatively or simultaneously. In integrative hybrids, one algorithm only contributes a little. It is mainly used to improve the main algorithm.

3.1. Example of combination based on the genetic algorithm

Article (L. Yichen, L. Bo, Z. Chenqian and M. Teng, 2020) and (J. Elhachimi and Z. Guennoun, 2011) show different hybrid genetic algorithms to solve frequency assignment problems [20,21]. The optimization methods help to manage frequencies when many frequency-using equipment work at the same time. The genetic algorithm is combined with tabu search, simulated annealing and ant colony algorithm separately to operate different frequencies. The algorithm is based on a genetic algorithm, the other methods are used to help individual values in every generation for the genetic algorithm to reach optimal solution faster. GA cross-combined with tabu search, TS, when genetic algorithm finishes calculating fitness value for the current generation, tabu search is performed on all individuals to generate new individuals. The new individuals form the next population and go through the genetic process again. Combing genetic algorithms with ACO and SA uses the same method. An internal combination of these algorithms is just performing GA firstly and followed by SA or ACO. So, each population in a generation can be optimized by two different methods to find the next generation. Overall, the optimal value is better and usually with a shorter convergence time except for ACO. And all three hybrid algorithms use fewer generations to find an optimal value. As Hybrid GAACO produces the most accurate optimal value, a similar convergence rate is no longer a big deal.

3.2. Example of combination based on PSO

AS-PSO is a combined algorithm of PSO and ACO, the output of ACO is optimized by PSO(N. Rokbani, A. Abraham and A. M. Alimi, 2013) (S. Kefi, N. Rokbani, P. Krömer and A. M. Alimi, 2016) [22,23]. The internal working principle of each algorithm does not change. It is a type of collaborative hybrid algorithm where two algorithms cooperate in parallel. Figure 2 shows how AS-PSO works.

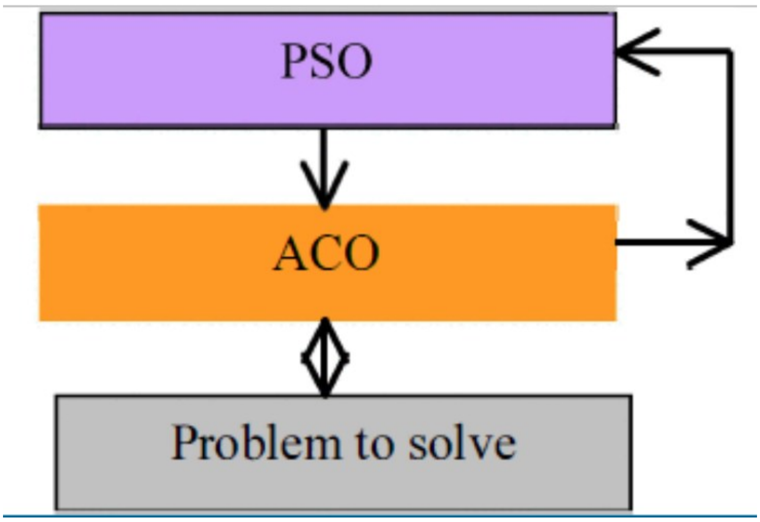

Figure 1. Working mechanisim of ACO and PSO (N. Rokbani, A. Abraham and A. M. Alimi,2016).

The results for TSP compare the ACO, AS-PSO, S-AS-PSO (simplified AS-PSO) (where particle convergence rate depends only on best global solution) and F-AS-PSO (Fuzzy AS-PSO). AS-PSO produces the shortest distance when number of cities is 10 and 20. When the number of cities keeps increasing, S-AS-PSO and F-AS-PSO start to produce shorter distances than the other two algorithms. However, the experiments are only tested 4 times for each algorithm. There should be more tests taken on these algorithms to demonstrate better behaviour on S-AS-PSO and F-AS-PSO.

4. Conclusion

In conclusion, meta-heuristic optimization is an advanced method that can solve complex and high-dimensional problems. There are some disadvantages to every method, however, every method inspired is evolutionary. This novel has concluded the most popular meta-heuristic optimization methods with their insprition as well as their basic mechnisim and how can they be used to solve daily problems. There are also examples of combinations of different methods to produce a better optimal value. Combining local search and global search can maximize the advantages of both methods and minimize the disadvantages. Results have shown that hybrid algorithms can always produce a better optimal solution compared to a single optimization method. How to make the combination of different algorithms more efficient is the topic that most of the researchers are focusing on. The future of meta-heuristic optimization thus has a trend of cooperation via two or more optimization methods that can solve complex, high dimensional problems in a relatively short time. There will be more articles with hybrid optimization methods, however the principle and basic mathmetical formula of each optimization method also require researchers to explain them to the public in detail. For example, if hybrid-genetic algorithm shows a better performance which means a better optimum value than genetic algorithm, how to prove that hybrid method is always better therotically and methmatically is still a mistery. More options of combinations should be explored and tested in future. And metaheuristic optimization is still a new technique so it requires more mathematical demonstration and more data to show that hybrid algorithms are always better.

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