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Comparative Analysis of Markowitz Model and Sharpe's Index Model: An Empirical Study on Singaporean Stock Market

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Abstract: In this paper, the researcher uses the Markowitz model and Sharpe's index model, two commonly used models in investment, to analyze the Singaporean stock market. The researcher selected 10 years of daily stock prices of 10 companies listed on the Singapore Stock Exchange, including Z74.SI, DBS, UOB, G07.SI, S68.SI, Flex, KLIC, V03.SI, U96.SI, and C07.SI. The Singapore Traits Index (STI) and the federal funds rate are included, respectively, as the market index and risk-free rate of return to simulate real-world diversified investment products better. Following data aggregation and processing, the researcher calculates the minimal variances and maximum Sharpe ratios by adjusting the weights of each stock within the portfolio separately under five constraints based on Markowitz and Sharpe's index models. In addition, the researcher uses the Solver Table in Excel to calculate the minimum variance frontiers, maximum return frontiers, and minimum return frontiers under each constraint and visualizes all frontiers in scatter diagrams. Lastly, based on the calculations and diagrams, the researcher proposes recommendations for prospective investors looking for investment opportunities in Singapore's stock market.

Keywords: Portfolio Optimization, Markowitz Theory, Sharpe's Index Model, Portfolio Frontiers

1. Introduction

1.1. Singaporean Market

Singapore has experienced rapid and steady growth since gaining independence in 1965. By 2022, Singapore's GDP (per capita) is US$82,807.6, making its economy among the strongest in the world, and highest in Asia [1]. Singapore is also a trusted hub for businesses worldwide, attracting thousands of global professionals yearly due to its political stability and forward-thinking strategies. According to a 2022 report by the Singapore Department of Statistics, the manufacturing industry accounts for 21.6% of Singapore's nominal GDP. Meanwhile, the financial services sector, which comprises 13.5% of nominal GDP in 2022, is the third-largest industry in Singapore and positions the country as a prominent financial center in the Asia-Pacific region. Other industries contributing significantly to Singapore's economy include transportation and storage, information and communications, and real estate [2].

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1.2. Singaporean Market Index

Straits Times Index or STI is a market capitalization-weighted index that records and monitors the daily changes of the 30 representative companies' stocks on the Singapore Stock Exchange. It is a crucial indicator of market performance in Singapore, and it enables investors to compare current stock prices with past prices and evaluate the performance of securities and investment portfolios relative to Singapore's stock market [3]. In this paper, a list of ten Singaporean stocks has been selected and added to an investment portfolio along with STI. The optimal investment strategies are identified by adjusting the individual stock weights in the portfolio through simulating the two models under five constraints.

2. Data Acquisition and Processing

2.1. Data Source

Table 1 lists STI, the market index, and 10 other stocks, including their abbreviations, full company names, and sectors they are in.

Table 1: Stocks used in this paper

Index	Abbreviation	Fill Company Name	Sector
1	STI	Straits Times Index	
2	Z74.SI	Singapore Telecommunications Limited	Communication Services
3	DBS	DBS Group Holdings Ltd	Financial Services
4	UOB	United Overseas Bank Limited	Financial Services
5	G07.SI	Great Eastern Holdings Limited	Financial Services
6	S68.SI	Singapore Exchange Limited	Financial Services
7	Flex	Flex Ltd.	Technology
8	KLIC	Kulicke and Soffa Industries, Inc.	Technology
9	V03.SI	Venture Corporation Limited	Technology
10	U96.SI	Sembcorp Industries Ltd	Industrials
11	C07.SI	Jardine Cycle & Carriage Limited	Industrials

2.2. Calculations of Key Indicators

After obtaining the daily closing prices of each stock, the risk-free investment for each day is calculated based on the federal fund rates using the following formula:

$$RFI_i = RFI_{i-1} \times \left(1 + \frac{FEDL01_i}{100 \times 252}\right) \quad (1)$$

Then, the returns of each stock on each day are calculated, and the month-end data from the daily data is extracted to satisfy normal distribution as a prerequisite for both models. The excess return rate is obtained by subtracting the risk-free rate from each stock's return rate. Subsequently, each stock's annualized average return and annualized standard deviation were calculated based on the excess return rate, the beta, and annualized alpha were calculated using least squares estimation, and finally, the annualized residual standard deviations were calculated from regression. Each indicator is calculated and displayed in Table 2, as below:

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Table 2: Calculations of Indicators

	STI	Z74.SI	DBS	UOB	G07.SI	S68.SI	Flex	KLIC	V03.SI	U96.SI	C07.SI
Annualized Average Return	-0.38%	-0.39%	7.42%	3.20%	-0.43%	6.53%	17.10%	19.56%	11.86%	8.13%	0.01%
Annualized StDev	15.28%	17.00%	21.95%	19.19%	14.57%	15.89%	33.04%	35.74%	23.63%	35.58%	25.23%
Beta	1	0.804	1.125	0.986	0.638	0.376	1.087	0.727	0.531	0.889	0.798
Annualized Alpha	0%	-0.08%	7.85%	3.57%	-0.19%	6.67%	17.52%	19.83%	12.06%	8.46%	0.31%
Annualized Residual StDev	0%	11.74%	13.64%	11.89%	10.83%	14.81%	28.56%	33.97%	22.20%	32.88%	22.08%

After calculating indicators, the covariance of the two stocks and the correlation coefficients between them is calculated, and the calculations are embellished to get a heatmap, as shown in Figure 1.

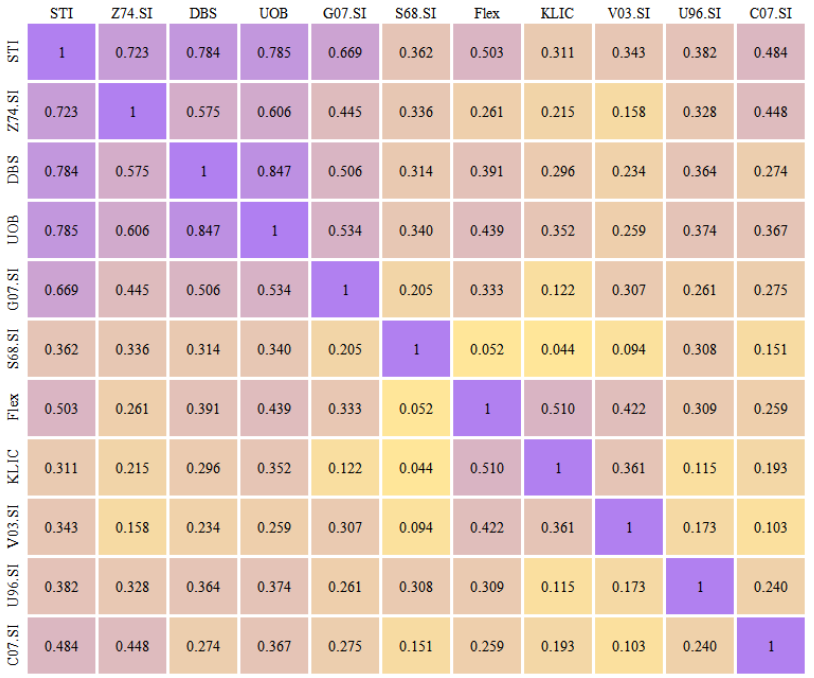

Figure 1: Heatmap of Correlation Coefficients (Photo credit: Origin)

3. Description of Markowitz Model and Sharp's Index Model

3.1. The Markowitz Model

In 1952, Harry Markowitz developed the Modern Portfolio Theory, which suggests that portfolio returns can be maximized or risks minimized by appropriately combining assets with different risks and returns characteristics. According to this theory, investors can reduce overall risk by diversifying their investments. Specifically, an investor can optimize the weight allocation of a portfolio to maximize expected return or minimize risk, thereby achieving the desired investment objective [4]. In Markowitz's model, the variance and standard deviation of return are used to measure investment risk. The variance $\sigma_p{}^2$ is calculated using the following formula:

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$$\sigma_p{}^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j C_{ij} \quad (2)$$

Where:
Wi = amount invested in asset i
W_j =a mount invested in asset j
C_{ij} = covariance between i and asset j
N = number of assets

3.2. The Index Model

In 1963, William Sharpe developed the index model, a statistical model of security returns. The model is based on the idea that stocks vary together, driven by the same economic forces in the stock market. Two sources of risk for a security's return are indicated in this model: systematic (market) risk and unsystematic risk [5]. Expected return of a security can be calculated using the following formula:

$$R_i = \alpha_i + \beta_i R_m + e_i \quad (3)$$

Where:
R_i = expected return on asset i
α_i = alpha coefficient
β_i = beta coefficient
R_m = rate of return of market index
e_i = residual return on asset i

4. Five Constraints

In this section, we considered additional constraints on the model to approach the realistic stock market.

4.1. Constraint 1

The additional constraint limits the sum of absolute values of each weight to be less or equal to 2. According to the Regulation T by FINRA, brokers can lend up to fifty percent of the total purchase price of a margin equity security to a customer for new purchases [6]:

$$\sum_{i=1}^{11} |w_i| \leq 2 \quad (4)$$

4.2. Constraint 2

Constraint 2 limits the absolute value of weight to be smaller or equal to 1. It simulates some of the arbitrary box constraints on weights:

$$|w_i| \leq 1, for\ \forall_i \quad (5)$$

4.3. Constraint 3

Constraint 3 is a "free" problem, with no additional constraints or any restrictions.

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4.4. Constraint 4

Constraint 4 simulates a typical situation in America's mutual fund industry: no short positions are allowed in the open-ended mutual funds [7]:

$$w_i \geq 0, for\ \forall_i \tag{6}$$

4.5. Constraint 5

Constraints 5 tests the broad index's effect on portfolio performance, and in our case, with and without the STI:

$$w_i = 0 \tag{7}$$

5. Comparison of Two Models with Five Constraints

5.1. Minimum Risk Portfolio and Maximum Sharpe ratio

The Excel Solver function is used to compute the minimum risks and maximum Sharpe ratios under each constraint based on the Markowitz and Sharpe index models. It is determined that both models have the lowest risks and highest Sharpe ratios when subjected to constraint 3, the 'free' problem.

5.2. Minimum Risk Frontier, Efficient Frontier and Inefficient Frontier

The minimum-variance frontier displays all the lowest portfolio variances at a certain expected return. Under a certain level of risk, the efficient (maximum return) frontier is a group of portfolios that yield maximum returns, while the inefficient (minimum return) frontier is a group of portfolios that yield minimum returns.

Using Excel's Solver and Solver Table functions, the minimum risk frontier, the efficient frontier, and the inefficient frontier are plotted by applying the Markowitz and index models under five constraints. Each case is displayed in Figures 2 to Figure 6.

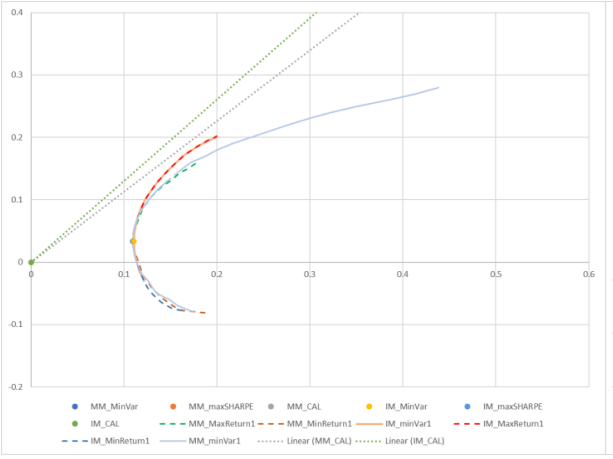

Figure 2: Constraint 1 (Photo credit: Origin)

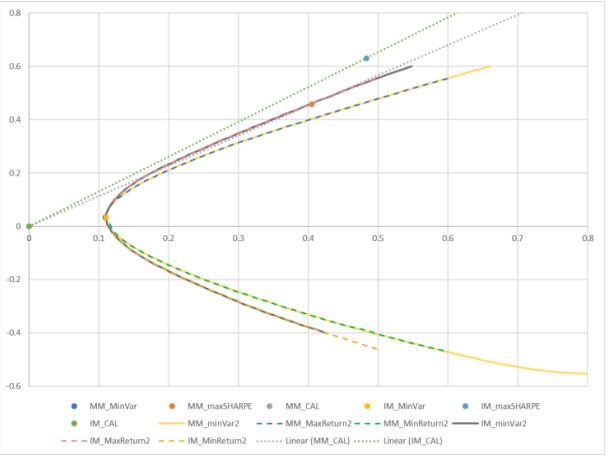

Figure 3: Constraint 2 (Photo credit: Origin)

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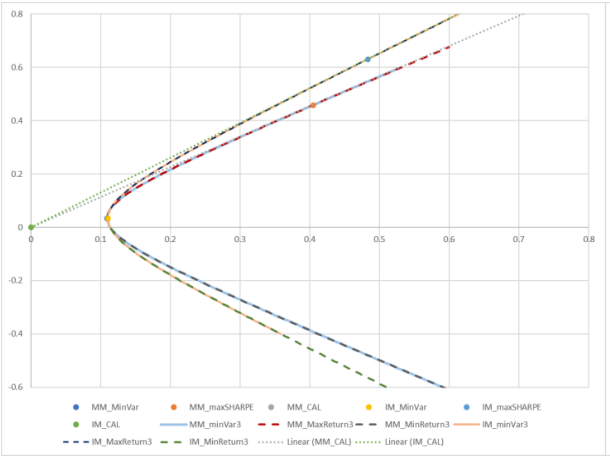

Figure 4: Constraint 3 (Photo credit: Origin)

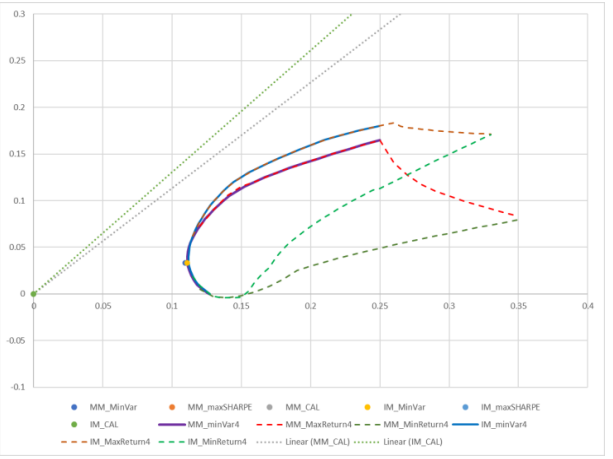

Figure 5: Constraint 4 (Photo credit: Origin)

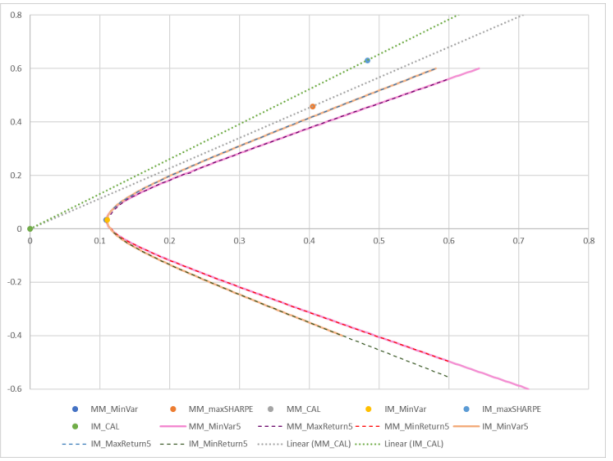

Figure 6: Constraint 5 (Photo credit: Origin)

As shown in the figures above, the index model calculates a greater risk and greater return for the optimal portfolio than the Markowitz model. This is likely because the index model does not consider the correlation between stock returns [8].

Figures 7 and 8 display the frontiers under 5 constraints calculated by the Markowitz and the index models, respectively. It is evident that in the absence of constraints, both models have an efficient frontier that outperforms all other efficient frontiers and an inefficient frontier that is lower than other inefficient frontiers. This suggests that for the same level of risk, additional constraints cause the portfolio's minimum returns to increase while causing maximum returns to decrease, resulting in a concentrated risk-return distribution of the portfolio [9,10].

Under constraint 4, the inefficient frontier of the index model runs significantly higher than that of the Markowitz model. This indicates that when there are no short positions in the portfolio, compared to the 'free' problem, the index model yields higher returns for the same level of risk.

The minimum variance frontiers of both models under constraints 2, 3, and 5 exhibit similar shapes and patterns: the three frontiers of the Markowitz model under these constraints are noticeably more concentrated towards the x-axis. For the same level of risk, the Markowitz model calculates a more concentrated risk-return distribution under these 3 constraints.

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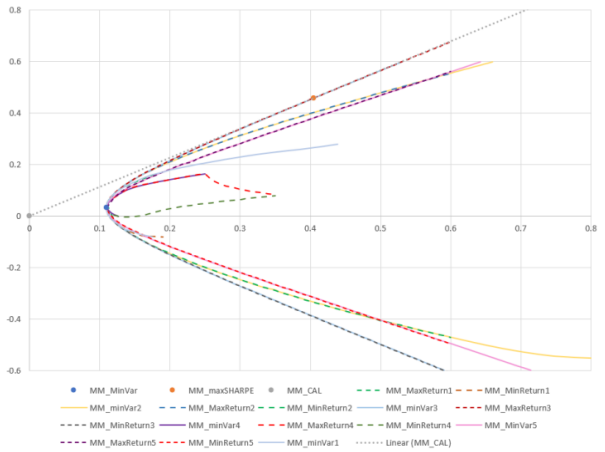

Figure 7: All frontiers under Markowitz model (Photo credit: Origin)

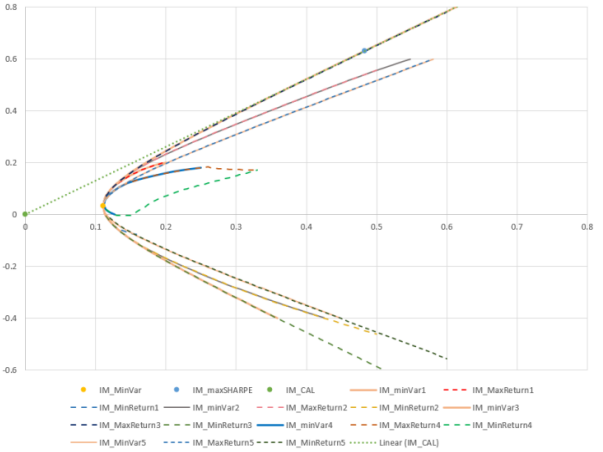

Figure 8: All frontiers under index model (Photo credit: Origin)

6. Conclusion

In this paper, the researcher utilizes the Markowitz and Sharpe index models to compare the optimal portfolios under different real-world scenarios by adding five additional constraints to the models. The researcher also simulates the impact of asset diversification by using 10 different stocks from the SGX. Asset diversification is a commonly used investment strategy that spreads assets across asset classes to reduce risks. Diversification of a portfolio is a personal choice that depends on one's goals for return and risk tolerance. The more diversified a portfolio is, the more predictable and less volatile the returns are.

According to the calculations and diagrams, it is concluded that both the Markowitz model and index model can be well applied to Singapore's stock market. Nonetheless, the index model is better than the Markowitz model because it calculates better results. The frontiers obtained by the Markowitz model show a more concentrated risk-return distribution, indicating a larger amount of idiosyncratic risk. Therefore, it is recommended that investors use the index model to build their portfolios.

Singapore has one of the strongest and most stable economies in the world. Table 1 shows that most stocks have positive excess returns. The betas of 9 out of 10 stocks are below 1.0, indicating that these stocks are less volatile than the market index. And the alpha of 8 out of 10 stocks is larger than 0, meaning these stocks outperform the market. The beta and alpha indicate that these stocks have historically performed well and stably.

Singapore may not be home to large tech giants such as those in the US, but many great companies are still worth investing in. Some of the stocks selected to use in this paper, such as DBS and UOB, are the largest banks in Singapore. These companies have strong exposure to both Singapore and the Asian region, which can provide stability to Singapore's stock market. Some of the stocks in the technology sector, such as FLEX and KLIC, have shown impressive returns in the past 10 years. Therefore, the researcher has reasons to believe that Singapore's stock market shows competency and prospects for investors looking for stable and long-term returns.

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