

# ***Research on the Optimization Strategy of Chinese Concept Stock Investment Portfolio under Multiple Constraints Based on Markowitz and Index Model***

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**Abstract:** This study analyses the strategic allocation of investment portfolios of Chinese concept stocks in the USS market, emphasizing the importance of these assets in global finance. It evaluates their historical performance and optimisation potential under different market constraints, using the Markowitz and Index Models as analytical frameworks. The research examines ten different Chinese stocks' data from the past decade to identify the best investment strategies in a sector that is inherently volatile and has growth potential. The analysis highlights the different risk-return profiles of the stocks, their independence by sector, and correlation dynamics. This provides a detailed plan for diversifying investments. The study demonstrates that Chinese concept stocks are relatively independent of the USS market and that both models provide solid risk-adjusted returns for the 10 Chinese concept stocks, even when the weighting of the market indexes is excluded. These findings enhance the theoretical discourse on portfolio optimization and offer actionable insights for investors navigating the complexities of international markets, particularly the nuanced Chinese concept stocks in the US. These insights are important for comprehending the integration and innovation in financial markets, particularly in light of China's increasing global economic influence.

**Keywords:** Chinese Concept Stock, Markowitz Model, Index Model, Optimal Portfolio

## **1. Introduction**

In August 1991, South Pacific Properties (NTP) was listed on NASDAQ, signaling the beginning of Chinese enterprises gaining listings in the United States. As per WIND statistics, 380 Chinese companies have been listed on the US stock market and raised \$79 billion as of December 1, 2023. Although the share of Chinese concept stocks in the US capital market may not be substantial, it presents an opportunity for US and international investors to benefit from China's rapid economic development and contribute to bolstering the global status of the US capital market. Theoretically, alterations in China's official interest rates and policies will directly affect the fundamentals of Chinese concept stocks, but the expectations of investors on the fundamental changes in the USS market will ultimately determine the trend of Chinese concept stock prices [1]. Investors must devise an effective investment strategy for Chinese concept stocks in a complex market environment.

Conducting an optimization analysis of a Chinese concept stock portfolio is crucial, considering various market, personal, and other limitations.

Markowitz's "Mean-Variance Model" was the forerunner of portfolio theory, paving the way for contemporary modern portfolio theory and laying the groundwork for subsequent portfolio decision-making, which advocates finding the efficient boundaries of a portfolio and determining an investor's final portfolio in relation to that investor's level of risk appetite, based on the two main assumptions of a ban on bond melting and risk-free, unlimited borrowing [2]. Sharp proposes a single index model that divides the uncertainty of stock returns into common market factors (systematic risk) and firm-specific uncertainty (unsystematic risk) and simplifies the Markowitz model substantially by using only the common market factors to be considered in the calculation of the covariance of the Markowitz model, assuming that the two types of risk are independent of each other and that unsystematic risk is independent of each other across firms [3]. Based on the theory that the Markowitz model analyses how an individual investor can acquire an optimal blend of assets, Sharp, Lintner, and Mossin introduced the Capital Asset Pricing Model (CAPM), which examines the following: If all investors make optimal asset choices in accordance with the Markowitz model, each investor tends to optimize their asset portfolio. As a result, the portfolios of assets will ultimately converge. This ensures that the market's asset portfolios are not only on the efficient frontier but also with the undifferentiated investment utility asset portfolio curve, which is tangent to the investment utility nondifferentiation curve [4-6]. As the assumptions of CAPM are theoretical and lack practical application, and the single-index model suffers from idealized presumptions about risk, Ross proposes the APT theory, which incorporates three assumptions - no persistent arbitrage in the market, adequate diversification of investor assets, and multiple risk indices. This theory optimizes applying single-index models of the CAPM in practice [7].

The above portfolio investment optimization model has been used to support empirical analyses of different stock market equities. Shchankina and Zou Ping conducted an empirical analysis of portfolio optimization strategies using the Markowitz model and the BEKK GARCH model with data from the Shanghai and Hong Kong stock exchanges, and the study aimed to determine the effectiveness of portfolios constructed from multivariate time series forecasts in reducing overall portfolio volatility and outperforming stock market indices, which confirmed the effectiveness of these models in portfolio optimization strategies [8]. Chakrabort and Patel compared the efficiency of the Sharpe Single Index Model and the Markowitz Model in maximizing investment returns and minimizing risk by analyzing the data of the stocks of India's Nifty 50 Index from September 2016 to September 2017. They demonstrated how to determine the weighting of each stock in an ideal portfolio and provided both theoretical and empirical foundations for making effective investment decisions in a volatile market [9].

The aim of this study is to investigate the efficient allocation of investment portfolios under strict market and personal constraints based on historical data on Chinese concept stocks. The study aims to identify optimal investment and minimum risk portfolios for different risk preferences and to reveal the unique behaviour and investment potential of Chinese concept stocks in the US market. Additionally, the study aims to provide empirical evidence for the practical use of the Markowitz and index models for making investment decisions on Chinese concept stocks.

## 2. Case Description

The dataset used in this study was obtained from Bloomberg. It includes 10 years of historical daily return data for 10 Chinese concept stocks listed in the USS. These stocks are categorized based on the nature of the company as either state-owned enterprises (SOEs) or private enterprises (PSEs), and based on the nature of the industry, they can be classified into four industries. Table 1 shows the list of ticker symbols and the corresponding areas covered in this study.

Table 1: Stocks, Industry and Owner

Ticker	Industry	Owner
NTES	Technology	PSE
SOHU	Technology	PSE
EDU	Service	PSE
LFC	Service	SOE
PTR	Oil	SOE
SNP	Oil	SOE
CEA	Aviation	SOE
ZNH	Aviation	SOE
HNP	Industrials	SOE
ACH	Industrials	SOE

The data for the 10 stocks was collected from May 12, 2008, to May 14, 2018. This period is significant as it represents the 10 years of China's economic prosperity and is considered the hottest 10 years for Chinese concept stocks. Therefore, it is suitable for analyzing Chinese stock investments. Additionally, the data for this study includes eleven risky assets, including the S&P 500 equity index, which reflects the overall market performance of US stocks and is a proxy for the risk-free rate, specifically the 1-month Fed Funds rate. In order to mitigate non-Gaussian effects, the study aggregates daily data into monthly observations by retaining data from the first day of each month. This approach helps to ignore abnormal fluctuations in daily data and obtain a smoother and more representative dataset.

The 10-year share price trends of the EDU are shown in Figure 1, together with the SP500 index, which is used to visualize the 10-year trend of the entire US stock market, and the HXC index (Nasdaq Golden Dragon China Index), which is used to provide an overview of the 10-year trend of the entire Chinese concept stock sector.

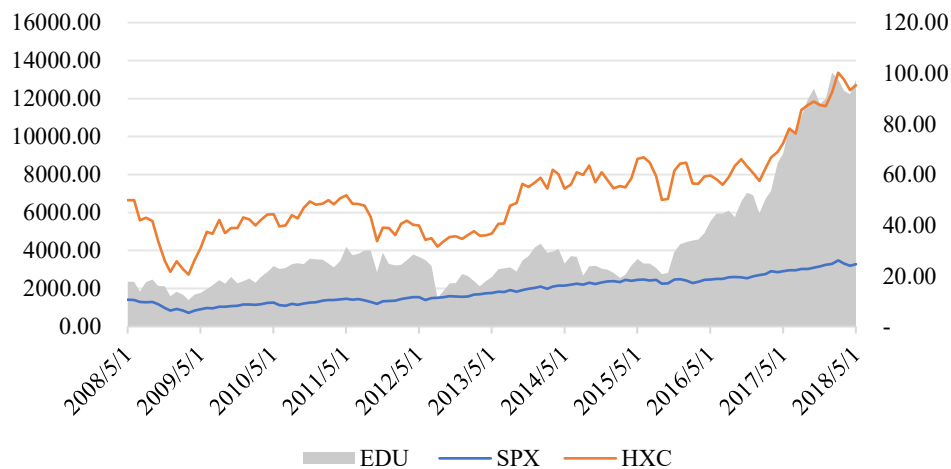


Figure 1: The 10-year share price trends of EDU, HXC and SPX (Photo credit: Origin)

This study requires calculating all appropriate optimization inputs for the full Markowitz Model (MM) and the Exponential Model (IM), including the Alpha and Beta coefficients and the correlation coefficients, based on monthly observations. Additionally, it is necessary to find the regions of permissible portfolios for some additional constraints.

### 3. Analysis on the Problem

#### 3.1. Data Preprocessing

The risk-free investment (with a base of \$1,000) is calculated using the 1-month Fed Funds rate. Next, the monthly and excess monthly returns are calculated for the 12 data points (10 stocks, the SPX index, and the risk-free investment). Based on this data, the annualized average return and annualized standard deviation are calculated.

$$R_{AA} = 12 * \overline{R_E} \quad (1)$$

$$\sigma_A = \sqrt{12} * \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}} \quad (2)$$

The Beta coefficient is calculated to measure the correlation between individual stocks or portfolios and the overall market volatility (represented by the S&P 500 index). This involves a linear regression analysis of the excess monthly return of a stock against the excess monthly return of the S&P 500 index.

$$\beta = \frac{n(\sum_{i=1}^n R_{SPX_i} R_i) - (\sum_{i=1}^n R_{SPX_i})(\sum_{i=1}^n R_i)}{n(\sum_{i=1}^n R_{SPX_i}^2) - (\sum_{i=1}^n R_{SPX_i})^2} \quad (3)$$

Annualized Alpha is calculated to measure the average overperformance of a stock relative to the market over a one-year period by obtaining the intercept of the linear regression line from the Beta calculation averaged over time.

$$\alpha_A = 12 * \frac{(\sum_{i=1}^n R_i)(\sum_{i=1}^n R_{SPX_i}^2) - (\sum_{i=1}^n R_{SPX_i})(\sum_{i=1}^n R_{SPX_i} R_i)}{n(\sum_{i=1}^n R_{SPX_i}^2) - (\sum_{i=1}^n R_{SPX_i})^2} \quad (4)$$

In addition, a stock's return may be influenced by company-specific or sectoral factors that the overall market movement cannot explain. To account for this, Residual Returns and Annualized Residual Standard Deviation must be calculated. High values for these metrics may indicate that a stock's performance has been impacted by factors unique to the company or sector that the market or model did not properly consider.

$$R_{R_i} = R_i - R_{SPX_i} * \alpha_{A_i} - \frac{\beta_i}{12} \quad (5)$$

Finally, the correlation coefficients between the 10 stocks and the S&P 500 were calculated and used to quantify the strength and direction of the linear relationship between the stocks.

#### 3.2. Markowitz Model

The main point of Markowitz's model is to minimize risk for a given rate of return - i.e., to minimize unsystematic risk through diversification, which also means that the investor tries to select assets with a low degree of correlation (measured by correlation coefficients or covariances) in the portfolio selection process. In theory, an investor can eliminate the unsystematic risk of a portfolio by reasonably allocating investment assets. This allows for a desired rate of return that only bears market risk.

The Markowitz model begins with an asset's own return volatility as a risk measure for asset selection. The risk of an individual asset is determined by its variance, while the overall variance of the portfolio determines the risk level of the portfolio. This defines the 'mean-variance model' boundaries in a two-dimensional plane with the expected rate of return as the vertical axis and the variance of the risk measure as the horizontal axis.

If a portfolio contains  $n$  assets, each with an expected rate of return  $R_i$  and a weight  $\omega_i$  (where  $i = 1, 2, \dots, n$ ), then the portfolio's expected return  $R_p$  can be calculated.

$$R_p = \sum_{i=1}^n \omega_i R_i \quad (6)$$

The risk of a portfolio is determined not only by the risk of the individual assets in the portfolio but also by the correlation between them. To determine the total risk of a portfolio, calculate the covariance or correlation coefficient between the assets. The equation below provides the standard deviation, which represents the portfolio's total risk.

$$\sigma_p^2 = \sum_{i=1}^n \sum_{j=1}^n \omega_i \omega_j \sigma_i \sigma_j \rho_{ij} \quad (7)$$

The next step is to locate the Efficient Frontier, a line that displays the maximum expected return that can be achieved for a given level of risk or the minimum risk that must be taken for a given level of expected return. To find the Efficient Frontier, mathematical optimization is required to solve the following problem: maximize the expected return, minimize risk, or some combination of the two, subject to constraints (e.g., asset weights summing to 1).

$$\begin{cases} \min \sigma_p^2 = \min \sum_{i=1}^n \sum_{j=1}^n \omega_i \omega_j \sigma_i \sigma_j \rho_{ij} \\ \sum_{i=1}^n \omega_i = 1 \quad \sigma_p^2 \leq \sigma_{\text{conditions}} \end{cases} \quad (8)$$

### 3.3. Index Model

According to the Markowitz Model, the Index Model only considers the common market factor (in this case, the stock index) when calculating the covariance. This simplifies the workload of calculating the covariance matrix. The regression equation is expressed as follows:

$$R_i = \alpha_i + \beta_i(R_{\text{market}} - R_{\text{free}}) + \varepsilon_i \quad (9)$$

$R_{\text{market}} - R_{\text{free}}$  is the excess return of the market index,  $\varepsilon_i$  denotes firm-specific uncertainty, which is assumed to have a mean of 0 in this model. The relationship between risk premium and beta for the mono-exponential model is:

$$E(R_i) = \alpha_i + \beta_i E(R_{\text{market}} - R_{\text{free}}) \quad (10)$$

It is possible to derive the variance and covariance for each stock by assuming that systematic and unsystematic risks are uncorrelated and that unsystematic risks are uncorrelated across firms.

Subject to the constraints  $\omega_i$ , the optimization procedure selects the portfolio point with the smallest standard deviation to form the efficient boundary of the portfolio of risky assets under the fixed return condition. Finally, the Sharpe ratio is computed to calculate the optimal portfolio.

$$S_p = \frac{E(R_p)}{\sigma_p} \quad (11)$$

The optimal portfolio of risky assets required for this study is the portfolio with the largest Sharpe ratio.

## 4. Suggestions

### 4.1. Performance of individual stocks

Table 2 shows the Alpha, Beta and Annualised Average Excess Return results for the 10 Chinese concept stocks and the SPX coefficients. This provides a preliminary picture of the performance of individual stocks over a 10-year period and the extent to which their returns are affected by the overall market.

Table 2: Alpha, Beta, and Other Analytical Data for 11 Risk Assets.

Risk Assets	Annualized Average Return	Annualized Standard Deviation	Beta	Annualized Alpha	Residual Returns Standard Deviation
SPX	9.752%	17.772%	1.00000	0.000%	0.000%
EDU	27.351%	44.379%	1.15752	16.063%	39.323%
NTES	32.116%	39.035%	1.07694	21.614%	34.021%
SOHU	0.879%	45.682%	1.11410	-9.986%	41.168%
PTR	2.540%	36.225%	1.02872	-7.492%	31.273%
SNP	10.587%	29.807%	0.77005	3.078%	26.479%
CEA	23.865%	63.324%	1.48077	9.425%	57.596%
ZNH	27.043%	52.329%	0.81159	19.128%	50.302%
LFC	5.146%	38.737%	1.13376	-5.910%	33.084%
HNP	6.136%	27.877%	0.15652	4.610%	27.738%
ACH	6.571%	46.825%	1.37055	-6.794%	39.991%

NTES has an annualised average return of 32.116% and an annualised alpha of 21.614%, the highest among all stocks. This indicates that NetEase has a significant excess return compared to the SPX after adjusting for market risk, demonstrating strong market performance and value.

CEA has a high Annualised Standard Deviation of 63.324%, which may be due to the cyclical and volatile nature of the airline industry. Its Annualized Average Return of 23.865% is also relatively high despite the high risk, which may reflect investors' optimism about its long-term growth potential.

LFC has a Beta of 1.13376, indicating higher price volatility than the market average. However, it has an annualised Alpha of -5.910%, indicating underperformance relative to market expectations, even after accounting for the higher Beta. This suggests that the company may not have managed its business risks effectively in the past.

In general, the 10 Chinese concept stocks demonstrate diverse risk-return features. NTES and ZNH offer significant excess returns after accounting for market volatility, possibly due to aggressive growth and effective market risk management. However, the negative Alpha values of SOHU and LFC suggest below-market expected performance. Furthermore, variations in Beta values reveal distinct sensitivities of individual stocks to overall market volatility. For instance, HNP's low Beta value suggests that its price volatility is significantly lower than that of the market, making it an attractive option for investors seeking low-volatility assets.

## 4.2. Stock Correlation

Figure 2 displays the correlation coefficient calculations for the 11 risky assets. These calculations quantify the strength and direction of the linear relationship between the stocks and the market and between the stocks themselves.



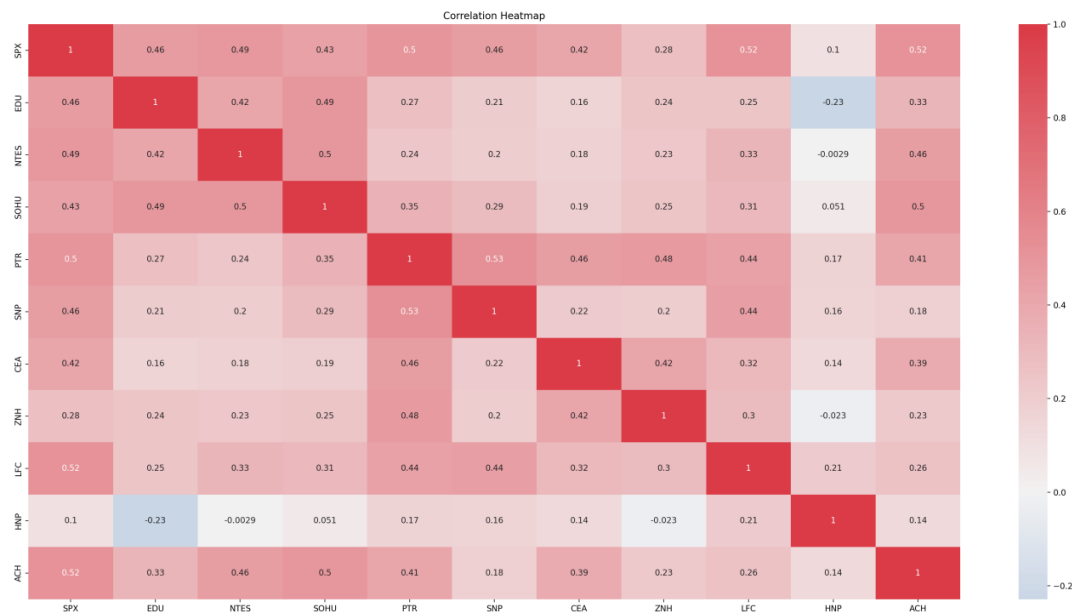


Figure 2: Correlation coefficient Heatmap for the 11 risky assets (Photo credit: Origin)

The heat map reveals that most Chinese concept stocks exhibit a moderate level of positive correlation with the SPX. This suggests that while the overall market trend influences the price movements of Chinese concept stocks, specific factors are also at play for each of them. Both factors have a comparable influence.

HNP, on the other hand, exhibits a low or negative correlation with most other stocks, particularly with EDU. HNP's price action appears to be out of sync with, or even opposite to, the rest of the market. This asset may act as a hedge in a portfolio and help reduce overall portfolio volatility.

When analysed from a sector perspective, the correlation between education stocks (e.g., EDU) and technology stocks (e.g., NTES) is not very strong. This may reflect the independent performance of different sectors under specific market conditions, providing a basis for sector diversification.

Investors should consider this correlation when constructing a diversified portfolio to avoid over-concentration in highly correlated assets, which can reduce systemic risk.

### 4.3. Optimization under constraints

Based on the results of Table 2 and Figure 2 above, the optimal asset allocation ratio for each portfolio under different constraints can be found, and combined with the Sharpe ratio in Equation 11, the optimal portfolio should have the maximum Sharpe ratio among all possible portfolios. This involves applying the Markowitz and exponential models to solve the optimization problems of maximum sharp rate, minimum standard deviation, and maximum extra return [10]. Table 3 shows the final results of the Markowitz model under five constraints, while Table 4 displays the results of the exponential model under the same constraints.

Table 3: Optimal risky portfolio of the Markowitz model under five constraints

Constraint	Max sharp rate	Max extra return	Min standard deviation
Total weight constraint	50.22%	34.34%	146.22%
Single asset weight constraint	32.33%	26.17%	123.51%
Unconstrained	22.09%	17.59%	125.57%
Non-negative weight constraint	34.73%	29.33%	118.42%
Index weight constraint	23.89%	21.12%	113.11%

Under the Total weight constraint, the Max sharp rate is 50.22%, indicating that the portfolio's risk-adjusted return is highest when leverage (i.e., borrowing or shorting) is allowed. Meanwhile, the Max extra return under this constraint is 34.34%, accompanied by a very high Min standard deviation of 146.22%, indicating a correspondingly high risk. Under the Single asset weight constraint, the portfolio's risk and return are reduced.

In the Unconstrained case, the maximum sharp rate decreases to 22.09% without additional constraints, reflecting a more robust investment strategy. Under the Non-negative weight constraint, where shorting is not allowed, the risk-adjusted returns remain relatively high.

Finally, when the weight of SPX is set to zero, both the portfolio's risk and return are relatively stable. This may indicate that not including a market index can achieve robust portfolio performance.

Table 4: Optimal risky portfolio of the Index model under five constraints

Constraint	Max sharp rate	Max extra return	Min standard deviation
Total weight constraint	54.18%	36.33%	149.14%
Single asset weight constraint	36.47%	29.56%	123.40%
Unconstrained	21.85%	17.30%	126.27%
Non-negative weight constraint	36.14%	30.11%	120.02%
Index weight constraint	26.03%	22.64%	114.98%

Compared to the Markowitz model (MM), the Index model (IM) has a higher maximum Sharpe ratio (54.18%) under the total weight constraint. This suggests that IM provides higher risk-adjusted returns when leverage is allowed. Similarly, IM demonstrates better risk-adjusted returns under the single asset weight constraint.

In the unconstrained case, the performance of the two models does not differ significantly. Under the Non-negative weight constraint, IM performs slightly better on Max extra return and slightly worse on Min standard deviation. Additionally, IM may provide slightly better risk-adjusted returns. When the market index weight is set to zero, IM outperforms MM in terms of both return and risk.

## 5. Conclusion

From an individual asset investment perspective, investors have different strategic choices based on the performance of individual stocks and their roles in the market. For instance, NTES and ZNH's better data performance may reflect positive growth within the company or effective market risk management, while SOHU and LFC's poorer data performance may imply that they are underperforming the market's expectations after adjusting for market factors. Some stocks, such as HNP, exhibit price volatility that is significantly lower than the market average, making them an attractive option for risk-averse investors.

Inter-stock correlation analysis can reveal potential diversification opportunities and provide a basis for hedging strategies. While market trends influence most Chinese concept stocks, they also have unique factors that impact their price movements. Some stocks, such as LFC, have a high degree of synchronisation with the market, making them suitable for a strategy that tracks market performance. On the other hand, the negative correlation between some stocks, such as HNP and EDU, suggests that they act as a hedge in the portfolio and help to reduce overall portfolio volatility.

Finally, comparing the results of the two models, MM and IM, under the total weight constraint, both achieve the highest risk-adjusted returns. However, this high return comes at a higher risk. Both models show high Max Sharp rates when shorting is not allowed, indicating that they are efficient investment vehicles under real-world investment constraints. Additionally, the performance of both



models remains robust even after removing the weight of the market index, suggesting that the market index is not the sole determinant of portfolio performance.

In today's globally interconnected economy, the performance of Chinese concept stocks reflects the state of the Chinese economy and serves as a barometer of the health of the global capital market. This study combines theoretical exposition with case-specific empirical analyses to explore the nuances of individual stocks and reveal the complex dynamics of Chinese concept stocks as a collective in the US market by meticulously analysing the data of 10 representative Chinese concept stocks. This study offers investors and policymakers a means of comprehending and utilising Chinese concept stocks to optimise their portfolios. It also provides insights into the unique role that Chinese companies can play on the global stage.

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