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# Critical Assessment of Markowitz and Index Models for Portfolio Optimization

**Xibin Jing**[1,a,*]

[1]*University of Nottingham Ningbo China, No. 199 Taikang East Road, Yinzhou District, Ningbo, China*
*a. smyxj4@nottingham.ac.uk*
**corresponding author*

***Abstract:*** This paper conducts a comparative analysis of the Markowitz Model (MM) and the Index Model (IM) for portfolio optimization, taking into account the realistic constraints of financial regulations and investor preferences. The analysis is grounded in 20 years of historical data from ten stocks across various sectors, coupled with the S&P 500 index and a proxy for the risk-free rate. By aggregating daily returns into monthly observations, we calculate inputs for both models, aiming to discern their effectiveness in navigating the complexities of market behavior and investment strategy formulation. This approach not only highlights the theoretical underpinnings of each model but also examines their practical applicability and performance in real-world settings, thereby offering insights into their respective strengths and limitations in achieving optimized investment outcomes.

***Keywords:*** Model Descriptions, Data Methodology, Comparative Analysis, Markowitz Model, Portfolio Management

## 1. Introduction

In the dynamic landscape of finance, portfolio optimization has emerged as a pivotal task for investors and asset managers, driven by advancements in society and technology. Harry Markowitz, in his groundbreaking work "Portfolio Selection" [1], introduced Modern Portfolio Theory (MPT). MPT underscores the significance of minimizing variance through the selection of assets whose returns are not correlated, aiming to strike an optimal balance between risk and expected return [2]. Markowitz's seminal research laid the foundation for contemporary portfolio theory. William Sharpe expanded the theory of diversification with the introduction of the Capital Asset Pricing Model (CAPM) in his 1964 paper, "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk." CAPM dissects the relationship between the return of individual stocks and a broad market index like the S&P 500, positing that a stock's return is influenced by both market-wide (systematic) risk and specific (unsystematic) risk inherent to the stock [3]. Through diversification across stocks with varying degrees of systematic risk, investors can better manage portfolio volatility and refine their risk management practices.

This paper aims to conduct a comprehensive analysis and comparison between the Markowitz Model and the Index Model. Leveraging historical daily total return data for both the S&P 500 index and ten selected U.S. stocks, this study seeks to calculate the necessary optimization inputs for both models. Furthermore, it aims to delineate the permissible regions of portfolios under five different sets of additional constraints. Through this comparative approach, the paper aims to shed light on the efficacy and practical applicability of these two prominent portfolio optimization models. Section 2 introduces the background of the companies and their stock prices, while Section 3 provides an overview of the Markowitz Model and the Index Model. Section 4 discusses the collected data, compares the MM and IM. This project aims to accomplish several goals: gaining insight into contemporary portfolio theory, with a specific focus on the Markowitz Model (MM) and the Index Model (IM), constructing the optimum portfolio based on historical stock prices, and comparing the benefits and drawbacks between these two models in order to conduct a comprehensive analysis. Through this approach, the aim is to reduce investment risk and maximize returns effectively.

## 2. Background of Selected Companies

For the upcoming analysis, a curated selection of stocks has been chosen, including major corporations such as Adobe Inc., IBM, SAP SE, Bank of America, Citigroup Inc., Wells Fargo & Company, The Travelers Companies, Inc., Southwest Airlines Co., Alaska Air Group, Inc., and Hawaiian Holdings, Inc., along with the S&P 500 index for a comprehensive market overview. This diverse lineup will be utilized to conduct a detailed historical data analysis, employing both the Index Model (IM) and the Markowitz Model (MM) to explore investment strategies and risk management techniques.

## 3. Portfolio Optimization Models and Data Analysis Tools

### 3.1. Markowitz

The Markowitz model, also known as Modern Portfolio Theory (MPT), serves as a mathematical framework for optimizing investment portfolios. Introduced by Harry Markowitz in 1952, it emphasizes diversification to mitigate risk by spreading investments across assets with varying risk levels and returns. Utilizing statistical analysis, the model determines the ideal asset allocation considering factors such as expected returns, risks, and asset correlations. Its objective is to maximize portfolio return for a given risk level or minimize risk for a targeted return. Markowitz's model is pivotal in modern finance, widely utilized by investors, analysts, and portfolio managers.

### 3.2. Markowitz Model

The Markowitz model, devised by Harry Markowitz in 1952, is commonly referred to as the mean-variance model. This approach to portfolio optimization seeks to construct the most efficient portfolio in terms of return relative to risk. It achieves this by evaluating different combinations of assets based on their expected returns (mean) and variability (variance), ultimately aiming to strike a balance between maximizing returns and minimizing risk. Following are four relevant formulas.

$$\min\sigma^2\left(r_p\right) = \sum\sum x_i x_j \mathrm{cov}(r_i, r_j) \tag{1}$$

The formula minimizes a portfolio's risk by optimizing asset weights and considering asset return correlations.

$$r_p = \sum x_i \overline{r_i} \tag{2}$$

The expected portfolio return, calculated by summing the products of asset weights and their expected returns.

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$$r_{it} - r_f = \alpha_i - \beta_i(r_{mt} - r_f) + \epsilon_i \quad (3)$$

The Capital Asset Pricing Model, linking an asset's expected excess return to its risk compared to the market.

$$r_i = E(\overline{r_i}) + m_i + e_i \quad (4)$$

A breakdown of an asset's return into expected return, systematic risk contribution, and random error.

### 3.3. Five Cases of the Additional Constraints

The first additional optimization is structured to emulate the Regulation T by FINRA [4], which permits broker-dealers to offer their clients the ability to hold positions, at least 50% of which are financed through the equity of the customer's account:

$$\sum_1^{11} |w_i| \leq 2 \quad (5)$$

The second optimization constraint aims to replicate certain arbitrary "box" constraints on weights, as specified by the client:

$$|w_i| \leq 1, \text{for } \forall i \quad (6)$$

Third, an unconstrained scenario demonstrates the appearance of the allowable portfolio space and, specifically, the efficient frontier in situations devoid of any limitations.

The fourth optimization constraint models common restrictions found in the U.S. mutual fund sector: a U.S. open-ended mutual fund is prohibited from holding any short positions [5].

$$w_i \geq 0, \text{for } \forall i \quad (7)$$

Finally, we aim to determine whether incorporating a broad index into our portfolio yields a beneficial or detrimental outcome.

$$w_i = 0 \quad (8)$$

## 4. Data Analysis and Comparison

### 4.1. Descriptive statistics and correlation analysis

We first use EXCEL software to calculate the corresponding Annual Average Return, Annual Standard Deviation, Beta, Alpha, and Residual Standard Deviation, respectively, of each stock. And then showing the descriptive statistical results of relevant parameters in Table 1. It can be seen that the average return of ALK stock is the highest with an average of 17.4%, while the average return of C stock is the lowest with an average of 1.0%. The larger the standard deviation of HA's return is 62.1%, indicating that the stock has the largest investment risk, while the smaller the standard deviation of SPX's return is 14.9%, indicating that the stock has the smallest investment risk. For the other parameters, the Beta value of HA stock is the largest (1.63), while that of SPX stock is the

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standard (1.00). The Alpha value of HA stock is the largest (0.15), while that of C is the smallest (-0.14). In addition, the Residual Standard Deviation value of HA is the largest (57.2%), the Residual Standard Deviation value of SPX was the smallest (0.0%). These parameters themselves nevertheless shed light on the weight composition, as we shall see that Alpha plays a paramount role in predicting maximum return portfolios.

The correlation coefficient, a statistical metric, mirrors the correlation level between stocks, influencing the variance within stock portfolios and thus their risk diversification potential. Lower correlation coefficients between two stocks result in decreased portfolio variance or standard deviation, indicating enhanced investment diversification benefits, and the reverse for higher coefficients. Additionally, the correlation coefficient's sign provides insight into the nature of the relationship between stocks; a coefficient above 0 suggests a positive correlation, while one below 0 indicates a negative correlation. Table 2's statistical data shows that the selected stocks generally have positive correlations. Notably, the correlation between SPX and C is high at 70.2%, indicating weaker diversification benefits. In contrast, a much lower correlation of 18.0% between HA and ADBE suggests stronger diversification advantages. Furthermore, the moderate correlation of 46.4% between ALK and SPX offers a reasonable diversification prospect. This correlation data is crucial for investors to evaluate the diversification potential of different stock combinations in a portfolio.

Table 1: Summary statistics of Stock Parameters

| | SPX | ADBE | IBM | SAP | BAC | C | WFC | TRV | LUV | ALK | HA |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Annual Average Return | 7.5% | 19.6% | 4.8% | 12.0% | 11.1% | 1.0% | 8.9% | 9.1% | 9.8% | 17.4% | 26.9% |
| Annual StDev | 14.9% | 31.8% | 23.2% | 33.9% | 39.3% | 42.5% | 28.1% | 20.0% | 31.8% | 37.7% | 62.1% |
| beta | 1.00 | 1.42 | 1.01 | 1.48 | 1.60 | 2.01 | 1.05 | 0.80 | 1.15 | 1.18 | 1.63 |
| alpha | 0.00 | 0.09 | -0.03 | 0.01 | -0.01 | -0.14 | 0.01 | 0.03 | 0.01 | 0.09 | 0.15 |
| residual Stdev | 0.0% | 23.8% | 17.6% | 25.8% | 31.4% | 30.3% | 23.4% | 16.0% | 26.8% | 33.4% | 57.2% |

Table 2: Correlation matrix

| | SPX | ADBE | IBM | SAP | BAC | C | WFC | TRV | LUV | ALK | HA |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **SPX** | 100.0% | 66.5% | 64.9% | 64.9% | 60.2% | 70.2% | 55.5% | 59.8% | 53.7% | 46.4% | 39.0% |
| **ADBE** | 66.5% | 100.0% | 45.5% | 53.4% | 42.3% | 46.3% | 29.8% | 45.2% | 38.8% | 23.3% | 18.0% |
| **IBM** | 64.9% | 45.5% | 100.0% | 58.5% | 31.3% | 42.0% | 26.7% | 38.2% | 34.7% | 35.7% | 24.6% |
| **SAP** | 64.9% | 53.4% | 58.5% | 100.0% | 33.1% | 43.4% | 29.8% | 37.5% | 31.8% | 28.2% | 14.4% |
| **BAC** | 60.2% | 42.3% | 31.3% | 33.1% | 100.0% | 82.6% | 76.1% | 39.3% | 42.8% | 27.5% | 33.8% |
| **C** | 70.2% | 46.3% | 42.0% | 43.4% | 82.6% | 100.0% | 70.3% | 51.2% | 42.8% | 30.4% | 34.3% |
| **WFC** | 55.5% | 29.8% | 26.7% | 29.8% | 76.1% | 70.3% | 100.0% | 34.5% | 40.6% | 34.7% | 35.8% |
| **TRV** | 59.8% | 45.2% | 38.2% | 37.5% | 39.3% | 51.2% | 34.5% | 100.0% | 40.7% | 36.0% | 24.0% |
| **LUV** | 53.7% | 38.8% | 34.7% | 31.8% | 42.8% | 42.8% | 40.6% | 40.7% | 100.0% | 51.9% | 42.2% |
| **ALK** | 46.4% | 23.3% | 35.7% | 28.2% | 27.5% | 30.4% | 34.7% | 36.0% | 51.9% | 100.0% | 40.4% |
| **HA** | 39.0% | 18.0% | 24.6% | 14.4% | 33.8% | 34.3% | 35.8% | 24.0% | 42.2% | 40.4% | 100.0% |

## 4.2. Five Constraints

### 4.2.1. Constraint1 (C1)

In the next phase, we aim to determine the most advantageous allocations for both the Markowitz Model (MM) and the Index Model (IM) for each stock under varied constraints aimed at solving three critical optimization issues: constructing portfolios to minimize expected variance (risk), to maximize

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expected Sharpe ratio (efficiency), and to achieve the highest possible expected excess returns. The predictive analysis for portfolios that adhere to the lowest variance (risk) and highest Sharpe ratio objectives are presented in Tables 3 and 4, showcasing the findings for both the Markowitz Model and the Index Model accordingly.

The first and second rows of Table3 and Table 4 show the prediction results under constraint 1 (C1). The estimation results of the Markowitz Model suggest that a portfolio can be constructed with a global minimum variance as low as 9.94%, and a maximum Sharpe ratio of 1.035, indicating a careful balance between risk and return. The Index Model predicts a minimum variance of 9.26% and a maximum Sharpe ratio of 0.433, revealing a significant difference in risk and efficiency outcomes when compared to the Markowitz Model.

### 4.2.2. Constraint2 (C2)

The imposition of the FINRA Constraint establishes a realistic boundary by limiting investor leverage to no more than twice their starting equity. This rule keeps the weight configurations of the minimum variance portfolios constant for both models under C2 as they were under C1. On the contrary, it does affect the maximum Sharpe ratio performance, with a minor adjustment downward to 1.035 in the Markowitz Model, and a more substantial drop to 0.433 in the Index Model.

### 4.2.3. Constraint3 (C3)

Arbitrary 'Box' Constraint, which imposes limitations on customers' potential positions, both models predict a minimum variance of 0.00% and a maximum Sharpe ratio of 0.892 for the Index Model, suggesting that the Index Model maintains a consistent performance edge over the Markowitz Model.

### 4.2.4. Constraint4 (C4)

Typical U.S. Open-Ended Mutual Fund Constraint, adheres to regulations that effectively prohibit both short and long positions exceeding certain thresholds. Under C4, both models give higher weights to traditionally lower-risk stocks and suggest similar weight allocations and optimizing subjects. The Index Model consistently outperforms the Markowitz Model with a minimum variance of 9.23% and a maximum Sharpe ratio of 0.892.

### 4.2.5. Constraint5 (C5)

Exclusion of the Index, illustrates the weight predictions if SPX was not considered as an option. Without SPX for diversification, the minimum risk achievable is higher for both models, with the Markowitz Model predicting a minimum variance of 10.67% and the Index Model predicting 10.07%.

In summary, the Markowitz Model tends to suggest portfolios with a lower variance, indicating a potentially lower risk profile. However, it's important to note that variance alone does not capture the complete picture of risk, especially concerning the potential for extreme market events. Therefore, portfolio managers should combine these predictions with an understanding of both systematic and unsystematic risks present in the market to make informed decisions. Systematic risks, which affect the entire market or industry, cannot be diversified away and require different strategies for management.

Table 3: Construction Predictions for Markowitz Model under Constrain1-Constrain5

| MM (Constr1): | SPX | ADBE | IBM | SAP | BAC | C | WFC | TRV | LUV | ALK | HA | Return | StDev | Sharpe |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| MinVar | 110.97% | -9.68% | 5.14% | -9.90% | 0.35% | -22.54% | 14.06% | 19.45% | -0.13% | -4.86% | -2.85% | 6.72% | 11.75% | 0.572 |
| MaxSharpe | 39.54% | 28.26% | -0.10% | 0.29% | 16.67% | -48.07% | 19.80% | 30.14% | -1.80% | 7.70% | 7.57% | 17.59% | 17.70% | 0.994 |
| MM (Constr2): | SPX | ADBE | IBM | SAP | BAC | C | WFC | TRV | LUV | ALK | HA | Return | StDev | Sharpe |
| MinVar | 100.00% | -8.53% | 8.25% | -9.24% | 0.66% | -23.03% | 16.19% | 22.93% | 0.28% | -4.76% | -2.75% | 6.97% | 11.79% | 0.591 |
| MaxSharpe | 50.09% | 35.81% | -22.32% | 7.19% | 29.74% | -66.14% | 20.56% | 35.57% | -14.83% | 13.45% | 10.87% | 22.07% | 21.33% | 1.035 |
| MM (Constr3): | SPX | ADBE | IBM | SAP | BAC | C | WFC | TRV | LUV | ALK | HA | Return | StDev | Sharpe |
| MinVar | 111.46% | -9.91% | 5.27% | -10.07% | 0.60% | -22.81% | 14.06% | 19.53% | -0.26% | -4.98% | -2.88% | 6.69% | 11.75% | 0.570 |
| MaxSharpe | 50.09% | 35.81% | -22.32% | 7.19% | 29.74% | -66.14% | 20.56% | 35.57% | -14.83% | 13.45% | 10.87% | 22.07% | 21.33% | 1.035 |
| MM (Constr4): | SPX | ADBE | IBM | SAP | BAC | C | WFC | TRV | LUV | ALK | HA | Return | StDev | Sharpe |
| MinVar | 83.61% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 16.39% | 0.00% | 0.00% | 0.00% | 7.79% | 14.61% | 0.533 |
| MaxSharpe | 0.00% | 50.27% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 17.56% | 0.00% | 19.52% | 12.66% | 18.24% | 24.75% | 0.737 |
| MM (Constr5): | SPX | ADBE | IBM | SAP | BAC | C | WFC | TRV | LUV | ALK | HA | Return | StDev | Sharpe |
| MinVar | 0.00% | 3.53% | 34.20% | -1.99% | 1.18% | -24.94% | 34.79% | 52.68% | 5.03% | -2.86% | -1.61% | 9.38% | 15.45% | 0.607 |
| MaxSharpe | 0.00% | 50.82% | -19.51% | 13.81% | 36.74% | -76.96% | 28.73% | 49.95% | -16.54% | 18.45% | 14.50% | 26.53% | 25.98% | 1.021 |

Table 4: Construction Predictions for Index Model under Constrain1-Constrain5

| IM (Constr1): | SPX | ADBE | IBM | SAP | BAC | C | WFC | TRV | LUV | ALK | HA | Return | StDev | Sharpe |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| MinVar | 138.69% | -9.85% | 0.03% | -9.68% | -8.19% | -15.39% | -0.28% | 11.23% | -2.17% | -1.77% | -2.62% | 6.07% | 11.96% | 0.508 |
| MaxSharpe | 48.18% | 35.88% | -14.54% | 2.04% | -0.61% | -34.85% | 3.97% | 28.36% | 3.60% | 17.73% | 10.24% | 18.90% | 21.04% | 0.898 |
| IM (Constr2): | SPX | ADBE | IBM | SAP | BAC | C | WFC | TRV | LUV | ALK | HA | Return | StDev | Sharpe |
| MinVar | 100.00% | -6.58% | 8.50% | -6.99% | -6.49% | -13.97% | 3.74% | 23.09% | 0.74% | 0.08% | -2.12% | 6.90% | 12.47% | 0.553 |
| MaxSharpe | 54.94% | 38.26% | -22.71% | 2.98% | -2.31% | -37.58% | 4.25% | 28.63% | 4.00% | 18.66% | 10.89% | 19.81% | 21.99% | 0.901 |
| IM (Constr3): | SPX | ADBE | IBM | SAP | BAC | C | WFC | TRV | LUV | ALK | HA | Return | StDev | Sharpe |
| MinVar | 144.39% | -10.70% | -0.63% | -10.37% | -8.62% | -15.69% | -1.35% | 10.96% | -2.97% | -2.27% | -2.75% | 5.85% | 11.95% | 0.490 |
| MaxSharpe | 54.94% | 38.26% | -22.71% | 2.98% | -2.31% | -37.58% | 4.25% | 28.63% | 4.00% | 18.66% | 10.89% | 19.81% | 21.99% | 0.901 |
| IM (Constr4): | SPX | ADBE | IBM | SAP | BAC | C | WFC | TRV | LUV | ALK | HA | Return | StDev | Sharpe |
| MinVar | 83.61% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 16.39% | 0.00% | 0.00% | 0.00% | 7.79% | 14.61% | 0.533 |
| MaxSharpe | 0.00% | 40.84% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 26.27% | 0.00% | 20.52% | 12.38% | 17.28% | 23.35% | 0.740 |
| IM (Constr5): | SPX | ADBE | IBM | SAP | BAC | C | WFC | TRV | LUV | ALK | HA | Return | StDev | Sharpe |
| MinVar | 0.00% | 2.68% | 29.08% | 0.61% | -1.69% | -10.08% | 15.22% | 50.40% | 9.09% | 5.38% | -0.70% | 9.26% | 16.64% | 0.556 |
| MaxSharpe | 0.00% | 52.54% | -20.56% | 8.64% | 0.79% | -41.18% | 9.50% | 42.11% | 8.47% | 25.23% | 14.46% | 23.79% | 26.68% | 0.892 |

Table 5: Predictions result Markowitz versus Index Model

| | SPX | ADBE | IBM | SAP | BAC | C | WFC | TRV | LUV | ALK | HA | Return | StDev | Sharpe |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| MM | 50.1% | 35.8% | - 22.3% | 7.2% | 29.7% | - 66.1% | 20.6% | 35.6% | - 14.8% | 13.5% | 10.9% | 22.1% | 21.3% | 1.035 |
| IM | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 100.0% | 26.9% | 62.1% | 0.433 |

## 5. Conclusion

In conclusion, this study has meticulously examined the impacts of employing the Markowitz and Index Models across a spectrum of realistic constraints, using an extensive dataset of 20 years of monthly returns from ten selected stocks, the SPX index, and the 1-month federal funds rate. The efficient frontiers for both models were delineated within five distinct constraints using Excel Solver, allowing a comparative analysis of the portfolios' returns, standard deviations, and Sharpe ratios to assess the influence of the different models and the varying constraints.

The findings unmistakably lean towards investments in firms like Adobe Inc. (ADBE), IBM, SAP SE, Bank of America Corp (BAC), Citigroup Inc. (C), Wells Fargo & Co. (WFC), The Travelers Companies Inc. (TRV), Southwest Airlines (LUV), Alaska Air Group (ALK), and Hawaiian Holdings (HA), which have reliably produced positive returns under our portfolio selection methodology. The Markowitz and Index models exhibit closely matched Sharpe Ratios and standard deviations, denoting similar risk-return considerations in the assessed portfolios. Nevertheless, the Index Model is prone to favor portfolios with a bit less variance, indicating a slight reduction in portfolio risk.

The investigation also highlights key differences between the two models. The Markowitz Model relies on complex mathematical statistical methods and requires comprehensive data to accurately optimize portfolios. It allows investors to anticipate expected returns and determine the minimum variance for different expected return combinations. In contrast, the Index Model simplifies the optimization process by considering fewer factors, thus facilitating more straightforward calculations but at the potential cost of predictive performance. Ultimately, the Markowitz Model can provide superior results, assuming the availability of extensive and precise data.

This research underscores the significance of both models in modern portfolio management, emphasizing the necessity of understanding and managing systematic and unsystematic risks to optimize investment decisions effectively. It is a reminder that while quantitative models serve as powerful tools for risk assessment and portfolio construction, the nuanced understanding of market dynamics remains essential for achieving investment objectives.

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