Proceedings of the 2nd International Conference on Management Research and Economic Development
DOI: 10.54254/2754-1169/84/20240805

Comparing Different Models in Forecasting the Series SP500

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Abstract: The drastic fluctuations in SP500 caused by the epidemic have become one of the topics of concern in the academic field. When analyzing and forecasting time series, many classic models of forecasting were widely used, including the exponential smoothing model, ETS, and ARIMA model. This article focuses on testing the effectiveness of these models on the specific time series SP500, using the data from the year 2010-2021 to forecast the possible results in 2022-2023 and afterward test its accuracy. According to the results, none of the three models capture the tendency in the years 2022 and 2023 well. This may be due to the abnormal plunge in 2022, which was never been observed before. At last, some possible accountabilities, including the influence of COVID-19, are provided to explain such a phenomenon. This study expands the application scope of the ARIMA model and has certain practical significance for the development of the stock market.

Keywords: SP500, Forecast, COVID-19, ETS, ARIMA

1. Introduction

1.1. Research Background and Motivation.

Through the long history of time series analysis, forecasting has often been seen as its objective, playing an irreplaceable role when applied in fields such as economics and finance [1]. In 2016, Mizumoto and Plonsky stated that one of the most pronounced advantages of R project is its reproducibility of data analysis [2]. Hence, utilizing R as the main carrier for its high integrating degree, the article attempts to gain analytical insights on one of the most well-known stocks in America to evaluate the effectiveness of forecasting by the three methods.

1.2. Literature Review

The literature cited includes wide-ranging essays that provide theoretical assistance or significant summaries for the dissertation. For instance, the essay by Makridakis et.al stated the basic definition of the exponential smoothing method, and the one from Dhamo and Puka introduced the deduction of it [3,4]. Both clarify the conception of the exponential smoothing method and guide the experiment on the exponential smoothing method. There are also some articles cited that present the meaningness of certain objects. Take the essay written by Pack as an example: Pack believed that forecasting is a product that relies on theory and empiricism [5]. This helps the conclusion since the statement corroborates the inevitability of the happening of the low-possibility events.

In conclusion, the research on the methods so far provides affluent theoretical contents and relevant experimental results, which largely benefit the comparing process.

1.3. Research Contents and Framework

The whole research focuses on testing the exponential smoothing model, the ETS model, and the ARIMA model in SP500 closing price series, with the conclusion and relevant explanation made afterward. During the analysis, related tables and images that show the experimental results are given, and some comparison is made which is mostly about the difference between the forecasting data and the actual one.

The article contains five parts. The introduction part gives basic background and literature information about the essay. The methodologies section introduces the historical and theoretical contents of the three models and how the data is processed. The experiment procedures are presented in the empirical results part, and the conclusion and thoughts are summarized in the discussion and conclusion part.

2. Methodologies

2.1. Data Selection and Processing

The series chosen in the essay is SP500, which is a typical index in the American financial market and to some extent reflects the tendency of its stock market. Instead of using all the data of SP500, the essay uses the past thirteen years (2010-2023) closing price at the end of every month to form a series, which allows the series to have a wide range while not losing

Notably, the command chooses frequency equals 12 because the Holt-Winter method requires an examination in seasonality, and hence the time series needs to be represented monthly. Here, the subset of data from January 2010 to December 2021 is defined to act as the part for forecasting while the one from the last two years is also set to serve as testing statistics.

Figure 1 illustrates how the closing price of the SP500 changes during 2021 and 2023. Overall, it can be observed that, despite some fluctuations, the index is increasing progressively.

Figure 1: The closing price of SP500 from January 2010 to December 2023

2.2. Model Building

2.2.1. Exponential Smoothing Model

The exponential smoothing model is an approach to forecasting that unceasingly increases its forecasting performance by weighting the previous values in a time series in an exponential manner [3]. After recognizing some flaws in the simple exponential smoothing model, Holt modified the simple exponential smoothing model by adding linear trends. This Holt model was again refined by Holt himself and his student Winters to consider seasonal factors. Hence, originally put forward in around the 1950s, these methods experienced modifications and extensions by various scientists and eventually evolved into 15 sub-methods [4]. Table 1 states all 15 exponential smoothing methods by pairing different trend components with seasonal components.

Table 1: The fifteen exponential smoothing methods

Trend Component	Seasonal Component		
	N (No seasonality)	A (Additive)	M (Multiplicative)
N (No trend)	N, N	N, A	N, M
A (Additive)	A, N	A, A	A, M
A_d (Additive damped)	A_d, N	A_d, A	A_d, M
M (Multiplicative)	M, N	M, A	M, M
M_d (Multiplicative damped)	M_d, N	M_d, A	M_d, M

2.2.2. ETS Model

The second model utilized is the ETS model. According to Lingzhi Qi et. al, a family of exponential smoothing models is summarized into a systematic theory about automatic forecasting through the efforts of many researchers, which is now called the ETS method (which stands for Exponential Smoothing, or Error, Trend, Seasonality) [6]. Compared with the simple exponential smoothing methods, the ETS model allows the past forecast error to play a role in the future prediction of values, meaning that the model tends to have some self-correcting character.

2.2.3. ARIMA Model

The third model, Autoregressive Integrated Moving Average model, also well-known as the ARIMA model, combines the former ARMA model with differencing. For the series that does not have strong seasonality, a suitable model ARIMA (p, d, q) can be considered, where "p" stands for autoregressive part order, "d" for the degree of first differencing involved and "q" for moving average order[4]. In 1970, Box and Jenkins proposed the seasonal ARIMA model [7]. The formula was given:

$$\Phi_p(B^s)\phi(B)\nabla_s^D\nabla^d X_t = \alpha + \Theta_Q(B^s)\theta(B)w_t \quad (1)$$

where$s$ represents seasonal lag, ϕ represents the coefficient for autoregressive process,Φ represents the seasonal autoregressive process coefficient, θ refers to the coefficient for moving average process, Θ refers to the coefficient for seasonal moving average process, B refers to the backward shift operator, and w_t is an uncorrelated random variable with mean zero and constant variance.

3. Empirical Results:

3.1. Exponential Smoothing Method

Table 2 states the smoothing parameters obtained through R.

Table 2: Smoothing parameters of the exponential smoothing method

Smoothing parameters	
$alpha$	0.8364
$beta$	0.0000
$gamma$	1.0000

At the same time, it will be straightforward to generate a graph comparing the original time series and the one after processing, the results of Holt-Winters exponential smoothing model forecasting are show in figure 2.

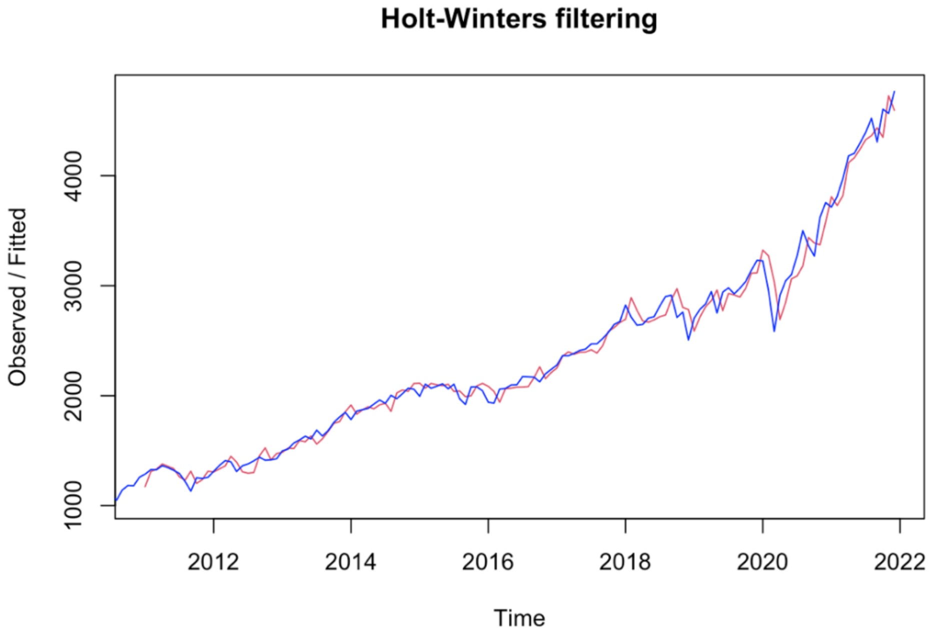

Figure 2: Holt-Winters exponential smoothing model forecasting
Note: blue line = the smoothed value, red line = the original value

Since the forecasting model is already well-established, it is time to test the accuracy of the method using the last two years' data. Based on previous observation results, this article derives prediction data for 2022 and 2023 based on predictive models. Afterwards, the residuals series was defined by taking the difference between the forecasted and factual data from Jan.2022 to Dec.2023. Since the residuals series is constructed, corresponding commends are applied to obtain the p-value of the Ljung-Box test.

Table 3: Ljung-Box test and relative information of established residuals of the exponential smoothing method

Test	Distribution	Statistic	p-value
Ljung-Box Q	$Q \sim chisq(20)$	40.9	0.0038

The p-value of the Ljung-box test is less than the significance level(take 0.05 in all three models). Hence, the null hypothesis is rejected, and the residuals cannot be considered white noise (see Table 3). According to Imoh Udo Moffat et.al, the white noise series displays an unchanged variance and the mean equal to zero, which guarantees that the residuals are randomly distributed and are not autocorrelated [8]. This non-white-noise character implies that the accuracy of the model is not high enough or the series in the last two years behaved anomalously.

Figure 3 illustrates the deviation of exponential methods from the actual data. One can easily tell that the deviation is quite apparent, exceeding 1000 dollars most of the time. Therefore, a conclusion can be drawn that the exponential smoothing method does not forecast the ongoing trend well.

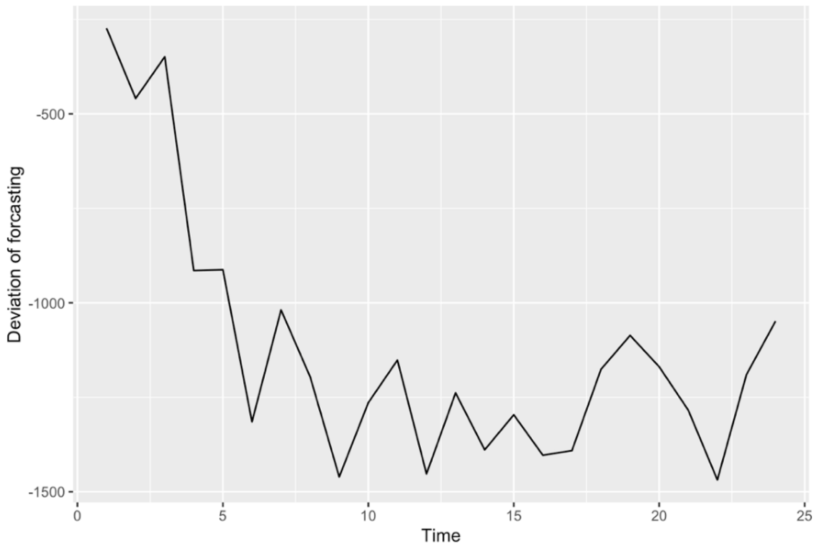

Figure 3: The series of errors between the simulation from the established model and the actual data

3.2. ETS Method

Now the data is examined by the ETS method. R is able to produce smoothing parameters, initial values, error analysis, and forecasting data for the next two years. It recommends the ETS (M, A, N) model, which stands for multiplicative errors, additive trends, and no seasonality (see Table 4).

Table 4: The parameters of ETS(M, A, N) model

Smoothing parameters	$alpha$	0.8199
	$beta$	1e-04
	$gamma$	0.0395

At the same time, it provides relevant information such as parameters, error measures, point forecasting, and forecasting intervals (see Table 5).

Table 5: The seven measures of error generated by ETS (M, A, N) model

ME	8.1141
$RMSE$	98.7012
MAE	67.9130
MPE	-0.0183
$MAPE$	2.9681
$MASE$	0.2256
$ACF1$	0.0586

Again, the following steps construct a series and test the autocorrelation of the errors. Firstly, a time series is formed by taking the point forecast of the ETS(M, A, N) model. The residual series then is defined to be the difference between the forecasting series and the series of actual data. Such a series is experimented to examine whether the residuals are white noise or not.

Table 6: Ljung-Box test and relative information of established residuals of the ETS(M, A, N) model

Test	Distribution	Statistic	p-value
Ljung-Box Q	$Q \sim chisq(20)$	39.24	0.0062

In observation, the p-value of the Ljung-Box test does not exceed 5%, confirming that the residuals do not perform white-noise character and hence have autocorrelations. As similar to the exponential smoothing model, the corresponding bias results for the ETS method are given in Table 6.

Figure 4 compares the predicted values with the actual ones. Here, since there is no seasonality observed in the previous eleven years, the ETS model suggests linear forecasting with a gently growing tendency. However, the actual data, with fluctuation, faces a decrease at the beginning of 2022 and a refluxing afterward.

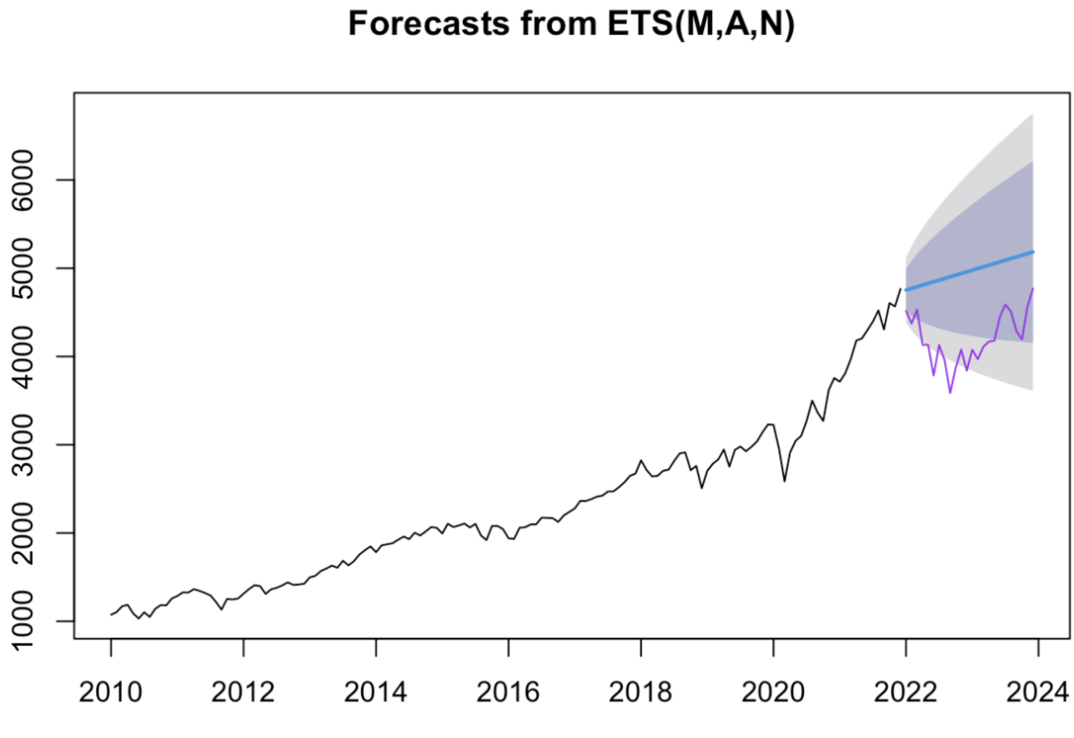

Figure 4: Forecasting the data from January 2022 to December 2023 using the ETS model
Note: blue line = forecast data by ETS, purple line = real data

3.3. ARIMA Model

Similar to the command summary () utilized in the ETS model, the package "forecast" provides a useful command auto.arima()to automatically find the most suitable ARIMA model and provides coefficients. The commend recommends ARIMA (1, 2, 2) with the following coefficients (see Table 7).

DOI: 10.54254/2754-1169/84/20240805

Table 7: Coefficients of ARIMA (1, 2, 2) model

	AR (1)	MA (1)	MA (2)
	0.5719	-1.7223	0.7542
s.e.	0.1860	0.1571	0.1603

Here, the intervals of the forecasting data are also given. According to Chatfield, such intervals of forecasting, which often contain an upper and a lower limit, capture the range in which the actual value of future data is expected to be [9]. The range was presented by the relatively dark and more light-colored area in Figure 5. The possibility of future data lying in this area is 80% in the dark area and 95% in the light one.

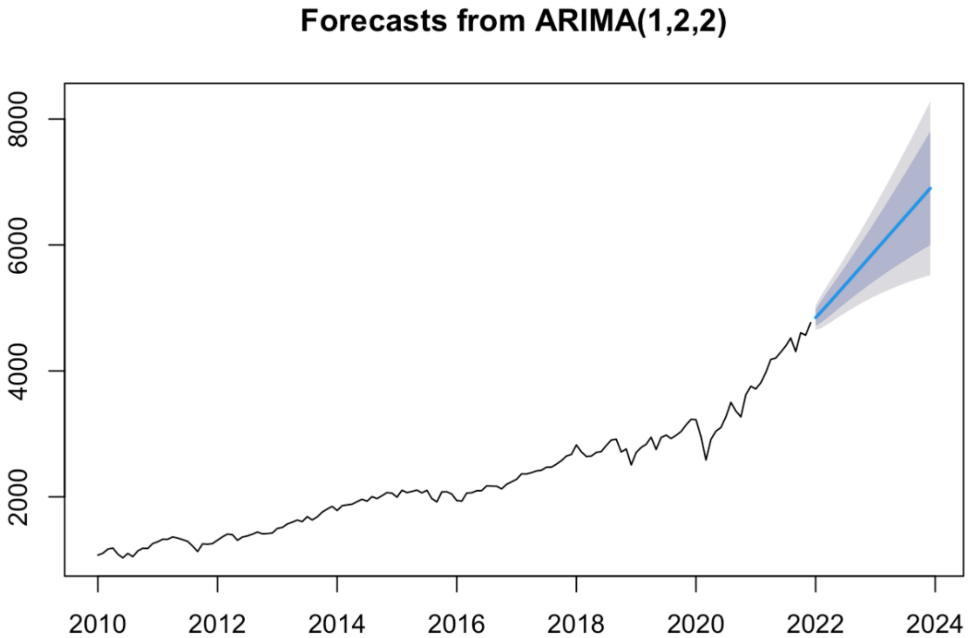

Figure 5: Using the ARIMA model to forecast the data

Furthermore, this article will analyze the predicted values for two years and test the residuals of the prediction model. Table 8 presents the results of Ljung-Box test.

Table 8: Ljung-Box test and relative information of established residuals of the ARIMA (1, 2, 2) model

Test	Distribution	Statistic	p-value
Ljung-Box Q	$Q \sim chisq(20)$	109.34	0*

Note: 0* represents that the figure is extremely close to zero.

The p-value does not reach 5%, leading to a rejection of the white-noise character of the series. Hence, the deviation of the prediction through the ARIMA model is also apparent.

Figure 6 shows how the error goes between Jan.2022 and Dec.2023. The deviation is exaggerated during the two years, from around 500 dollars at the beginning to about 2500 at the end of 2023. Notably, the closing price in Dec.2023 was only 4800 roughly, implying that the deviation is over 50% of its current value.

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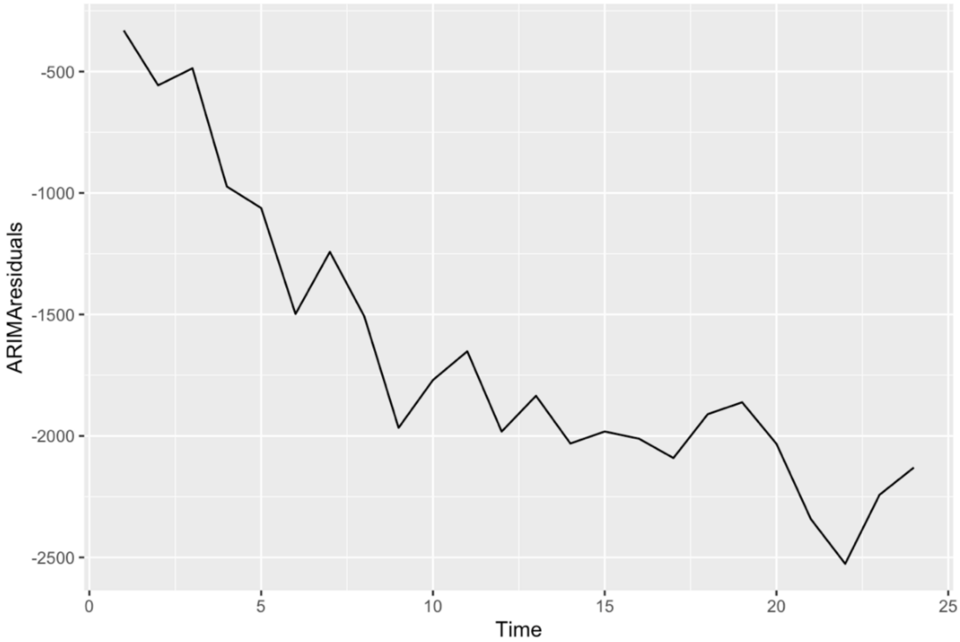

Figure 6: The errors generated by ARIMA(1,2,2) model

Figure 7 visualizes the difference between the two series. One can see that there is no forecasting data point that is in the shadowed area, i.e., the possibility of such an event is nearly zero, indicating the strong ineffectiveness of the ARIMA model.

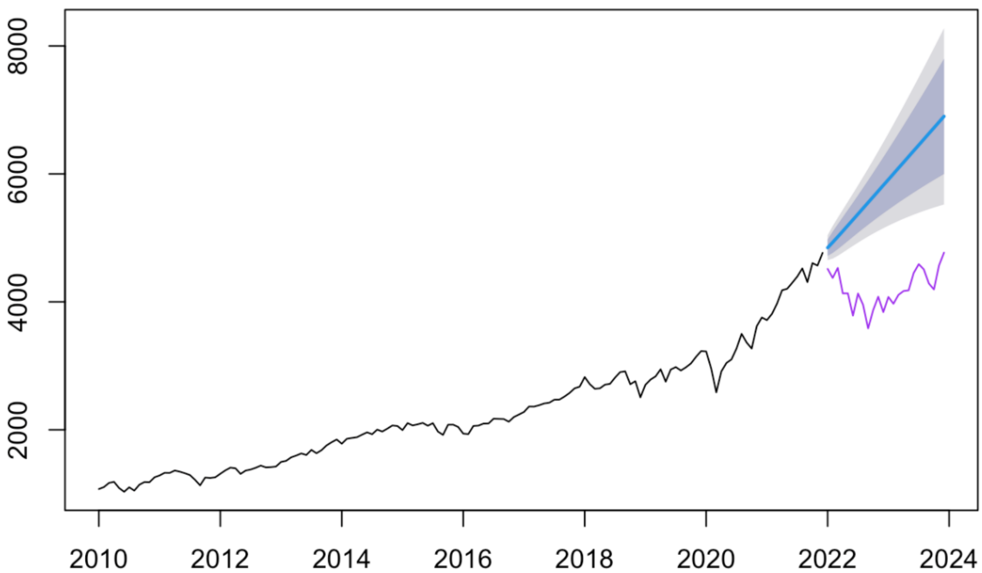

Figure 7: Forecasting the data from January 2022 to December 2023 using ARIMA model
Note: blue line = forecast data by ARIMA, purple line = real data

4. Discussion

It may seem strange to conclude that none of the three methods above provides a precise forecast of the SP500 closing price between the years 2022 and 2023. This abnormal phenomenon may be attributed to the outbreak of COVID-19. According to Scott R. Baker et.al, an unprecedented boost in the volatility of the stock market in the United States was witnessed after the start of the pandemic [10]. This may be a critical factor illustrating the downward trend of the SPX500 index during the year 2022. Therefore, since the volatility of the stock market sharply increased, it is justifiable that the previous forecasting does not match the real data nicely. In the case that the volatility did not change, the three models tend to predict more accurately.

It is helpful to mention the empiricism in the time series forecasting. Pack stated that the forecasting itself is a mixture of both theory and empiricism[5]. Both sides are unalienable and play a role during the forecasting process. The drop of SP500 in 2022 itself is unexpected and is beyond the scope of previous empiricism which expects it to rise with fluctuation. Hence, all the models applied failed due to this systematic reason.

5. Conclusion

So far, the empirical results of the essay suggest that none of the three models is suitable to simulate the SP500 closing price series from 2010 to 2023 because the severe inaccuracy. However, this does not mean that those three models are questionable: The data between 2020-2021 chosen to capture the future tendency does not incorporate an unprecedented typical dive witnessed only in 2022 and hence cannot fully be seen as an example that forebodes the future trend. This, however, is not to be blamed since the data itself is random. One cannot fully guarantee what the future will be, and it is sometimes the case that a significant difference is witnessed from the hypothesis.

Nonetheless, there still exist two main limits of the research. Firstly, the research lacks universality since it only analyzed one stock of the US. Though SP500 is one of the most well-known stocks, it cannot fully represent the whole US financial market. Besides, the article only analyzed the closing price of SP500, resulting in the analysis not thoroughly inspecting SP500. Secondly, the research only examined the data without providing any new models or assistance to resolve the problem of not forecasting accurately, leading to paucity in the renovating significance of the essay.

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