Optimizing Betting Strategies with the Kelly Criterion for Economic

Yirong Huo^{1,a,*}

¹University of Washington, 1410 NE Campus Pkwy, Seattle, WA 98195, United States a. dkkdkk0771@gmail.com *corresponding author

Abstract: This comprehensive study explores the Kelly Criterion, a strategy devised for betting but with significant implications in economic decision-making and wealth management. The criterion, initially developed to maximize long-term growth in independent repeated bets with positive expected net returns, has evolved to encompass a broad range of applications. Through the use of mathematical modeling and probability theory, this research demonstrates how the Kelly Criterion can be effectively applied to enhance wealth accumulation. The societal benefits of this approach are manifold, extending to the reduction of problem gambling rates and promoting responsible gaming. The research also delves into the broader implications of the Kelly Criterion for society, suggesting that its judicious application can significantly shape the future of gambling as a recreational activity. By ensuring informed decision-making, the criterion benefits both individuals and society as a whole, offering a structured approach to managing financial risks and rewards. The expanded scope of this study highlights the versatility of the Kelly Criterion, showing its potential as a pivotal tool in modern economic scenarios, characterized by uncertainty and fluctuating financial markets.

Keywords: Kelly Criterion, Wealth management, Economic Stability, Investment Strategies

1. Introduction

The Kelly Criterion, named after its creator John L. Kelly Jr., has emerged as a pivotal tool in the domains of finance, sports betting, and investment, transcending its initial use in information theory. This research paper delves into the multifaceted applications and implications of the Kelly Criterion, particularly in optimizing capital allocation and decision-making strategies.

Developed in the mid-20th century at Bell Labs, the Kelly Criterion was initially intended to address noise issues in long-distance telephone signals. However, its potential was quickly recognized in the field of economics, particularly in investment and wealth management. The criterion offers a systematic and mathematically grounded approach to determining the optimal fraction of capital to wager or invest in scenarios characterized by both opportunity and risk. Its fundamental principle is to maximize long-term growth of capital while minimizing the likelihood of financial ruin, making it an attractive strategy for both individual investors and institutional fund managers.

The motivation behind this research lies in the challenges posed by contemporary financial markets and gambling sectors. In today's economic landscape, traditional investment strategies often fall short in terms of consistency and long-term growth. The Kelly Criterion, with its focus on positive expected

^{© 2024} The Authors. This is an open access article distributed under the terms of the Creative Commons Attribution License 4.0 (https://creativecommons.org/licenses/by/4.0/).

net returns in repeated independent bets, presents a promising alternative. This research aims to bridge the gap between theoretical probability models and practical economic applications, highlighting the Kelly Criterion's potential in revolutionizing traditional investment approaches. The study explores the criterion's robustness in optimizing investment strategies and its effectiveness in promoting responsible gambling behaviors, ultimately benefitting both individuals and the wider society.

Moreover, this research offers a comprehensive exploration of the Kelly Criterion's historical background, its evolution, and its current applications in various economic scenarios. By doing so, it contributes significantly to the fields of probability, decision theory, and economics. The expanded introduction aims to provide a detailed understanding of the Kelly Criterion's role in addressing the challenges of modern financial markets and gambling. It examines the criterion's applicability in maximizing long-term capital growth and effectively managing risk, thereby providing new perspectives on its role in economic and financial strategies.

In summary, this study contributes to the academic understanding of the Kelly Criterion and seeks to demonstrate its practical applications in real-world financial scenarios. It is poised to influence economic industries, shaping investment strategies, and fostering responsible gambling practices, thus benefiting individuals and society as a whole.

2. Related Work

The betting and investment strategies field has witnessed extensive research and applications, with the Kelly Criterion being a central focus [1]. It has been applied in various contexts, including finance and sports betting. The Kelly Criterion's mathematical foundation systematically optimizes capital allocation based on expected returns and risk.

Numerous studies have explored the applications of the Kelly Criterion in practical scenarios. It has been used in finance to enhance asset allocation in investment portfolios, often outperforming traditional methods. The criterion has proven effective in determining optimal wager sizes in sports betting. However, the practical implementation may encounter challenges such as accurate return estimations and assumptions of constant parameters over time [2].

3. Methodology

This study delves into applying the Kelly Criterion, specifically optimizing investment strategies while maximizing H, which represents happiness, rather than merely focusing on return. The core problem revolves around maximizing long-term wealth while accounting for the inherent uncertainty associated with financial investments. We aim to formulate a systematic approach that utilizes the Kelly Criterion to address this problem, providing a robust framework for wealth management.

The essence of the Kelly Criterion lies in the idea that it allows us to maximize H, which refers to happiness, rather than simply concentrating on return. To illustrate this point, consider an investment with a 99% probability of gaining 100 times the initial capital and a 1% chance of losing the entire investment. If our goal were to maximize return, we would allocate 100% of our capital to this investment. However, there is a significant risk of losing everything in the long term. Therefore, we choose to maximize the logarithm of return, which is represented as H [3].

Our approach builds upon the foundation principles of the Kelly Criterion, which offers a mathematically grounded framework for optimizing capital allocation while considering the relationship between expected returns and risk [4]. Unlike traditional investment methods, which often lack a systematic approach, the Kelly Criterion emphasizes a balance that minimizes the risk of capital erosion while maximizing long-term growth and, as a result, the logarithm of wealth.

We begin by formulating a mathematical model to implement the Kelly Criterion effectively. Suppose the betting ratio as f. We consider a game with n possible outcomes, each associated with a net return (R_i) and a probability of occurrence (P_i) . The Kelly Criterion is founded upon the mathematical expectation of the increment (logarithmic growth rate) of the principal investment, which can be calculated using the Equation 1:

$$H = \sum_{i=1}^{n} (P_i * \ln(1 + R_i * f))$$
(1)

We further derive the optimal betting ratio (f) by differentiating H with respect to f. The solution for the extreme value of f, which maximizes H, forms the basis of our approach.

To elucidate the framework of our approach, we utilize figures and diagrams. These graphical representations illustrate the relationship between investment ratios, expected returns, and the associated gains, making the principles of the Kelly Criterion more accessible.

Incorporating the Kelly Criterion into investment involves precise mathematical details. It is imperative to grasp the formula's core components, including the probabilities of outcomes, the net returns, and the logarithmic growth rate equation [5]. This paper will provide in-depth mathematical explanations and derivations to ensure a thorough understanding of the proposed approach, which ultimately prioritizes happiness and long-term financial well-being over simplistic return maximization.

4. Discussion

This study's results substantiate the foundation principles of the Kelly Criterion, demonstrating its efficacy in systematically optimizing capital allocation for investment scenarios. The strategy's core strength is maximizing long-term growth while concurrently mitigating excessive risk. This aligns with Phatarfod's exploration of the Kelly Criterion, particularly in its gambling application, where it provides a mathematically grounded approach to wealth management and offers precise, systematic methods for capital allocation [6][7].

The study's identification of an "average luck zone" provides critical insights for individuals seeking responsible investment strategies, underscoring the Kelly Criterion's potential to offer a risk-adjusted approach to investing. This is echoed in the findings of Hung, who emphasized the Kelly Criterion's capacity to consistently outperform alternative methods, showcasing its robustness for sustainable long-term growth [8].

However, the practical application of the Kelly Criterion is challenging. The strategy's effectiveness heavily relies on accurately estimating probabilities and returns. This sensitivity to estimation errors can be a significant impediment, especially in volatile financial markets where predictions are inherently uncertain [9]. Such challenges are further accentuated by the assumption of a static environment, an aspect that may not hold in the dynamic, real-world scenarios of economic markets. This limitation is highlighted by Kong, who discusses the application of the Kelly Criterion in varying economic conditions [8].

Another critical aspect to consider is the exclusion of real-world factors, such as transaction costs and taxes, which can significantly affect the practical application of the strategy. This omission underscores the need for a more comprehensive approach that considers these external variables, as suggested by MacKay and Citron in their analysis of utilizing the Kelly Criterion under capital constraints [10].

The potential of the Kelly Criterion in economic decision-making is undeniable, yet its sensitivity to estimation errors and the assumption of a static environment pose significant limitations. Future research should focus on improving estimation accuracy and adapting the strategy to dynamic economic scenarios. Addressing external factors such as transaction costs and taxes could significantly enhance the practicality and applicability of the Kelly Criterion in real-world situations.

In conclusion, the Kelly Criterion stands as a robust strategy for wealth management and investment decision-making. However, a nuanced understanding of its strengths and weaknesses is essential for responsible and informed decision-making in the realm of investment and finance. This research contributes to this understanding, laying the groundwork for future exploration and adaptation of the Kelly Criterion in diverse economic contexts [7].

5. Conclusion

This study has explored the application of the Kelly Criterion in wealth management and investment. Our findings underscore the effectiveness of this systematic approach, with an optimal investment ratio of approximately 25% of the principal, consistently maximizing long-term wealth accumulation while balancing risk. Additionally, the identification of an "average luck zone" within the 0% to 50% investment range highlights the strategy's ability to provide a balanced risk-return profile.

Our contributions extend to a comprehensive understanding of the Kelly Criterion's principles, practical applications, and insights into addressing estimation errors in dynamic investment scenarios. The significance of this research lies in its promotion of informed and responsible decision-making in wealth management, benefiting individual investors and contributing to economic stability.

Looking ahead, future research can focus on refining return and probability estimations, addressing practical challenges like transaction costs and taxes, and exploring the integration of the Kelly Criterion into automated investment platforms. These avenues hold promise for advancing the utility of this strategy in modern financial decision-making, offering a systematic and mathematically-grounded approach for wealth management and investment.

References

- [1] Thorp, Edward O. "Understanding the Kelly criterion." The Kelly capital growth investment criterion: theory and practice. 2011. 509-523.
- [2] Rotando, Louis M., and Edward O. Thorp. "The Kelly criterion and the stock market." The American Mathematical Monthly 99.10 (1992): 922-931.
- [3] Thorp, Edward O. "The Kelly criterion in blackjack sports betting, and the stock market." Handbook of asset and liability management. North-Holland, 2008. 385-428.
- [4] Baker, Rose D., and Ian G. McHale. "Optimal betting under parameter uncertainty: Improving the Kelly criterion." Decision Analysis 10.3 (2013): 189-199.
- [5] Piotrowski, Edward W., and Malgorzata Schroeder. "Kelly criterion revisited: optimal bets." The European Physical Journal B 57 (2007): 201-203.
- [6] Phatarfod, Ravi. "Some aspects of gambling with the Kelly criterion." Mathematical Scientist 32.1 (2007): 23-31.
- [7] Phatarfod, Ravi. "Further aspects of gambling with the Kelly criterion." Mathematical Scientist 36.1 (2011).
- [8] Hung, Jane. "Betting with the Kelly criterion." Massachusetts Institute of Technology (2010).
- [9] Kong, Xinyi. "Kelly Criterion and its Application." Highlights in Business, Economics and Management 9 (2023): 630-635.
- [10] MacKay, James A., and Gary P. Citron. "Utilizing the Kelly Criterion to Select the Best Projects When Capital is Temporarily Constrained." (2016): 357-362.