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Research on the Optimal Strategy of Investment Portfolio Based on Markowitz Model

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Abstract: Nowadays, investment is becoming increasingly common. Under the globalization of the economy, investors are given more investment opportunities and choices. Investors need to select excellent assets and allocate the selected asset portfolio on weight during the investment process. The mean-variance model proposed by Markowitz plays an important guiding role in investment and risk management. This model can effectively evaluate investors' portfolio risk and return decisions and significantly impact their decision-making choices. This study uses the data of the annual average return and variance from 11 risky assets and 1 risk-free asset in about 20 years to construct an investment portfolio in a fixed group based on the Markowitz Model. The study calculates the changes in the allocation of the maximum Sharpe ratio and the minimum variance portfolios under three different constraints and one condition with the addition of risk-free assets, respectively, and then analyzes changes in the outcome of data. The capital allocation lines are introduced to analyze the outcome of an investment portfolio with risk-free assets. Compared with the changes from three constraints, reasons are explained why these changes happen under three constraints. Then, the above limitations are taken as a premise, and the study proposes recommendations for the optimal investment portfolio selection for different investors according to the investment cycle, the preference of investors based on the situation of different investors, and market constraints. The study guides investors' future investment decisions.

Keywords: Markowitz Model, Asset Allocation, Portfolio Strategy

1. Introduction

In the era of economic globalization, investors are given more investment possibilities. Today, more and more investors are willing to invest in stocks, bonds, funds, and other high-risk but high-return assets, different from low-risk and low-return assets such as treasury bills. To some extent, high returns may cause high risks. With the increase in the investment frequency of investors and the improvement of their cognitive level relating to investment, investors have a deep comprehension of the market. At the same time, many investors know that they not only pay attention to the high-return assets but ignore high risks. They want to find an asset or portfolio that can balance the risk and return, which means that when investors choose a high-return asset or portfolio, they also want to avoid the impact of risk on returns. During the early 18th century, Adam Smith created the principles of classical economics and the famous "The Wealth of nations" bringing the economy to life [1]. Since

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then, some investors have begun to think about how to invest when future expected returns cannot be predicted accurately to make wealth reach the maximum while risks are achieved at the minimum. In the 20th century, scholars such as John Maynard Keynes and Irving Fisher explored the investment field, attracting more attention. Until Markowitz's "Portfolio Selection was published in the "Financial Journal" in 1952, it laid the foundation for modern portfolio theory [2]. After that, the Markowitz model was widely used in investment and risk management. Markowitz's model effectively links risk and return and says that investors diversify risk by investment portfolio, which means investors could avoid nonsystematic risks to reduce the portfolio's risk and achieve the optimal return [2]. This model effectively evaluates the decisions on risk and return in the investors' portfolio, which significantly impacts their decision-making. Many scholars have analyzed investment portfolios from different fields based on the Markowitz model through data. For example, Cao Hongbo uses strategic thinking and risk control requirements to screen funds in order to provide reference methods for product allocation in the fund advisory business, taking an example of the optimal allocation for all market funds combined with the local fund advisory business practices and based on Markowitz model [3]. Meanwhile, scholars such as Zhou Tingsen and Ma Tian selected actual stock data in the market as analysis samples to analyze different investment portfolios under the Markowitz model [4,5]. Chen Hongbin used the Markowitz model to consider the specific strategies of commercial banks in the bond investment portfolios and then established the commercial strategy of investment portfolios in bank bonds [6]. Four strategy schemes are proposed according to different preferences of the investments in the commercial bank. Other scholars must also select relevant market data in other industry fields to develop investment plans and recommendations. The Markowitz model has practical significance, which is to solve for the optimal return of a portfolio at a certain level of risk and obtain the optimal investment weight of a single asset under certain conditions of the portfolio's expected return to minimize the risk of the entire portfolio. Most scholars generally define the market as a free market to research the allocation problem of investment portfolios based on the Markowitz model. On the contract, fewer scholars add some constraints, such as setting the weight above 0, which means there is a possibility of short selling in the investment portfolio analysis. However, it is rare for investors to have no constraints on their investment portfolio when investing (including banks, companies, etc.). Therefore, based on the previous research, this paper further analyzes the decision-making process of optimal investment portfolios. First, this paper selects the data of ten stocks from companies in the United States and one risk asset as the research sample. Second, it increases market restrictions on investment portfolios. Third, analyze how investors make investment decisions under different constraints and propose the optimal investment strategy. The paper defines the optimal investment strategy as the portfolio with the highest return under low-risk conditions.

2. Case Description

2.1. The Markowitz Model

Harry Markowitz introduced the Markowitz model in 1952 [2]. It is also known as the mean-variance model, a portfolio optimization model. It is to create the optimal investment portfolio with the return ratio of risk by analyzing various portfolios through the expected return and risk of the assets equal to mean and standard deviation, respectively.

There are three assumptions premise.

1. Investors are rational, which means at a certain level of risk, investors expect the portfolio to have maximum return; correspondingly, at a certain level of return, investors hope the portfolio has minimized risk.

2. The distribution of asset returns is close to a normal distribution.

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3. It is merely based on the risk and the return of assets.

The Markowitz model formula based on the above assumptions is as follows:

Assume there are n types of different assets in the market (1,2,…,n). For one asset_i, the expected return rate of this asset_i is Ri, and σi is the standard deviation of Ri.

1. The sum of weight 1 plus weight 2 plus weight N equals 1
2. The expected return of portfolio
3. $E(r) = \sum_{t=1}^{n} wi \times E(ri)$ The variance

$$\sigma 2 = \sum_{t=1}^{n} wi \times wi \times cov(Ri \times Ri) \tag{1}$$

The formula used in the following paper

1. Annual average return

$$AAR = \frac{1}{n}\left(\sum_{t=1}^{n} R_e\right) \times 12 \tag{2}$$

2. Annual standard deviation

$$ASD = \sqrt{\frac{1}{n}\left[\sum_{t=1}^{n}(R_e - AAR)^2\right] \times 12} \tag{3}$$

3. Correlation coefficient

$$Corr(X, Y) = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2 \sum(y-\bar{y})^2}} \tag{4}$$

X and Y stand for one asset, respectively, and x and y mean the return rate of one asset. x¯, y¯ means the mean of each asset's expected return

4. Shape ratio

Sharpe ratio is the target for risk-adjusted indicators for measuring fund performance.

The main idea of Sharpe's ratio is to assume that rational investors will invest and hold a portfolio that is most beneficial to themselves. It is an investment portfolio with a certain level of risk but maximizes or minimizes expected returns [7,8].

$$S\_p= E\ (R\_p)-R\_f\ /\sigma\_p \tag{5}$$

R_p =expected annualized rate of return r_f = annual interest rate of risk-free asset σ_p =annual standard deviation

2.2. Data Selection

Most investors generally choose the investment portfolio combining risk-free assets and risky assets. The paper mainly selects ten stocks of risky assets for 20 years; the risk-free assets are treasury bills.

The reason for selecting the return of 10 stocks over the past 20 years is that due to the impact of the pandemic in 2020 and 2021, stock prices are affected by market factors and fluctuate from the other years. In order to prevent abnormal data, which means special market factors for predicting the future return rate of the portfolio, the paper selects 20 years of data to mitigate this impact, as shown in Table 1, according to Yahoo! Finance.

Table 1: Data for 10 stocks and one (S&P 500) equity index

	SPX	NVDA	CSCO	INTC	GS	USB	TDCN	ALL	PG	JNJ	CL
Annualized Average Return	7.542%	32.802%	9.714%	8.905%	10.825%	9.878%	11.010%	10.080%	9.437%	8.464%	7.105%
Annualized StDev	14.850%	55.774%	30.809%	30.503%	29.572%	23.680%	18.134%	24.884%	14.587%	14.785%	15.350%

3. Analysis on the Problem

The study further analyzes the optimal investment portfolio based on different constraints under portfolio allocation. This study calculates the following two sets of data under different constraints: the minimum variance portfolio and the maximum Sharpe ratio portfolio. The reason for choosing to calculate these two sets of data is:

1. The reason for calculating the minimum variance portfolio is based on the assumption that investors are all risk averse and generally willing to give up high returns to invest in low-risk investment portfolios.
2. The reason for calculating the maximum Sharpe ratio of an investment portfolio is that the Sharpe ratio is an effective index to measure the efficiency of asset portfolio allocation. When the Sharpe ratio of an asset portfolio reaches its highest level, it can be estimated to be the optimal asset portfolio allocation [2].

3.1. Portfolio Weight Allocation in a Free Market

A free market refers to a portfolio of assets allocated without any constraints (excluding risk-free assets)

Table 2: The stocks variance and sharpe

Free	SPX	NVDA	CSCO	INTC	GS	USB	TDCN	ALL	PG	JNJ	CL	Return	StDev	Sharpe
Min Variance	38.3667%	-2.968%	-2.893%	1.327%	-5.899%	-0.305%	19.415%	11.481%	25.931%	18.822%	19.674%	7.51%	10.95%	68.54%
Max Sharpe	-109.975%	22.457%	0.893%	8.186%	12.725%	13.213%	46,460%	7.897%	53.498%	42.717%	18.201%	16.99%	16.48%	103.11%

As shown in Table 2, this study shows that when the risk reaches the minimum, the weight of many stocks with large variance, which means high risk is less than 0, that is, short-selling high-risk stocks and borrowing assets to invest in other stocks to offset the risks brought by high-risk stocks and to reduce risks.

Under the premise of constraints, when the Sharp ratio reaches the highest (meaning that the portfolio is the best allocation in this case), it can be found that the variance of the portfolio is not the lowest. At the highest Sharp rate, the variance reaches 16%. If the investor is risk-averse, investors prefer to choose the minimum variance combination rather than the maximum Sharp rate. In comparison, risk-neutral and risk-preference investors are inclined to choose the largest Sharp rate portfolio, which does not mean that the return reaches the highest, while it only illustrates that there is the most optimal point between the return and the portfolio's risk. Risk-preference investors may also seek higher returns and increase risk (this case is considered separately).

3.2. Portfolio Weight Allocation when the Absolute Value of the Sum of Weight is Less Than 2

The absolute value of the sum of weights is less than 2, which means that the regulation in real investment Regulation T by FINRA allows broker-dealers to allow their customers to have positions, 50% or more of which are funded by the customer's account equity.

Table 3: The absolute value of the sum of weights

$\sum_{t=1}^{n} w \le 2$	SPX	NVDA	CSCO	INTC	GS	USB	TDCN	ALL	PG	JNJ	CL	Return	StDev	Sharpe
Min Variance	38.3667%	-2.968%	-2.893%	1.327%	-5.899%	-0.305%	19.415%	11.481%	25.931%	18.822%	19.674%	7.51%	10.95%	68.54%
Max Sharpe	42.736%	15.745%	1.148%	-6.110%	3.252%	-81.291%	35.289%	1.068%	45.714%	30.002%	12.447%	14.01%	13.95%	100.42%

As shown in Table 3, under this constraint, there is no strict constraint on investors who are not able to make a short selling on stocks and borrow money to invest in other stocks. By contrast, it only limits the sum of weights. Therefore, the minimum variance portfolio under this limit is consistent, whether it is the weight, return, or variance compared with the free market in Table 2. The difference is the portfolio with the highest Sharp rate. The portfolio with the highest Sharp rate under this limit is lower than the portfolio with the highest Sharp rate under the free investment market. The reason is that when the sum of the portfolio's investment weights is limited, investors can't invest too many assets with high returns or short some high-risk stocks to borrow assets to invest in other stocks with high returns. The allocation of stock weights is relatively average compared with the free market. In the free market, there are extreme configurations, such as the weight of -1.0994. Therefore, although the risk of the optimal portfolio under this limit has decreased, the Sharp ratio has also declined.

3.3. Portfolio Weight Allocation When the Weight of One Asset is Greater than or Equal to 1

The purpose of this constraint is to simulate the typical limitations existing in the U.S. mutual fund industry: a U.S. open-ended mutual fund is not allowed to have any short positions.

Table 4: The stocks constraint's variance and sharpe

W>=0	SPX	NVDA	CSCO	INTC	GS	USB	TDCN	ALL	PG	JNJ	CL	Return	StDev	Sharpe
Min Variance	9.491%	0	0	0	0	0	19.849%	0	28.905%	20.622%	21.134%	8.88%	11.27%	78.8%
Max Sharpe	0	10.948%	0	0	0	0	23.727%	0	42.561%	16.169%	6.597%	12.06%	13.12%	91.90%

As shown in Table 4, under this constraint, investors are not allowed to borrow assets to invest in other stocks by shorting stocks, so investors cannot reduce the risk by shorting high-risk stocks and borrowing assets to invest in low-risk stocks. Therefore, the variance under the minimum variance portfolio under this limit is higher than that of the first two portfolios of variances, but the return is also higher. At the same time, this limit is based on the weight limit of a single stock, which limits the downline of the proportion of stocks invested in the portfolio. Thus, the possibility of optimizing the portfolio by shorting stocks is limited. Therefore, the Sharp rate of the optimal portfolio under this limit is the lowest based on the above portfolio in Table 2 and Table 3 with two restrictions. Meanwhile, restricting the investment ratio of high stocks results in a reduced return and lower risk.

3.4. Portfolio Weight Allocation when Increase Risk-Free Assets (U.S. Treasury Bonds)

The above data are all portfolios of risk assets. Investors may also consider adding risk-free assets to reduce risks in real investment. Therefore, this study calculates the increase of different proportions of risk-free assets, the variance of the portfolio, and the change of return, as shown in Figure 1. Capital Allocation Line combines free-risk assets and one or more risk assets in different proportions. Each point online represents an investment portfolio consisting of risky and risk-free assets. The study illustrates the Capital Allocation Line's situation in the free market.

When the portfolio is added risk-free assets under the same risk, the income corresponding to the points on the Capital Allocation Line is greater than or equal to the return of the points on the effective frontier. The points on the Capital Allocation Line are all portfolios with the maximum Sharp rate (the optimal portfolio studied in this article). The point that the effective frontier is tangent to the Capital Allocation Line is the optimal risk portfolio.

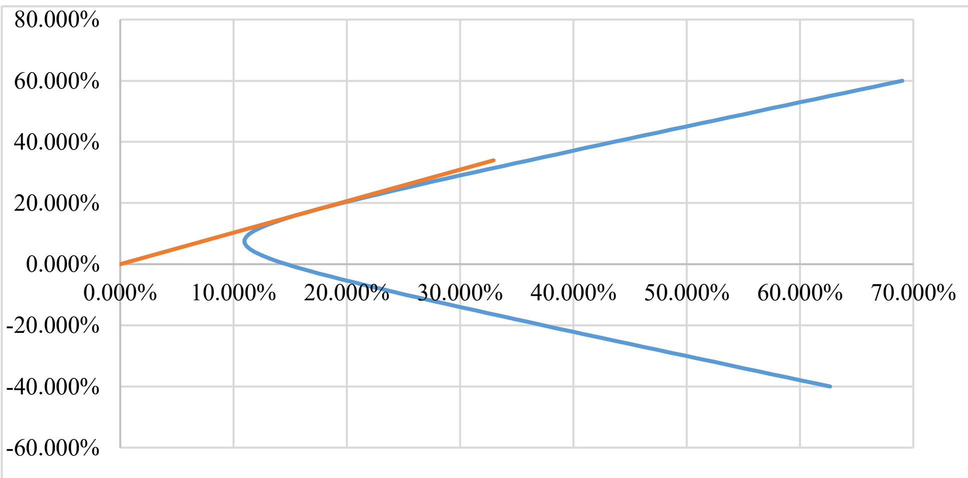

Figure 1: The Capital Allocation Line (Photo credit: Origin)

4. Suggestions

In real investment, different investors have different risk tolerance abilities. According to Elton, the response to risk can be quantified as a parameter: the risk aversion coefficient, which can measure the degree of aversion of risk-averse individuals [9]. Investors can be divided into three types based on risk aversion: risk averse, risk neutral, and risk preference investors. Different investors may have different allocation methods on weight for the same asset portfolio due to their different risk aversion preferences. This study provides corresponding suggestions for different selection problems of investors based on the four different constraint conditions mentioned above.

4.1. The Investment Portfolio with All Risk Assets

Based on the first constraint, the situation in the free market is mainly aimed at ordinary investors in real life. For ordinary investors, there are constraints on investment and asset selection based on individual rather than market restrictions. In a free market, investors can freely invest. Their preferences are one of the most important consideration factors for them.

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Risk-averse investors invest most of their funds in low-risk assets, and appropriately adding some medium to low-risk assets with slightly higher returns is recommended. At the same time, they can sell corresponding high-risk assets to reduce risk. As shown in Table 2, risk-averse investors are recommended to increase the proportion of low-risk assets such as INTC, TDCN, PG, NJ, and CL. From Table 1, it can be found that the annual variance of these assets is all below 20, indicating low risk. When choosing asset weighting ratios, risk-averse investors can try to add low-risk and high-return assets, as shown in Table 2. Although the risk level of TDCN is lower than that of ALL, the return rate of TDCN is slightly higher than that of ALL. Therefore, when investing in TDCN, the investment weight ratio can be appropriately increased to achieve the effect of low risk and high return. Due to the need to consider building a diversified investment portfolio to reduce risk, other asset weights can be added in small amounts with a certain proportion of weights, but excessive diversification and concentration should be avoided. Risk preference investors, who accept high risks in order to increase returns, invest relatively more in high-risk assets such as NVDA in Table 1, which has the highest risk among the 11 assets and is also the asset with the highest return among the 11 assets. From the portfolio with the maximum Sharpe ratio, it can be seen that the investment ratio of this asset is relatively high. At the same time, the maximum Sharpe ratio balances risk and return, achieving a high return asset ratio at relatively low risk, which is a good choice for risk-preference investors. In addition, as the market is constantly changing, high-risk stocks are prone to drastic fluctuations due to market changes. Investors also need to regularly adjust the weight ratio of assets according to market changes. Risk neutrality refers to investors who do not have requirements for risk changes. If risk-neutrality investors want to achieve better returns, an investment portfolio with the highest Sharpe ratio, similar to those with high-risk preferences, is chosen as an optimal portfolio.

The main restriction under the second restriction is the constraint on investment by economic traders. Economic traders should consider different factors when selecting investment portfolios, such as risk preference, which is similar to the above, investment period, and investment objectives, for investment cycle issues. If the investment period is short, economic traders can choose to increase the proportion of high-yielding stocks, such as NVDA. Due to the short-term investment cycle, assets are less affected by market fluctuations and can increase the proportion of high-risk and high-return assets to achieve high returns. It is noted that some high-risk assets may not necessarily have high returns [10]. An investment portfolio with the maximum Sharpe ratio, which can achieve high returns while controlling risk, can be considered in this situation. This is also suitable for risk-averse investors. In the case of a long investment cycle, investors first need to pay attention to risk diversification, and it is recommended to increase the proportion of low-risk assets to ensure returns. In this case, investors can consider the minimum variance portfolio. Although risk-averse and risk-neutral investments are not as risk-averse as risk-averse investors, the market greatly affects asset volatility when the investment cycle is long. Therefore, it is recommended to appropriately increase the proportion of low-risk assets to ensure a certain rate of return and achieve long-term appreciation.

The third limitation is to simulate investors' investment in certain situations where they cannot short positions. In a market without shorting, investors still need to follow the above aspects when determining the proportion of stocks and examine the future development and good fundamentals of the stock. At the same time, in the absence of shorting, market fluctuations are usually more intense, so investors need to pay more attention to short-term market fluctuations. The proportion of high-risk assets should be properly reduced to ensure higher returns; as can be observed in Table 4, the investment weights for high-risk assets are all 0.

4.2. The Investment Portfolio with One Free-Risk Asset

In this situation, a capital allocation line determines how investors allocate their capital when making investment decisions [2]. Investors' decision-making is mainly based on the level of risk preference, as shown in Table 5. For risk-averse investors, the risk of the allocated asset portfolio should be low (meaning that the proportion of risk-free assets invested is relatively large). For risk-averse investors, they can reduce their portfolio of free risk assets. At the same time, they still need to invest based on their conditions and market conditions to reduce the probability of high-risk portfolios taking risks. The points on asset allocation are the same for risk neutrality. If you want to achieve higher returns, the point where the asset allocation line is tangent to the effective front line can be chosen better.

5. Conclusion

The Markowitz model is the cornerstone of modern portfolio theory and plays an important guiding role for investors. Many scholars have explored the market's applicability and specific use of the mean-variance model. This study is also based on the Markowitz model to analyze portfolio allocation and optimal portfolio selection for investors under different market constraints further. This study mainly selected eleven risk assets and one risk-free asset and used these eleven risk assets as a fixed investment portfolio. Based on the Markowitz model, the optimal asset allocation problem on weight in the portfolio was analyzed under three constraints and one condition of adding risk-free assets. The study calculated the weight allocation scheme for the minimum variance and maximum Sharpe ratio. In a free market, the Sharpe ratio is the highest among the maximum Sharpe ratio portfolios, and due to different weight limitations, the Sharpe ratio decreases sequentially. The variance is the smallest in the combination of free market and the minimum variance under the constraint based on economic traders. Due to the influence of the market not allowing short positions, the minimum variance in the portfolio is higher than the other two constraints. Based on the four limitations mentioned above, suggestions are proposed for investors to choose the optimal investment portfolio. Investors mainly choose the optimal investment portfolio plan through risk preference, investment cycle, and investment constraints. The study provides certain guiding significance for investors to invest. However, the Markowitz model still has certain limitations in practical applications. The financial market environment in the real world is very complex and has high volatility, making it difficult to evaluate the specific volatility of each investment asset. Therefore, risk assessment may have certain biases. In addition, the study's data is based on past values to infer future situations, which have certain inaccuracies, so further research is required.

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