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Empirical Study of Monte Carlo Simulation in SSE 50 ETF Options Pricing

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Abstract: Option is one of the most basic financial derivatives, and the pricing of option has always been a significant research topic in financial engineering. Reasonable pricing of options is the premise of options to play an important role in the financial market, for avoiding market risks, stabilizing the financial market, sustainable development of the derivatives market has vital practical significance. Monte Carlo Simulation is an important simulation method in option pricing and it based on probability theory and mathematical statistics, by simulating the path of asset prices to predict a complete average return and obtain an option price estimate. In this paper, the Monte Carlo simulation is used to make an empirical analysis of China's financial derivative SSE 50 ETF option, and the simulation price is compared with prices simulating by other models, and finally compared with the current price of SSE 50 ETF option. The results show that the SSE 50 ETF call option is recommendable to be bought, and has strong feasibility.

Keywords: Option Pricing, Monte Carlo Simulation, SSE 50 ETF

1. Introduction

Options are an important financial derivative product with rich pricing theories, including the B-S-M model proposed by Black and Scholes, the Merton model proposed by Merton, and the Heston stochastic volatility model proposed by Hull and White. On February 9, 2015, the first stock option in China, the SSE 50ETF option, was officially listed, marking the beginning of the option era in the Chinese securities market [1]. The SSE 50ETF option, as the pioneer for stock options in Chinese capital market, occupies an important position in the Chinese option market, and the importance of its pricing is self-evident. This article uses Monte Carlo simulation for empirical analysis of SSE 50ETF options, compares them with current prices, and provides investment suggestions.

2. Literature Review

Option pricing is an important and difficult issue in the financial derivatives market. Regarding the pricing theory of options, as early as 1900, Bachelier used Brownian motion to describe the changes in prices of stocks. He was the first person to propose an option pricing model and obtain an analytical solution for option prices. Then, Ito proposed the Ito [2] process and Ito lemma. On this basis, in 1973, American economists Fischer Black and Myron Scholes [3] proposed the B-S option pricing model. Subsequently, Robert Merton [4] introduced risk-neutral representation on the basis of this model and

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further improved it into the B-S-M model. In order to reduce the impact of uncertainty factors on option pricing calculations, some scholars have applied the Monte Carlo method to expiration option pricing. Boyle [5] first used the Monte Carlo simulation to price European stock options with a single asset as the underlying asset.

At present, there is still relatively little research on pricing SSE 50 ETF options in China. For example, Wang Peng and Yang Xinglin [6] considered the pricing problem of SSE 50 ETF options under time-varying volatility. Fang Yan [7] conducted an empirical comparative analysis on the performance of Black-Scholes model and the method of Monte Carlo simulation in the pricing of SSE 50 ETF options. This paper is based on the Monte Carlo simulation to price the SSE 50 ETF option and then compares the theoretical price with the current price, providing reasonable suggestions for whether to purchase the option.

3. Models and Methods

In order to price options properly, a dynamic model of the underlying asset price is essential [8]. Monte Carlo simulation is one of the most important algorithms in finance. It plays a crucial role in option pricing and risk management [9]. The principle of Monte Carlo simulation of option pricing is to simulate the price movement of the underlying asset according to a given price movement process. When the number of simulations reaches a certain number, the expectation is obtained by taking the mean. According to the law of large numbers, Monte Carlo simulation results ultimately satisfy convergence. The specific process is as follows [9]:

(1) Assuming that the change process of the underlying asset SSE 50 ETF follows a risk neutral geometric Brownian motion, its continuous form is

$$dS = S\mu dt + \sigma dz. \tag{1}$$

where μ is the expected return value of the SSE 50 ETF in a risk neutral world, σ is volatility and dz is a Wiener process.

According to Ito Lemma,

$$S(t+\Delta t) = S(t)\exp\left[(\mu - \frac{\sigma^2}{2})\cdot \Delta t + \sigma\varepsilon\sqrt{\Delta t}\right], \tag{2}$$

where z is a normally distributed random number with an expected value of 0 and a standard deviation of 1.

(2) If we divide the time between the validity period of the option from 0 to T into n equal intervals, $\Delta t = T/n$. Then we need to independently extract n normally distributed random numbers ε, Construct a possible path of change in the price of the underlying asset, i.e. obtain a sequence of $S(t+\Delta t)$. Starting from the initial moment S, randomly select one ε. The value of S at time Δt can be calculated, and then the value of S at time $2\Delta t$ can be calculated using the value of S at time Δt, repeated n times to obtain the price of the asset at maturity T. The estimated values of call and put options at time T are:

$$C_T = \max(0, S_T - K) \tag{3}$$

$$\text{and } P_T = \max(0, K - S_T); \tag{4}$$

(3) Repeat step 1 and step 2 to simulate enough times to obtain a large number of sample option return values;

(4) So, the estimated price of European call options obtained from these n simulations is

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$$C = \frac{1}{n}\sum e^{-rT} \times \max(0, S_T - K) \tag{5}$$

Similarly, the estimated price of European put options can be obtained as

$$P = \frac{1}{n}\sum e^{-rT} \times \max(0, K - S_T) \tag{6}$$

4. Empirical Analysis

This paper selects the daily closing price of SSE 50ETF from January 6, 2022 to September 6, 2023 as the research object, with a total of 408 sample points, sourced from Wind. From the model, it can be seen that five parameters need to be used, namely: the current price S of SSE 50ETF, the exercise price K of the option, the expiration date $(T - t)$ of the option, the risk-free interest rate r for continuous compounding, and the volatility of SSE 50 ETF σ [1].

(1) Calculate the volatility of SSE 50ETF.

Calculate the volatility of the SSE 50 ETF using historical estimation method [1], determined by the standard deviation of historical logarithmic returns. The selected inspection period is from January 6, 2022 to September 6, 2023.

Obtain values:$\mu = -0.00044$,$\sigma^2 = 0.000135$,$\sigma = 0.01163533$。The obtained volatility is the daily volatility of the closing price of the SSE 50ETF, which is now converted into annualized volatility:$\sigma_{year} = \sigma_{day} * \sqrt{250} = 0.1839707$

(2) Determine the current price S of SSE 50 ETF.

Determine the current price S of SSE 50 ETF

The closing price of the SSE 50 ETF on September 6, 2023 was 2.6370.

(3) Term of Option Contract $(T - t)$.

The latest expiration date of the SSE 50ETF options on the market is September 27, 2023, so there are still 15 days left until the expiration date $(T - t) = 15/250$.

(4) Determine the strike price of options K.

Assuming that the price of SSE 50ETF follows a lognormal distribution, then

$$E(S_T) = Se^{\mu(T-t)} \tag{7}$$

Then, $E(S_T) = Se^{\mu(T-t)} = 2.6370 * e^{-0.00044\times\frac{15}{250}} = 2.63693$,This equation indicates that after 15 days, the expected value of the SSE 50ETF is 2.63693, which is approximated as the strike price of the option. So $K = 2.63693$.

(5) Determine risk-free interest rates.

The yield of China's 10-year treasury bond is selected as the risk-free interest rate, and the data is sourced from www.chinamoney.com.

Due to the limited disclosure of data, the arithmetic mean within a year is calculated here as the risk-free interest rate, resulting in $r = 0.027679044$, Then convert to risk-free continuous compound interest:$r_f = 0.0273029$.

This obtains all the parameters required to calculate the call option price:

$$\mathrm{S} = 2.6370, \mathrm{K} = 2.63693, \mathrm{r} = 0.0273029, \sigma = 0.1839707, \Delta t = 0.06.$$

(6) Using Python to calculate the price of call options.

By simulating the price of the underlying asset 100000 times using Monte Carlo, the theoretical price for 50ETF to purchase September 2650 options is 0.0474. By continuously increasing the number of simulations, the following table can be obtained:

Table 1 shows that the more times the simulation is performed, the theoretical price of the option tends to converge [10].

Table 1. European call option price under different simulation times

Times of simulations	Theoretical price
100	0.0516
1000	0.0455
10000	0.0478
100000	0.0474

Using the Black-Scholes option pricing model, this theoretical option price derived is 0.0212, which is sourced from www.eastmoney.com. As of September 7, 2023, the latest quote for this option is lower than the theoretical price, so it is recommended to buy this call option.

5. Conclusion

This article uses the European option pricing method of Monte Carlo simulation to analyze the pricing of the September 2650 option purchased by the SSE 50 ETF, and compares it with the current price. The empirical results show that the more simulation times, the convergence of the option price. By comparing the theoretical price to the actual price, it is recommended to buy the call option. This empirical analysis process has a guiding role for options trading behavior.

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