

A Critical Comparison of SEUT and Prospect Theory

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Abstract: This article explores two key theories in behavioral finance: Subjective Expected Utility Theory (SEUT) and Prospect Theory, outlining their development as responses to historic economic paradoxes, challenging traditional notions of rationality in economic decision-making. SEUT extends the concepts of expected value and rational choice, incorporating individual utility functions and subjective probabilities. Despite its comprehensive approach, SEUT struggles to explain certain behaviors under risk and uncertainty. Prospect Theory, conceived by Kahneman and Tversky, offers an alternative. It introduces decision weighting, focusing on how individuals differently perceive gains and losses relative to a reference point. This theory explains human biases like loss aversion and framing effects, which SEUT overlooks. Through practical examples and empirical studies, the article demonstrates scenarios where human behavior deviates from SEUT predictions, influenced by psychological factors. However, both theories provide insights into economic decision-making, they have their respective strengths and limitations. The article suggests a more integrated approach, recognizing the complexities of human decision-making in economic contexts.

Keywords: Prospect theory, behavioral finance, financial history

1. Introduction

It is undeniable that Adam Smith is the pioneer, and often called “the founding father”, of political economy, whose book *The Wealth of Nations* marked the birth of modern capitalism. In that book he put forward several theories, which underlies the Neoclassical Economics, including the famous metaphor of an invisible hand representing individual’s behavior of bettering their own conditions can carry on the whole society to wealth and prosperity [1]. In such expression, Adam not only focused on the market participants’ tendency of maximizing their wealth as well as the consequence of such rational behavior, but also pointed out that such behavior was driven by the unseen instincts to strive for order and opulence [2]. The mechanism of the latter was more detailed discussed in his first book *The Theory of Moral Sentiments*, which was treasured as one of the roots of Behavioral Economics.

Based on Adam’s work along with the efforts of other great minds, Neoclassical Economics developed a whole framework of analyzing the economical world using mathematical approaches, however one of the underlying theory which Neoclassical Economics was built on, the hypothesis of rational man, was frequently criticized [3]. The Subjective Expected Utility Theory (henceforth,

SEUT) and Prospect Theory [4] are two millstones brought up respectively to solve the St. Petersburg Paradox [5] and Allais Paradox [6], which were two great challenge to such hypothesis. The two theory hare the similarity in attempting to explain the decision-making process of individuals under risk or uncertainty, however they have divergent views on the hypothesis mentioned above [7].

2. From the St. Petersburg Paradox to Von Neumann-Morgenstern Utility Function

The pre-SEUT economists generally agree that factors determine choices under risk and uncertainty is the expected value of different state of nature, which is computed by summing all possible gains respectively multiplied to the probability of their occurrence, and a rational man should choose the offer with higher expected value as expected by Bernoulli. As is shown in the equation below the $E[X]$ stands for the expected value, x_i and p_i stand respectively for the value and possibility of i th circumstance.

$$E[X] = \sum(x_i * p_i) = x_1 * p_1 + x_2 * p_2 + \dots + x_k * p_k \quad (1)$$

However, such theory of simply comparing the expected value is confronted with a great challenge call St. Petersburg paradox. This is a coin-tossing game between A and B, and B commits to pay A 1 ducat if it is head at the first throw, 2 ducats if it is head at the second throw, and 2^{n-1} ducats if it is head at the n th throw. And A needs to decide the amount of money he is willing to buy the ticket to start the game. Since it is a game with an infinite sum, as is shown in the 2nd equation, the expected value illustrates that the rational choice of A is to willingly pay B any amount of money, which is strongly contradict to the reality.

$$E[X] = 1/2 * 1 + 1/4 * 2 + \dots + \dots = \infty \quad (2)$$

Therefore, Bernoulli attributes people's choice under risk and uncertainty to the moral values yield by the monetary gains rather than the gains itself and supposed the satisfaction to be the square root of its mathematical quantity, concluding that the price A would like to pay is converging to a finite number, as is shown in the 3rd equation:

$$E[\sqrt{X}] = 1/2 * \sqrt{1} + 1/4 * \sqrt{2} + \dots + \dots = 1/(2-\sqrt{2}) \quad (3)$$

The very method was enhanced by Von Neumann and Morgenstern [8] and summarized to be the equation bellow, where $U(X)$ is the utility function of the monetary gain X .

$$E[U(X)] = \sum p_i * U(x_i) = U(x_1) * p_1 + U(x_2) * p_2 + \dots + U(x_k) * p_k \quad (4)$$

Moreover, the expected utility function mention above is subject to several axiomatic constraints, which define a rational decision maker and guarantee their choice under uncertainty is in accordance with the prediction of Von Neumann-Morgenstern (henceforth VNM) utility function, including the Completeness, Transitivity, Independence of Irrelevant Alternatives and Continuity [8].

- Completeness assumes an individual has a clear preference between two alternatives.
- Transitivity assumes the decision made according to the 1st axiom is also consistent.
- Independence of Irrelevant Alternatives assumes the decision between two alternatives do not depend on other alternatives.
- Continuity assumes if an individual prefers A to B and B to C, he should be indifferent between B and the linear combination of A and C.

3. Subjective utility and Subjective Expected Utility Theory

Beside the Bernoulli-VNM tradition of decision theory, SEUT has another root from the tradition of subjective probability which was treated as both a mathematical and psychological problem [9].

What is treated as default by the pre-SEUT economists is that the probability of the occurrence of different alternatives is an objective feature of the physical world which does not depend on human beliefs, just like the probability of tossing a fair coin [10]. Such objective probability should be known to all [11] and people's belief is described by Bayes' Law [12]. Just like what Keynes [13] believed that probability is not subjective but the logical relation between pairs of propositions and in some sense perceived. However, this was questioned by Ramsey [14] who argued that probability is the degree of belief of a particular individual, so different people may have different subjective probability towards the same event, and this could be measured by the lowest odds the individual would accept in a proposed bet.

Savage's [15] Subjective Expected Utility Theory, bridged the two traditions by adopting VNM utility function in a subjective probability framework, arguing individuals' choice is according to the expectation of their utility values, where the probability and the utility function vary among different people.

4. Allais Paradox and Prospect Theory

Many years after Savage's work, Machina [16] pointed out that SEUT might lead to several anomalies, including Allais paradox which was investigated by Kahneman and Tversky in their Prospect Theory.

The Allais paradox describes a situation as in the following table, where participants are asked with 2 questions, each containing 2 prospects.

Table 1: The Allais Paradox

Question 1			
Prospect A		Prospect B	
Value	Probability	Value	Probability
1 Million	100%	1 Million	89%
		0	1%
		5 Million	10%
Question 2			
Prospect A		Prospect B	
Value	Probability	Value	Probability
0	89%	0	90%
1 Million	11%	5 Million	10%

And Allais found that the majority answers chose prospect A in the first question while prospect B in the second one, which violates SEUT because there is no function U could satisfy both:

$$U(100) > 0.1 * U(500) + 0.89 * U(100) + 0.01 * U(0) \quad (5)$$

which equals to (0 value generates 0 unit of utility)

$$0.11 * U(100) > 0.1 * U(500) \quad (6)$$

and

$$0.11*U(100)+0.89**U(0)<0.1*U(500)+0.9*U(0) \quad (7)$$

which equals to

$$0.11*U(100)<0.1*U(500) \quad (8)$$

Just like the St. Petersburg paradox, Allais paradox also revealed that SEUT was not adequate enough to verify the rationality of individual's decision under risk, and Kahneman and Tversky solved such anomaly by replacing the probability in SEUT with a hypothetical decision weighting function towards different probability which is nonlinear and discontinuous around 0% and 100%. As is illustrated in the following figure, people are believed to have a preference for certainty, namely having a tendency to overweight outcomes that could be gained for sure compared with those are merely probable.

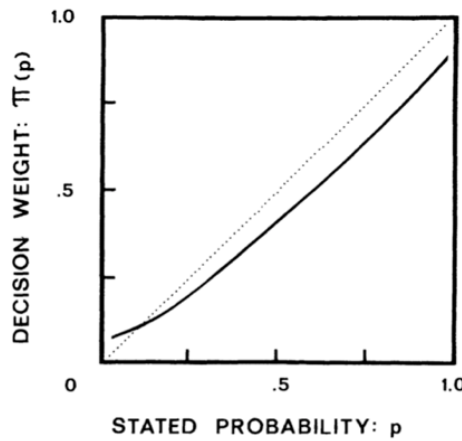


Figure 1: A hypothetical weighting function

Furthermore, Prospect Theory defines utility function separately over gains and losses, which are respectively assumed to be concave and convex. Such nonlinearity and asymmetry arising from people's heuristic of certainty preference, could better describe the different risk attitude towards different probability.

There are other essential features of the Prospect Theory: (i) the losses and gains are not necessarily the absolute value an individual is losing or winning, but rather a relative loss or gain compared to the Reference Point which is the expectation in their minds; (ii) the nonlinear and asymmetric utility function implies that losses loom larger than gains; (iii) such effect is attributed to the human heuristic of loss aversion. And the following figure could give us an more direct impression:

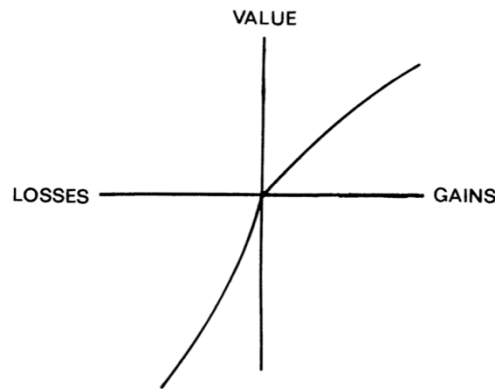


Figure 2: A hypothetical value function

5. Examining SEUT and Prospect Theory with Real-life Cases

The Prospect Theory argues that loss aversion would lead to human behavior deviating from what SEUT predicts, causing excessive risk aversion, Endowment effect, Disposition effect, Framing effect and etc., which we will discuss in the following studies.

One of most distinct examples illustrating the divergence between SEUT and Prospect Theory is the excessive risk aversion, stemming from loss aversion, which drives people to reject a lottery of winning \$11 with 50% probability and losing \$10 with 50%, however a rational individual should accept such lottery since it's subjective expected utility is positive. Benartzi and Thaler [17] investigated the so-called equity premium puzzle referring to the fact that the historical difference between the average return of equities and risk-free assets are believed to be too large. And such a high return, which means high potential subjective utility, should attract more investors building portfolios to arbitrage and hence reducing itself. They believed such phenomenon is also attributed to the excessive risk aversion generating from people's loss aversion. Similarly, the phenomenon of high saving ratio and low level of wealth diversification in China is also deemed to be caused by people's loss aversion [18].

Besides, the experimental test of the Endowment effect [19] also serves as a great example of loss aversion, where the measure of willingness to accept greatly exceed the measure of willingness to pay, because even if different people may have different preference, the same consumption objects (e.g. coffee mugs) should generate similar utility for two groups of randomly selected people in general. A similar effect, Disposition Effect, also reveals that people's behavior in real market is not as what SEUT predict. Genesove and Mayer [20] pointed out that condominium sellers in Boston avoid realizing nominal losses which is consistent with the result of Odean's [21] finding in the stock market.

Additionally, Camerer et al. [22] studied the labor supply of New York City cabdrivers and found out there is not a significant positive relationship between hours supplied and transitory changes in wages, which should be the result if individuals rationally maximizing their utility, namely working more during high-wage period and having more leisure in contrary situation. They argued that it was because they set a loose daily income target, which is the reference point to compare their losses or gains with, and once achieving such goal they will have leisure instead. Moreover, Hossain and List [23] examine such heuristic from another angle with a natural field experiment in a Chinese manufacturing facility – Malata. By merely change the expression of how the workers' bonuses will be, which is also known as the Framing Effect, they witness a 1% more productivity from the group whose bonus is posed as loss.

6. Conclusion

The SEUT and Prospect Theory both trying to explain people's decision-making process under risk and uncertainty, however, they are based on different assumptions of individual's rationality. They both adopt the VNM utility function and take the influence of individuals' preference into consideration, however, the former one needs more axioms to protect itself which makes SEUT not adequate enough in real-life cases. Still, we can regard SEUT as a salient and primitive model which is a first order approximation of the reality. Similarly, Prospect theory can be treated as a higher order amendment to SEUT, because on one hand, its focus on the asymmetric changes around the reference point can better explain the irrationality behind people's choice and on the other hand, it is still stemmed from the linear VNM utility function.

Furthermore, both theories are appealingly simple compared to the latest development, for example the subjectively weighted utility (SWU) [24], took people's information processing performance into consideration, and as for the original version of Prospect Theory, later research enhanced it with the study of people's overconfidence [25], probability misjudgment [26], self-control problem [27], social preference [28], and etc.,

Lastly, comparing the two theories may not be like a horse race, picking the theory which could explain the majority of the data and discard the other, because there might be several latent decision-making process jointly generate the choice we make and human might not be rational enough to inspect every prospect before making a decision.

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