

Research on the Pricing Model of Financial Derivatives

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Abstract: Based on the importance of the pricing model of financial derivatives, this paper points out the defects and risks of the current model and proposes some possible solutions. The research method is mainly to summarize the framework of the existing model by searching the academic literature and commercial examples related to the pricing model. The final conclusion is that an obvious flaw of the pricing model is that it is based on an idealized market state and cannot predict hidden market trends, but through random volatility, consideration of market friction, and the development of mathematical technology, the market can refine the pricing model. Research and forecast risk premium, market liquidity and asset price changes in advance when considering pricing. In addition, the market is also supported to consider the application of large machines in the portfolio as well as the issue of options panels, so as to deepen the understanding of the movement of financial products.

Keywords: pricing model, Financial derivatives, risk management.

1. Introduction

Although the research on financial derivatives has been very rich, there are still some research gaps and challenges:

Pricing and risk management of complex derivatives: While many models have been used to price standard derivatives such as options, futures and swaps, effective pricing and risk management methods for more complex derivatives such as structured products and hybrid derivatives are still inadequate.

Derivative pricing in incomplete markets: Most existing pricing models are based on the perfect market assumption, but in real markets, this assumption often does not hold. For example, there may be friction in the market, such as transaction costs, restrictive trading, or the behavior of market participants may deviate from rational expectations. For these cases, this paper needs to develop new theories and methods.

The systemic risk of derivatives: The financial crisis showed that derivatives transactions can affect the stability of the entire financial system. However, our understanding of how derivatives transmit and amplify systemic risk remains limited.

Regulation of derivatives: Effective regulation of derivatives markets is an important issue that requires further study. In particular, issues such as the transparency of the over-the-counter derivatives market, the derivatives trading practices of hedge funds and other non-bank financial institutions, and the clearing and guarantee mechanisms for derivatives need to be studied in depth.

Application of Big Data and artificial intelligence in the derivatives market: With the development of technology, big data and artificial intelligence have begun to play a role in financial markets. How to use these new technologies to improve the pricing, trading and risk management of derivatives is a new research direction.

These are just a few of the possible research gaps and challenges, the study of financial derivatives is an ever-evolving field, with new questions and opportunities always emerging. This essay will discuss the pricing model of financial derivatives, including its types, deficiencies and solutions.

2. Pricing Model of Financial Derivatives

The pricing of financial derivatives is an important research area of financial mathematics and financial engineering. The research on the pricing model of financial derivatives mainly focuses on the theoretical and empirical research of the following models:

Black-Scholes Model: Developed in 1973 by Fischer Black and Myron Scholes, this pricing strategy is among the most widely used [1]. The model assumes that interest rates and volatility are both known and constant and that asset values follow geometric Brownian movements in an idealized market. The Black-Scholes model remains the foundation for both theoretical research and empirical analysis, despite the fact that these assumptions are not entirely true in the real market.

The Binomial Model is a discrete option pricing model that was first put forth by Cox, Ross, and Rubinstein in 1979 [2]. The Binomial model, in contrast to the Black-Scholes model, enables the simulation of asset price movements in a sequence of discrete steps, beginning in the future. The asset price may increase or decrease at each stage, forming a binary tree. Even while the Binomial model has a higher computational complexity than the Black-Scholes model, it is more adaptable and can handle a wider range of scenarios, including American-style option pricing.

Stochastic volatility models: These models relax the assumption of constant volatility in the Black-Scholes model, allowing volatility to change over time. The Heston model is a very effective stochastic volatility model. Such models can better capture the volatility smile phenomenon in the market.

Jump diffusion models: This type of model introduces a jump component into the asset price dynamics, that is, prices can have large discontinuous movements at certain moments. Such models capture extreme events in the market, such as stock market crashes.

The above model is only a part of the pricing model of financial derivatives, in fact, there are many other complex models, such as stochastic volatility jump model, stochastic interest rate model and so on. They are designed to better match the price of financial derivatives in the actual market and improve the accuracy of pricing. However, the complexity of the model increases the difficulty of the calculation, so it is necessary to find a balance between the accuracy of the model and the efficiency of the calculation in practical applications.

Methods: The tools and methods to study the pricing model of financial derivatives mainly include mathematical tools, econometric methods, programming languages and software. Here are some specific tools and approaches:

Mathematics and probability theory: Financial derivatives pricing models are mostly based on stochastic process theory, so it is necessary to master advanced mathematics, probability theory, stochastic process and other relevant knowledge. In addition, mathematical tools such as partial differential equations, Monte Carlo simulations, and optimization methods are often used to price derivatives.

Meaning: Financial derivatives play an important role in the global financial market, so the study of them is very important. Specifically, the importance of studying financial derivatives can be seen from the following aspects:

Risk management: Financial derivatives are the main tools for enterprises and individuals to manage risks. By understanding and studying financial derivatives, this paper can help enterprises and individuals design effective hedging strategies to reduce the impact of market risk, interest rate risk, credit risk, etc., on their wealth and business.

Investment decision: Financial derivatives are also an important tool for investors to obtain income. By studying financial derivatives, this paper can help investors understand the nature and risks of these products and make better investment decisions.

Market stability: The 2008 global financial crisis showed that derivatives transactions can have an impact on the stability of the entire financial system. The study of financial derivatives can help us to understand this impact and design better regulatory policies to prevent the risk of financial instability.

Market efficiency: Financial derivatives provide investors with a variety of trading strategies, which helps to improve the liquidity and efficiency of the market. At the same time, the existence of the derivatives market may also affect the behavior and nature of the spot market, such as price volatility, trading volume, etc.

Theoretical development: In terms of mathematical models, the financial derivatives market provides a large number of theoretical and empirical questions, which helps to promote the development of financial theory. For example, the Black-Scholes model is not only a basic strategy for setting a price but also has an important impact on other financial fields.

Regulatory policy: The development of the derivatives market has brought a series of regulatory issues, such as market transparency, systemic risk, etc. This requires a deep understanding of the derivatives market in order to develop effective regulatory policies.

In general, the study of financial derivatives helps us to understand the operation mechanism of financial markets, improve the ability to manage risks, forecast and explain asset prices, and formulate effective financial policies.

3. Examples of Models

Next, this paper will introduce in detail the two most classic financial derivatives Pricing models: Black-Scholes Model and Binomial Option Pricing Model, including their theoretical basis, as well as their respective advantages and disadvantages.

The Black-Scholes model, also known as the Black-Scholes-Merton model, assumes that there is no friction in the market and that the logarithmic returns of stock prices follow a normal distribution. According to these assumptions, the model can deduce the analytical pricing formula of European option.

Advantages: The biggest advantage of Black-Scholes model is that it can give the analytical pricing formula of option, which is convenient to calculate the option price quickly in the actual business. In addition, the theoretical basis of the model is solid, which is helpful to understand the factors affecting option prices.

Disadvantages: However, the assumptions of the Black-Scholes model are too strict and often do not correspond to reality. For example, it assumes that the logarithmic return on stock prices follows a normal distribution, but in fact, the distribution of the logarithmic return on stock prices tends to have a fat tail. Moreover, the model assumes that there is no friction in the market, allowing for unlimited risk-free arbitrage, which is also impossible in reality.

Black-Scholes model gives the theoretical price of European options under the condition of no arbitrage.

The binary tree model, also known as the Cox-Ross-Rubinstein model, simulates the process of stock price changes by constructing a binary tree, and then calculates the price of options by backward induction.

Advantages: The main advantage of the binary tree model is that the model is simple, easy to understand, and suitable for various types of options. The model does not require analytical solutions, only numerical solutions, so the assumptions of the model can be more flexible.

Disadvantages: The disadvantage of the binary tree model is that the calculation amount is relatively large, especially for long maturity periods or multi-asset options, it needs to build a large binary tree, and the calculation efficiency is low. In addition, the approximation degree of the binary tree model to the continuous-time model depends on the choice of steps, too many steps will increase the amount of computation, too few steps will reduce the accuracy.

This is a discrete option pricing model that calculates option prices by constructing a binary structured stock price model.

The above two models are the most basic and classic models to study the pricing of financial derivatives. They have their own advantages and disadvantages and are suitable for different situations. Understanding these two models can not only help us understand the basic principles of option pricing, but also lay the foundation for subsequent research on more complex derivative pricing models.

Hull-White Model: The Hull-White model is a pricing model for interest rate derivatives that assumes that the evolution of short-term interest rates is a random process.

Heston Model: The Heston model is a stochastic volatility model that assumes that volatility itself is a stochastic process.

The above four examples are only a small part of the pricing models of financial derivatives, in fact, there are many other pricing models, such as jump diffusion model, GARCH model, LIBOR market model, etc., each has its own scope of application and advantages and disadvantages.

4. Drawbacks

The pricing model of financial derivatives is very important in practical applications, but there are some shortcomings. These problems arise mainly from the hypothesis of this model and the complexity of the real financial marketplace.

Idealization of assumptions: Many derivatives pricing models, such as the Black-Scholes model, are based on idealized assumptions such as perfect market efficiency, friction-free, no market risk, and normal distribution of logarithmic returns. However, these assumptions often fail in real-world market conditions. Empirical studies (such as Lo, A. W.'s "The Adaptive Markets Hypothesis") [3] find that markets are not always efficient, price fluctuations often have a "fat tail" characteristic, and there is a lot of friction in financial markets.

Ignoring market liquidity: Many models do not take market liquidity into account when pricing, but liquidity has a significant impact on prices in real markets. For example, Amihud, Y., & Mendelson, H. [4] proposed in the article "Asset pricing and the bid-ask spread" that poor market liquidity would lead investors to demand extra returns for holding these assets, thus affecting asset prices.

Undertreatment of underlying factors: For example, many models do not adequately account for the result of important factors such as margin of interest and stock market fluctuation. In Hull, J. C., & White, A., [5] "The pricing of options on assets with stochastic volatilities" the authors point out that the Black-Scholes model does not think over the volatility effect on option prices, which makes the pricing results of this model often proved to be biased by the market.

Complexity and computability: When pricing more complex derivatives, such as complex derivative structures or multi-asset derivatives, high-dimensional mathematical calculations or complex numerical methods are involved, which undoubtedly increases the complexity and computational cost of the model.

These are some of the shortcomings of the pricing model of financial derivatives, and one of the goals of academic research is to try to overcome these shortcomings and develop more accurate and practical pricing models.

5. Solutions

In order to solve the shortcomings of the financial derivatives pricing model, academia and industry are taking various measures to improve:

Relaxation or change of assumptions: For example, the Heston model [6] introduced random volatility based on the Black-Scholes model, bringing the model price closer to the market standard.

Direct consideration of market friction: for example, the introduction of transaction costs into the model, or the effect of introducing market liquidity. For example, Daniel C. Jefferies [7] in "Option Pricing with Transaction Costs Using a Markov Chain Approximation" article considers the effect of transaction costs on option pricing.

The development of efficient numerical methods such as Monte Carlo simulation [8], wavelet analysis [9], and finite difference methods [10], among others, have made the pricing of complex derivatives possible.

Leveraging machine learning and Big Data: For example, Dixon et al. [11] in "Sequence Classification of the Limit Order Book Using Recurrent Neural Networks" has published an article on the use of deep learning methods to price financial derivatives.

Risk-neutral pricing: Converting the various risks in the market into a unified risk premium, making the pricing of derivatives more concise. For example, Geman, H., El Karoui, N. & Rochet, J.C [12] in "Changes of Numeraire, Changes of Probability Measure and Option Pricing" is discussed in detail.

These are some of the methods currently being used to improve pricing models for financial derivatives. Each of these methods has its advantages and disadvantages, and no method can completely solve all problems and often needs to be combined. Up to now, the pricing of financial derivatives is still one of the hot topics in finance research.

Expect those methods, it is necessary to improve the pricing model of financial derivatives from the following aspects:

Add more practical factors: Basic derivatives pricing-setting models, such as the Black-Scholes model, often contain idealized assumptions, such as frictionless markets and a normal distribution of stock price fluctuations. To get closer to reality, this paper can add more practical factors to the model, such as transaction costs, market liquidity, jumping behavior of asset prices, random volatility, etc.

Leveraging data-driven approaches: With the development of big data and machine learning technologies, data-driven approaches are becoming more widely used in the pricing of financial derivatives. Using a large amount of historical data can help us estimate model parameters more accurately and also help us capture richer market information. For example, a deep learning approach can be used to learn a model that automatically captures a variety of complex market dynamics.

Developing more efficient numerical methods: Many financial derivatives pricing problems rely on numerical methods, such as Monte Carlo simulation, finite difference method, binary tree method, etc. To solve these problems more efficiently, this paper need to continuously develop and improve these numerical methods [13].

Interdisciplinary knowledge and technology: finance, mathematics, physics, computer science and other aspects of knowledge and technology can be used to improve the pricing model of financial derivatives. For example, many methods in quantitative finance are derived from physics, such as quantum finance, thermodynamic models, and so on. In addition, optimization methods, high performance computing and other computer science techniques have also played an important role in the pricing of financial derivatives.

A deep understanding of the market: Improving the pricing model of financial derivatives is not only a technical problem, but also a problem of understanding the market. Only with a deep understanding of the operating mechanism of the market can this paper develop a pricing model that is closer to reality.

6. Conclusion

Financial derivatives are a long-term subject, with the development of The Times, this paper also needs to put forward methods to keep pace with The Times, and constantly improve its pricing model system. In addition to this, this paper also has many other research directions, such as risk premium, the possibility of big data and machine learning techniques for dynamic portfolio selection, the problem of parameter inference and dynamic state recovery from options panel data, and the method of option pricing using fast Fourier transform. All in all, financial derivatives is an area that all researchers need to pay attention to at all times, and actively face new problems and new challenges that constantly arise.

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