

Multi-factor Models in Finance

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Abstract: This literature review explores the significance of multi-factor models in finance, emphasizing their dynamic nature and pivotal role in understanding global market patterns. The multi-factor framework, adept at capturing the complexities of asset pricing, incorporates various variables, enabling a comprehensive analysis of relationships impacting market conditions. The Fama-French Three-Factor Model, an influential extension of traditional models, introduces size and value components, enriching the understanding of asset valuations. Assumptions underpinning multi-factor models, such as market efficiency and rational investor behavior, form essential foundations. Despite their advantages, these models face challenges, including the risk of overfitting and the complexity of incorporating numerous variables. The study underscores the need to recognize these limitations while appreciating the models' contributions to portfolio management and risk assessment, ultimately contributing to the ongoing evolution of financial analysis.

Keywords: Fama-French Three-Factor Model, Asset Pricing, Market Efficiency

1. Introduction to Multi-Factor Models in Finance

A multi-factor framework is an efficient framework in the dynamic realm of finance modeling. It makes use of a variety of indicators to grasp the intricacies of market occurrences. It provides a thorough understanding by considering the simultaneous influence of several elements on the results of specific assets or portfolios. Essentially, a multivariate model uses financial modeling to examine the complexities of asset price [1]. The method stands out because it incorporates many variables, allowing for a more in-depth analysis of the relationships among the parameters in question and how those relationships impact market circumstances [2]. Through this forecasting approach, the key variables affecting the financial performance of particular stocks and portfolios may be thoroughly examined.

A multi-factor model's ability to depict an investment's return (R_i) as a function of numerous elements constitutes one of its essential features [3]. The model is encapsulated in the formula:

$$R_i = a_i + \beta_i(m) \times R_m + \beta_i(1) \times F_1 + \beta_i(2) \times F_2 + \dots + \beta_i(N) \times F_N + e_i$$

Where:

- R_i is the return of the security,
- R_m is the market return,
- $(1,2,3\dots)F(1,2,3\dots N)$ represents each of the factors used,
- β denotes the beta concerning each factor, including the market (m),
- e_i is the error term,

- α_i stands for the intercept. [1]

This formula outlines the intricate relationship involving a stock's return and the wide range of factors impacting its value, making it a valuable tool for financial study and portfolio development. The beta measures a security's risk profile in connection to the stock marketplace overall. It emphasizes how crucial it is to understand how every factor influences asset prices [3].

The Fama-French three-factor model, an expansion of the capital asset pricing approach, is a noteworthy example in this field. This framework provides a more complex understanding of the shifting nature of asset price by including size and value elements in addition to the marketplace risk component [4]. The multi-factor model is a potent tool in financial modeling, revealing the complex relationship between asset values and improving our capacity to build portfolios with a more profound comprehension of the relationship between risk and reward [5].

2. Assumptions of a Multi-Factor Model

The multi-factor model upholds the hypothesis of an effective market, which postulates the efficiency of financial markets. This suggests that asset values generally consider every relevant detail, making it difficult for traders to regularly beat the market using knowledge from the past [6].

The approach makes the logical assumption that investors make informed decisions. When investors act rationally, they properly assess facts and make choices that optimize their anticipated benefit. This presumption is essential for the model to represent real-world investors' tastes and behaviors accurately.

The framework assumes that projections about future earnings, dangers, and other pertinent economic factors are comparable among market players. This presumption guarantees that the model's asset evaluation and comparison framework is constant [7].

Generally, multi-factor approaches presuppose the lack of substantial transaction expenses. By eliminating the extra complication of trading costs, this presumption streamlines computations and allows a more precise examination of the variables influencing asset values [8].

The framework assumes that the chosen components and returns on assets have a linear relationship. Although this linearity makes modeling easier, it might not account for more intricate, nonlinear interactions that can occur in finance.

The model assumes that the analysis's components show stationarity or that their statistical features are steady across time. This presumption makes it easier to apply previous data to forecast asset prices in future years [6].

The model assumes no marketplace impacts, such as taxes, limitations on short sales, or liquidity constraints. These obstacles may induce prejudices in the model and affect investors' capacity to execute trading plans if these obstacles are not considered.

Every model component is supposed to represent a certain aspect of systematic risk. The components are selected to provide a thorough evaluation of the causes of financial risk; their selection relies on their capacity to clarify differences in the returns of assets [9].

The model assumes that diversification across several variables reduces unsystematic risk, making only systematic risk worthy of examination. This is consistent with the idea that idiosyncratic risks may be eliminated by an adequately diversified portfolio, freeing the model to concentrate on more general market patterns.

The model assumes that the inexplicable component of returns on assets, or residuals, has a typical distribution. This presumption makes it easier to do statistical studies and hypothesis testing about how well the model captures and explains the patterns of the market [8].

In order to assess the findings of a multi-factor model and appreciate its limits in encapsulating the intricacies of actual financial markets, it is imperative to comprehend and acknowledge these presumptions.

3. Development of Multi-factor Model

As it seeks to account for a broader range of impacts on asset prices, the Multi-Factor Model (MFM) in finance represents a substantial divergence from conventional single-factor designs, such as (CAPM). At the beginning of the 1990s, the work of monetary economists Eugene Fama and Kenneth French, who saw the shortcomings of the CAPM in describing the intricacies of yields in the stock market, laid the groundwork for the creation of the MFM [6].

The Fama-French Three-Factor Model is a crucial addition to the MFM paradigm. Building on the fundamental ideas of the (CAPM), this approach adds two more parameters to the analytical framework: value (HML - High Minus Low) and size (SMB - Small Minus Big). By adding size and value components to the marketplace risk variable, the Fama-French model attempts to offer a more profound rationale for inconsistencies in the valuations of assets [5]. The model's formula, which reflects a thorough grasp of the complex relationship between risk and reward in finance, describes an asset's yield as an indicator of size, worth, and market danger [4]. Since then, the Fama-French Three-Factor Model has established itself as an essential component in the literature on pricing assets, impacting later studies and assisting in the broader use of multifaceted frameworks for financial evaluation.

The formula for the Fama-French Three-Factor Model is:

$$R_i = a_i + \beta_i(R_m - R_f) + \beta_i(\text{SMB}) + \beta_i(\text{HML}) + e_i$$

Where:

- R_i is the return of security i ,
- R_m is the market return,
- R_f is the risk-free rate,
- SMB is the size factor (Small Minus Big),
- HML is the value factor (High Minus Low),
- β_i represents the sensitivity of the security to each factor,
- a_i is the intercept,
- e_i is the error term.

The purpose of adding size and value components to the Fama-French Three-Factor Model was to correct for inconsistencies seen in the CAPM, namely the propensity of value and small-cap companies to show aberrant returns [8]. This incident underscored how crucial it is to consider various factors outside of the market when attempting to explain why asset values vary.

The MFM has progressed to the point where other components, such as momentum, liquidity, and macroeconomic indicators, are being investigated. In addition to the Fama-French model, the Carhart Four-element Model takes previous price patterns into account by incorporating a momentum element (UMD, or Up Minus Down):

$$R_i = a_i + \beta_i(R_m - R_f) + \beta_i(\text{SMB}) + \beta_i(\text{HML}) + \beta_i(\text{UMD}) + e_i$$

This enlarged paradigm captures not just market danger and size and value impacts but also momentum-driven deviations, reflecting a more thorough grasp of the complex structure of asset pricing.

4. Types of Multi-Factor Model Categories

Approach-wise, multi-factor models fall into three main categories: statistical, fundamental, and macroeconomic. Providing a comprehensive understanding of stock market dynamics, macroeconomic models examine a security's return in connection to more general economic variables such as job creation, inflation, and interest rates [7]. On the other hand, fundamental models investigate the inherent connection between the return on investment of an investment and its economic characteristics, such as market value, amount of debt, and profits, offering valuable

perspectives on the fundamental robustness of the asset [9]. Statistical models make comparing stocks based on statistical measurements easier, which use historical data to evaluate each security's performance individually [2]. These many classifications demonstrate the adaptability of multi-factor models, accommodating a range of analytical inclinations and offering a refined comprehension of the complex dynamics impacting asset returns inside the financial environment.

5. Advantages of Multi-Factor Models

Multi-factor models are widely used in financial analysis because they provide distinct benefits. One of their main advantages is their ability to give a more sophisticated view of asset pricing by taking into account several variables at once. This method overcomes the drawbacks of single-factor models, such as (CAPM), and enables an improved assessment of the many factors affecting profits. Like the Fama-French Three-Factor Model, multi-factor models add aspects like size and value variables to classic models to increase their descriptive ability [4]. Investors may better capture and manage risk with the help of this greater power to explain, which results in better-informed portfolio creation. These models can also detect abnormalities that may go unnoticed by more basic approaches and adjust to changes in the marketplace, making them a more powerful tool for investors looking for a thorough and fluid knowledge of asset pricing [8].

Furthermore, the models recognize that risk is multifactorial, enabling researchers and investors to identify and measure the causes of systematic danger in addition to market beta. Finally, the Fama-French Three-Factor Model explains the reasons that cause anomalous returns and has advantageous uses in asset pricing and performance assessment [9]. These benefits highlight the model's usefulness and long-lasting influence on the financial analysis community.

6. Limitations

Even though they provide a more thorough method of comprehending asset pricing, multiple-factor models have shortcomings. Multi-factor models exhibit several limitations, three of which are particularly striking. In a book by Darolles et al., under the section "Why Factor Models in Finance?" The first weakness is revealed to be a correlation and unobserved factors [10]. Essentially, these factor models assume a correlation with portfolio performance, yet the presence of unobserved factors or high correlation could lead to a misleading explanation of financial returns. Another limitation shared among many financial scholars concerns data limitations and transparency. Darolles et al. further warn that limited transparency into underlying investments' entire composition and evolution, as observed in hedge funds, undermines the multi-factor models' capacity to accurately estimate factor exposures [10]. According to Nakagawa et al. (2019), a lack of transparency also affects the interpretability of predictions, making it unsuitable for use in actual investment practice. A third notable limitation is inherent in modeling assumptions [11]. Multi-factor models are only as effective as their reliance on specific assumptions, such as the Gaussian distribution of factors and idiosyncratic risks [10]. Any deviations from these assumptions could render a model inaccurate and unreliable.

7. Conclusion

The literature study demonstrates the history, benefits, and constraints of multi-factor models in finance, highlighting their dynamic environment. These models, which range from the primary (CAPM) to the complex Fama-French Three-Factor Model and beyond, have influenced how global markets' patterns of returns and risks are understood. The benefits of multi-factor approaches highlight their importance in the valuation of assets and portfolio administration, as they provide increased capacity for explanation and a more sophisticated evaluation of asset price. However, the research also emphasizes how critical it is to recognize the limits, including the possibility of

multicollinearity, data needs, and overfitting. The amalgamation of many models and approaches signifies a continuous dedication to enhancing financial analysis, providing professionals with an adaptable set of instruments to maneuver through the intricacies of contemporary financial markets. The examined literature offers a starting point for future studies. It motivates more investigation into multi-factor models to better comprehend and maneuver the complex terrain of price determination and making investment choices.

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