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The Influences of Ties in Preference on Market Matching

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Abstract: The ties of preference and priorities are commonly seen and widely used in daily life. When the agent considers the priority order of some selected objects to be the same, the final matching result may be affected. People sometimes require strict preference ranking in order to facilitate the use of matching models, which can cause controversy and dissatisfaction with people in the matching. In order to effectively solve the above problems, this paper surveys what these impacts are and how they affect the final result of matching. According to the analysis, it is proven that when there is a preference of indifference, Pareto efficiency is unstable and the deferred acceptance algorithm cannot be used for market matching. In addition, the Pareto-improving draft mechanism has proven to be powerful without imposing a strict preference. This mechanism can also avoid the complex process and other negative consequences of handing ties arbitrarily.

Keywords: Stable matching, Ties, Preference, Pareto-improving draft

1. Introduction

The matching problem refers to the problem that appears in establishing a suitable pairing relationship between a group of participants. In the matching process, the preferences between participants are involved, that is, the degree to which each participant prefers the other participants. Solving the matching problem involving preference is an important research field, which has a wide range of applications in real life, including marriage matching, student and school matching, doctor and hospital matching, etc. However, people's preferences are influenced by multiple factors during the selection process. These factors are called ties. It can be seen that ties in preference are a classic problem in market design and society. As an illustration, some inefficiencies exist in the current curriculum allocation mechanism. Generally, students have parallel preferences in courses, that is, they hold the same degree of preference for multiple courses. However, the current curriculum allocation mechanism requires students to arrange their curriculum preferences in a strict order, which leads to a loss of efficiency. Students are unable to express their relative preferences for multiple courses, which may result in a situation where they are assigned to courses that are not their ideal ones. In order to resist these unfair factors, this article analyzes the cause of club selection and the mechanism called the Pareto-improving draft by Li [1] to study the results of ties appearing in preference and how to handle these results. This paper contributes to the development of market matching and provides more methods for people facing allocation problems in the future.

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2. Solutions of Matching with Ties in Preference

2.1. A Stable Improvement Cycle

So far, all proposed methods for improving Pareto efficiency have certain drawbacks. For example, it is not possible to simplify the calculation after adding preferences. Moreover, the simple matching problems involving preferences can be divided into two cases: bilateral preference and unilateral preference. Bilateral preference refers to the existence of a preference relationship between both groups of participants, such as marriage pairing problems between men and women. Unilateral preference is when only one group of participants has a preference for another group of participants, such as a student's choice of school. In order to solve the matching problem involving preference, researchers have proposed various matching algorithms and mechanisms. The most famous algorithm is the Gale-Shapley algorithm, which is a stable matching algorithm and can ensure that the matching result will not be unstable. The algorithm iterates so that each participant can find the most satisfactory match. However, this traditional matching mechanism may not be effective in dealing with the preference of indifference. The stability and efficiency of matching results also are affected. The specific application of the mechanism of Erdil and Ergin's stable improvement cycles in school selection is to improve the efficiency of school selection and student welfare by executing a stable improvement loop [2]. Although this algorithm can improve the efficiency of selection and the welfare of students, allowing more students to obtain their preferred schools, it still can not simplify the calculation. First of all, the algorithm relies on the initial matching result of the random decision. If the initial matching result is not ideal, it may cause the subsequent stable improvement cycle to fail to achieve the optimal matching. Secondly, the stably improved loop algorithm may require several loops to reach the optimal match, which may increase the computation time and computational complexity. In addition, a stable improvement loop algorithm may not be able to solve all optimization problems, especially in the case of a more complex priority structure, there may be no stable improvement. To sum up, the steady improvement cycle algorithm has some advantages in school selection, but it also has some disadvantages. In practical application, it is necessary to weigh the advantages and disadvantages of the algorithm and choose the appropriate algorithm to select the school according to the specific situation.

2.2. Pareto-Improving Draft

Another solution is the Pareto-improving draft by Menling Li. It is a course allocation mechanism aimed at improving allocation efficiency and student welfare. In course allocation, the Pareto-improving draft mechanism ensures Pareto efficiency by assigning courses to students one by one and eliminating augmentation paths and loops in the allocation algorithm of each intermediate step. This mechanism is stable and pareto-effective, and it is strategic for students. The Pareto-improving draft mechanism plays a role in improving the distribution efficiency and students' welfare in course allocation. It is a step-by-step mechanism that assigns courses to each student according to their priority group. Within each priority group, students can simultaneously select multiple courses and rank them. Therefore this mechanism improves overall allocation efficiency by allowing students to have a tie in preference expression. However, the disadvantage of the Pareto-improving draft mechanism is that it does not take into account the strategic behavior among students. While the mechanism is stable and Pareto efficient, students may choose not to report their course preferences in the true order of preference in order to get a better allocation outcome. This may lead to information asymmetry and potential loss of allocative efficiency. In addition, the Pareto-improving draft mechanism can only assign courses to students one by one

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during the allocation process, which may result in some students getting poorer course choices during the allocation process.

3. Consequences of Matching with Ties in Preference

3.1. Positive Consequences

The famous Gale-Shapley algorithm is a stable matching algorithm and can ensure that the matching result will not be unstable. The algorithm iterates so that each participant can find the most satisfactory match. But it is based on a strict preference ranking. This is contrary to the phenomenon of indifference in preference studied in this article, so the positive results of preference matching can be highlighted by listing the shortages of the Pareto model which is based on a strict preference ranking. The consequence of a strict preference ranking is to restrict students from expressing weak preferences in course assignments, when, in fact, many students may have weak preferences. This can lead to a decrease in the efficiency of the allocation mechanism, as students are unable to accurately express their preferences for courses. Strict preference rankings can also lead to unfair distribution results, as they may not meet the preferences of all students. Therefore, allowing students to express weak preferences can improve the efficiency and fairness of course allocation. Furthermore, in an article by Kaplow and Shavell [3], the pursuit of equity often leads to undue sacrifices to human well-being and may even leave everyone worse off. The Pareto principle can affect distributive justice because it requires that the welfare of everyone is taken into account in the allocation of resources and that unfair results are avoided. However, the Pareto principle is not the only theory of equity, and it does not come up very often in practice. The reference also points out that conflicts of Pareto principles do not occur often but arise in the selection of normative standards. Therefore, the Pareto principle is not the only factor that determines distributive justice, and personal preferences also need to be taken into account. Pareto principle means that in resource allocation, as long as one party can benefit from resource reallocation while no other party suffers, it should be reallocated. However, this principle ignores the fairness of resource allocation and social justice considerations. In some cases, resource allocation, according to the Pareto principle, may result in the well-being of some people being improperly sacrificed while the well-being of others is improperly enhanced. The Pareto principle only focuses on the change of relative well-being between individuals while ignoring the initial well-being level of individuals. This means that even if some people have a very low level of well-being, as long as their well-being is not impaired, there is no need for resource redistribution according to the Pareto principle. This can lead to the well-being of some vulnerable groups being neglected, while resources are concentrated mainly on those whose well-being is already at a high level. The Pareto principle does not take into account the fairness of resource distribution and the requirements of social justice. Chang [4] realized that in real life, it is generally believed that resources should be distributed according to certain principles of fairness to ensure social justice and equality. However, the Pareto principle focuses only on efficiency and economic interests, while ignoring considerations of fairness and justice.

To sum up, the Pareto principle may lead to the undue sacrifice of human well-being because it ignores the requirements of fairness in resource allocation and social justice and focuses only on the changes in relative well-being between individuals. So the existence of preference relationships can increase people's well-being. Preferential matching can also improve the fairness, efficiency, and reliability of the matching process. Specifically, research by Abdulkadiroglu et al. [5] has shown that in the delayed acceptance mechanism proposed by students, if students are only guaranteed to be assigned to a particular school in which they rank first, the best strategy for students is either to rank that school first and then submit the rest of their preferences in the true order of preference, or

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to select as many preferred schools in the preference list and rank them in the true order of preference.

3.2. Negative Consequences

To begin with, preference apathy may affect the Pareto validity of matching results. Pareto effective matching means that in a matching market, there is no other matching that can make at least one participant more satisfied while not making other participants less satisfied, and an optimal matching state is reached. In this match, no other match increased satisfaction for all participants at the same time. When participants do not have a clear preference for different choices or are unable to distinguish between them, it may result in matching results that are not Pareto-valid. In this case, even if the presence of other matches increases the satisfaction of some participants without reducing the satisfaction of others, since participants cannot distinguish between different options, they may accept the current match outcome and not actively seek a better match. Therefore, preference apathy may result in the matching results that are not Pareto-valid. In an example given by Erdil and Ergin [6], each worker's preference for the company is indifferent, while each company's preference for the worker is the same. In this case, there are multiple stable matches, but only one is Pareto-valid. However, due to preference apathy, this Pareto-efficient match may not be achieved, while other stable matches lead to inefficiencies. Cseh and Heeger [7] found that the maximum size weakly stable matching problem is hard even in very dense graphs, which may be of independent interest. Secondly, Aziz [8] considers an additional feature of the distribution problem, namely, that agents' preferences involve uncertainty. The result of undifferentiated preferences is that there is no clear preference order or preference difference between the different options in resource allocation. Robert [8] also proved that preferences with relationships may lead to an increase in the complexity of matching algorithms. For example, in the stable marriage problem, the specific form and ranking of preference relationships may affect the final matching outcome, which may require more complex algorithms to handle. Besides, the preference with relation may increase the uncertainty of the matching result. Due to the transitivity and uncertainty of the preference relationship, the matching result may be affected by more factors, which may lead to a matching result that is less fair or less optimized. This means that it is impossible to determine which option is superior or to distinguish between the advantages and disadvantages of different options. In this case, there may be multiple options that are considered the optimal solution, increasing the uncertainty and complexity of resource allocation. Thus, the result of undifferentiated preferences may make the resource allocation process more difficult and require specific algorithms and models to solve. The disadvantages of undifferentiated preference are mainly reflected in the increased uncertainty and complexity of resource allocation results and the difficulty in satisfying the best interests of all agents. Since there is no clear order of preference or difference between the different options, it is impossible to determine which option is superior, making it difficult to determine the optimal solution. There may be multiple options that are considered optimal solutions, making the resource allocation process more difficult and possibly not in the best interest of all agents. Therefore, the disadvantages of undifferentiated preferences are mainly reflected in the increased uncertainty and complexity of resource allocation results and the difficulty in satisfying the best interests of all agents.

4. Conclusion

This paper mainly discusses the influence of the preference relationship on market matching. The paper points out that participants' preferences during the matching process are affected by a variety of factors, which are called preference relationships. This paper also studies the positive and

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negative effects of the preference relationship on matching results and the limitations of the Pareto principle in resource allocation. The effect of the connections in preferences on market matching is examined. It is found that when agents treat the priority order of certain selected objects as the same, the final matching result may be affected. Sometimes, people require strict prioritization in order to use the matching model, but this can cause controversy and dissatisfaction. Therefore, the main purpose of this study is to investigate these effects and how they affect the final outcome of matching. The research proves that Pareto efficiency is unstable when there is uncertainty, and the delayed acceptance algorithm cannot be used for market matching. In addition, the Pareto-improved draft mechanism has proven to be powerful without the need to impose strict preferences. Such a mechanism could also avoid the complex process of arbitrarily handling connections and other negative consequences.

References

[1] *Li, M. L. (2020). Ties matter: Improving efficiency in course allocation by allowing ties. Journal of Economic Behavior & Organization, 178, 354-384. https://doi.org/10.1016/j.jebo.2020.07.030.*

[2] *Erdil, A., & Ergin, H. (2008). What's the Matter with Tie-Breaking? Improving Efficiency in School Choice. The American Economic Review, 98(3), 669–689. http://www.jstor.org/stable/29730091.*

[3] *Kaplow, L. and Shavell, L. (2003). Fairness versus Welfare: Notes on the Pareto Principle, Preferences, and Distributive Justice. The Journal of Legal Studies, 32(1). https://doi.org/10.1086/345679.*

[4] *Chang, H. (1999). A Liberal Theory of Social Welfare: Fairness, Utility, and the Pareto Principle. Yale Law Journal, 110(2000a), 173-235.*

[5] *Abdulkadiroğlu, A., Pathak, P. A. and Roth, A. E. (2009). Strategy-Proofness versus Efficiency in Matching with Indifferences: Redesigning the NYC High School Match. The American Economic Review, 99(5), 1954–1978. http://www.jstor.org/stable/25592543.*

[6] *Erdil, A. and Ergin, H. (2017) Two-sided matching with indifferences. Journal of Economic Theory, 171, 268-292. https://doi.org/10.1016/j.jet.2017.07.002.*

[7] *Cseh, A. and Heeger, K. (2020). The stable marriage problem with ties and restricted edges. Discrete Optimization, 36, 100571. https://doi.org/10.1016/j.disopt.2020.100571.*

[8] *Aziz, H. (2019). Pareto Optimal Allocation under Uncertain Preferences: Uncertainty Models, Algorithms, and Complexity. Artificial Intelligence, 276, 57-78. https://doi.org/10.1016/j.artint.2019.08.002.*