Proceedings of the 3rd International Conference on Business and Policy Studies
DOI: 10.54254/2754-1169/79/20241753

Research on the Optimal Pricing for Life Insurance

Yuxi Huang[1,a,*]

[1]Ruamrudee International School, Bangkok, T10510, Thailand
a. gokwai666@gmail.com
**corresponding author*

Abstract: Term life insurance is a policy that guarantees the buyer of the policy insurance benefit upon their death for a period of time in exchange for a premium paid by the insurance buyer to the seller yearly for a set number of years. Purchasing life insurance has become an important, if not necessary, financial investment in everyone's life, as it protects their family from a potentially devastating financial loss if they unexpectedly pass away. This paper combined multiple variables in order to compute a pricing formula for life insurance. The final result is that as the age of purchase increases, the premium, or, in other words, the price of the life insurance using my pricing formula, increases exponentially, while the profit received decreases exponentially.

Keywords: Insurance benefits, Premium, Expected profit, Account value, Expected account value

1. Introduction

When selecting insurance products, consumers must recognize their specific needs and actual situation, consider the advantages and potential risks of the product, and make a rational choice. This paper will be mostly focused on finding the expected profit and the pricing using different variables such as the probability of death, the value of the insurance benefits, the rate of investment returns, and the proportion of the account taken as profit. This paper used Microsoft Excel and programming language Python to calculate the results using matrix and arrays.

2. Methodology

First, this paper has to obtain the probability of death for each age from 0 years old to 100 years old. It would be almost impossible for me to find the probability of death without spending years collecting data so instead this paper used the probability already calculated by the Social Security Administration of the probability of death of male [1, 2]. However, the probabilities provided are not exactly what this paper needed. The probability provided in the Actuarial Life Table is the probability of death of an American male at any age knowing that they've survived until that age. The probability this paper need is the probability of an American male dying at this specific age without any other conditions. Fortunately, this can be calculated. To make the following formula easier to read, this paper calls the probability of a person dying at age d Pd (0) and the probability of a person dying at age d knowing that they've survived until age d Yd. The formula for finding Pd (0):

$$P_0(0) = Y_0 \qquad \text{for} \qquad d = 0 \tag{1}$$

DOI: 10.54254/2754-1169/79/20241753

$$P_d(0) = \frac{P_{d-1}(0)}{Y_{d-1}} \cdot (1 - Y_{d-1}) \cdot Y_d \qquad \text{for} \qquad 1 \le d \le 100 \tag{2}$$

$$P_{\ge 101}(0) = 1 - \sum_{d=0}^{100} P_d(0) \qquad \text{for} \qquad d \ge 101 \tag{3}$$

Using this formula, this paper was able to find the probability of a person dying at any specific age.

Once this paper had the array of probability that it needed, it can now find the probability of a person dying at any specific age d knowing that they've survive until a specific age p Pd (p). The probability is calculated by dividing the probability of dying at d Pd (0) by the 1 minus the sum of Pd (0) that comes before p. The formula looks like this.

$$P_d(p) = P_d(0)/(1 - \sum_{d=0}^{p-1} P_d(0)) \tag{4}$$

This gives us all the possibility that this paper will be using in order to calculate the expected profit and the pricing.

Now this paper will introduce my simplified concept of the how life insurance works and how the insurance company profit from the sales[3, 4]. In the concept, a person pays the insurance company a set amount of premium every year for a set number of years and gets an insurance benefit of B upon their death before reaching their 100th birthday. The policy expires after their 100th birthday and there will be no pay-out if they die after the policy has expired. Only people who are 50 or below are allowed to purchase this policy. The insurance company upon receiving the premium payment will immediately invest the money and receive an investment return of r every year hypothetically. Each year, the insurance company will also take a proportion from the account value as profit [5]. The formula for the account value for a policy at age d that was purchased at age p is as follows:

When $\text{Age of Deat}h = \text{Age of Purc}h\text{ase}$:

$$\text{Account Value}_d(p) = \text{Premium} \cdot (1 - \text{Proportion Taken}) \tag{5}$$

When $\text{Age of Deat}h - \text{Age of Purc}h\text{ase} < 15$

$$\text{Account Value}_d(p) = (\text{Account Value}_{d-1} \cdot (1 + r) + \text{Premium}) \cdot (1 - \text{Proportion Taken} \tag{6}$$

When $\text{Age of Deat}h - \text{Age of Purc}h\text{ase} \ge 15$

$$\text{Account Value}_d(p) = \text{Account Value}_{d-1} \cdot (1 + r) \cdot (1 - \text{Proportion Taken}) \tag{7}$$

For the sake of this research, this paper will assume r = 0.07, B = $300,000, Proportion taken as profit to be 1% and the number of years that the client has to pay to be 15 years, assuming the premium is 100 dollars, we would have $99 in the first year because we took 1% from the account for profit. The second year we would have the original $99 plus the second year's $100 plus 99 * 0.07 = $6.93 from investment returns and have the whole thing multiplied by 0.99. All of that would give us the account value of the second year, which is 203.8707 dollars. This repeats until the client has paid 15 times. On the 16th year, however, the formula becomes slightly different; since the client is not required to pay any more premium, there will no longer be, for example, the additional $100 each year.

Now that this paper has the account value, this paper can find the net account value by simply subtracting $300,000 from the account value we already have to get the net account value.

DOI: 10.54254/2754-1169/79/20241753

$$\text{Net Account Value}_d(p) = \text{Account Value}_d(p) - 300{,}000 \quad (8)$$

With the net account value calculated, we can multiply the net account value with their corresponding possibility to get the expected net account value. Then we can find the sum of the expected net account value for a specific age of purchase that we will call Expected Net Account Value (p). The formulas are shown below:

$$\text{Expected Net Account Value}_d(p) = \text{Net Account Value}_d(p) \cdot P_d(p) \quad (9)$$

$$\text{Expected Net Account Value}(p) = \sum_{i=p}^{100} \text{Expected Net Account Value}_i(p) \quad (10)$$

For the Expected Net Account Value (p) we want it to be as close to zero as possible. This way we would be minimizing the loss while also minimizing the price of the premium to maximize sale.

Now that this paper has the formula, this paper has to find the price of the premium at which the Expected Net Account Value (p) would be close to 0. This paper did this by writing a Python program that will, starting from Premium = 0, finding the Expected Net Account Value (0) when the premium = 0, and check to see if the Expected Net Account Value (0) is less than 0. If it is, try again after increasing the Premium by 1. Repeat this process until the Expected Net Account Value (0) is greater than or equal to 0. Once this is done, move on to Expected Net Account Value (1). This would give us the optimal premium for every age of purchase up until age 50. The following figure 1 gives you a basic idea of how the program functions in detail.

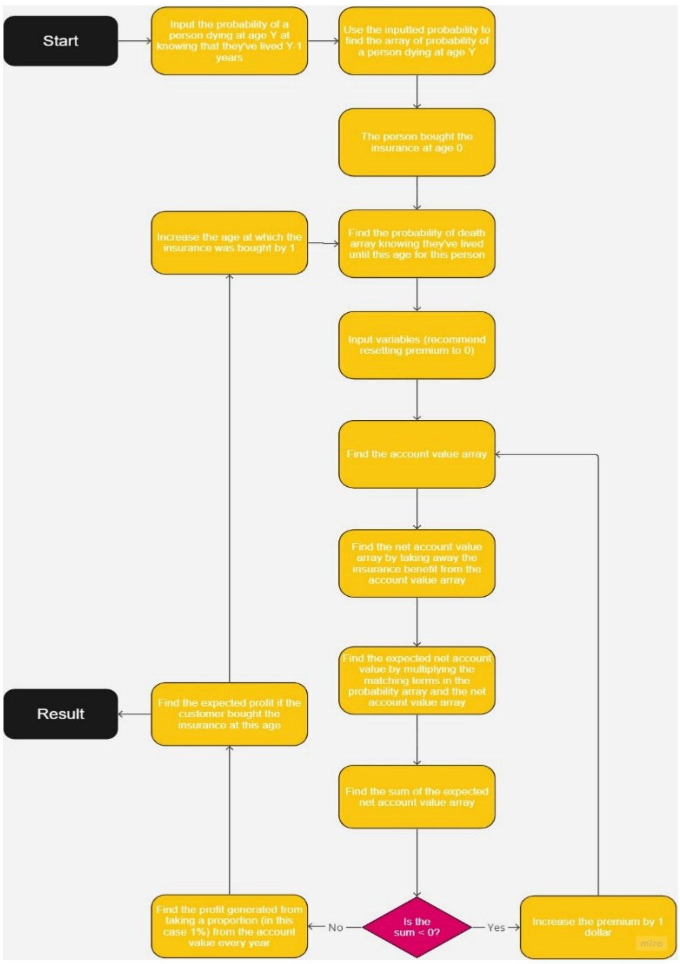

Figure 1: A basic idea of how the program functions in detail

Once we've found the value of premium that we need for each age of purchase, we can use that to find the expected profit for each age of purchase. The formula for the profit at age of purchase p and age of death d is:

$$\text{Profit}_d(p) = \sum_{i=p}^{d} \frac{\text{Account Value}_d(p)}{1-\text{Proportion Taken}} \cdot \text{Proportion Taken} \tag{11}$$

This gives us the 1% that we took from the account value as profit. And since the profit is cumulative, for every year, the profit is the proportion we took this year plus the proportions we took every year before that. Now we can do the same thing as the account value and multiply it by its corresponding probability to get the expected profit. Then we can find the sum of the expected profits in a specific age of purchase to find the expected profit that would be generated through the sale of the policy at that age of purchase.

$$\text{Expected Profit}_d(p) = \text{Profit}_d(p) \cdot P_d(p) \tag{12}$$

$$\text{Expected Profit}(p) = \sum_{i=p}^{d} \text{Expected Profit}_i(p) \tag{13}$$

3. Results

The following figure 2 and figure 3 should show the premium of each age of purchase and their respective expected profit.

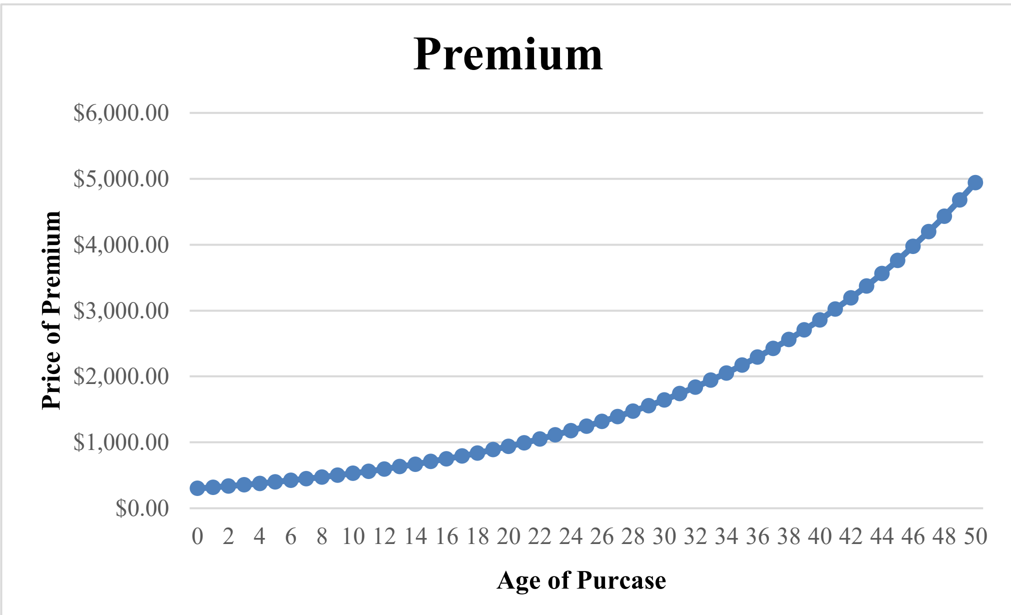

Figure 2: The premium of each age of purchase

This paper then made, using the data calculated, a graph of the price of premium for each age of purchase and another for the expected profit for each age of purchase.

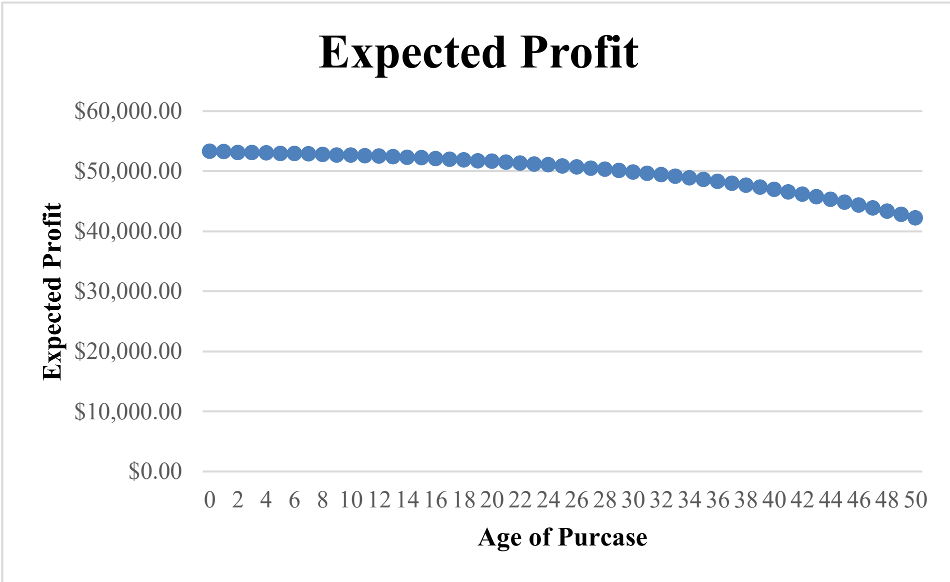

Figure 3: Respective expected profit.

4. Conclusion

As this paper mentioned before, the current formulas are simplified conceptions of how the expected profit might work in ideal scenarios and are obviously much more complex in real life. For example, is it nearly impossible to get the constant 7% investment return that this paper assumed in order to make formulating and calculating easier. In reality, investment is to some extent probability-dependent and would thus require me to find the expected value of investment returns using more data. Doing so would make it more realistic, but working on limited time, this paper opted not to do so.

This research provided a pretty solid concept as to how life insurance operates and how each of the variables plays into the overall calculation of life insurance. If this paper had more time to work on this project, it would have attempted to add variables such as market fluctuations and how that would affect the investment return rate. Despite being a simplified version of reality, the model runs smoothly and can be used as a basis for future related research.

References

[1] Balmford, B., Annan, J. D., Hargreaves, J. C., Altoè, M., & Bateman, I. J. (2020). Cross-country comparisons of Covid-19: policy, politics and the price of life. Environmental and Resource Economics, 76, 525-551.

[2] Social Security Administration. (n.d.). Table 4.C6 - Period Life Table: 2019. Retrieved from https://www.ssa.gov/oact/STATS/table4c6.html

[3] Poufinas, T., & Michaelide, G. (2018). Determinants of life insurance policy surrenders. Modern Economy, 9(8), 1400-1422.

[4] Investopedia. (n.d.). Underwriting Income. Retrieved from https://www.investopedia.com/terms/u/underwriting-income.asp

[5] Shavell, S. (2018). On liability and insurance. In Economics and Liability for Environmental Problems (pp. 151-163). Routledge.