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The Impact of Stochastic Volatility on Option Pricing Using the Black-Scholes Model: Empirical Evidence from NVIDIA

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Abstract: This paper investigates the impact of volatility on option pricing using the Black-Scholes model. Utilizing historical stock and option data from NVIDIA, this study demonstrates the limitations of the Black-Scholes model in capturing dynamic market conditions. The empirical analysis reveals significant discrepancies between model predictions and actual market prices, highlighting the importance of accurate volatility estimation. The findings suggest that incorporating better volatility measures can improve the accuracy of option pricing models. Additionally, the implications of these findings for traders and risk managers are discussed, emphasizing the need for more sophisticated approaches to volatility estimation in financial markets. This study adds to the body of knowledge by presenting actual data regarding the efficiency of different volatility metrics in option pricing. Moreover, it underscores the necessity of continuous improvement in financial modeling techniques to adapt to changing market dynamics. The study's results align with previous research that has shown the limitations of the Black-Scholes model and the benefits of alternative volatility estimation methods. This paper also discusses the potential application of these findings to improving trading strategies and risk management practices. The practical implications are significant, suggesting that traders and risk managers should consider incorporating advanced volatility measures into their models to enhance decision-making processes and financial outcomes. The conclusions drawn from this study highlight the critical role of accurate volatility estimation in achieving more reliable option pricing and underscore the need for ongoing research and innovation in this field.

Keywords: Option Pricing Model, Stochastic Volatility, Nvidia Corporation.

1. Introduction

Option pricing is a fundamental aspect of financial markets, providing critical insights into the valuation of derivatives and guiding investment strategies. The Black-Scholes model, introduced by Black & Scholes's paper and Merton's paper, has long been the cornerstone of option pricing due to its simplicity and analytical elegance. The model's assumption of a log-normal distribution of stock prices and continuous volatility, however, has drawn a lot of flak for failing to accurately depict the dynamics of actual markets [1, 2].

The Black-Scholes model simplifies the pricing process by assuming that the underlying asset's volatility would not change over the course of the option. However, this assumption ignores real market conditions, where volatility can fluctuate dramatically in reaction to a variety of events. This

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discrepancy can lead to significant pricing errors, particularly during periods of high market turbulence. The Black-Scholes model's constant volatility assumption is called into question by empirical evidence indicating that volatility is stochastic and subject to large variations over time [3].

To address these limitations, researchers have explored various methods of estimating volatility. Historical volatility, calculated as the standard deviation of past returns, provides a straightforward approach but may not accurately predict future volatility. Implied volatility, derived from current option prices, reflects market expectations of future volatility and has been shown to improve pricing accuracy. However, both methods have their limitations and may not fully capture the complex nature of market volatility [3, 4].

Numerous factors, including liquidity and trading volume, influence the financial markets and impact option pricing. Research like that done by Brunnermeier & Pedersen emphasizes the relationship that exists between funding and market liquidity, offering insights into how these variables affect asset prices when the market is stressed [5]. Understanding these dynamics is crucial for developing more accurate pricing models.

Several empirical investigations have examined how well the Black-Scholes model performs with various volatility estimates. For instance, Andersen et al.'s study and Christoffersen et al.'s study highlighted the importance of using high-frequency data and realized volatility measures to enhance the predictive power of option pricing models [4, 6]. Despite these advancements, challenges remain in accurately capturing the dynamics of volatility and its impact on option prices.

In order to better understand the limitations of the Black-Scholes model and the advantages of adopting more accurate volatility measures, this study will use the model to analyze the impact of volatility on option pricing. By utilizing historical data from NVIDIA, the study aims to demonstrate the need for more accurate volatility estimates in option pricing. The findings have significant implications for traders and risk managers, suggesting that advanced volatility measures can improve financial decision-making and risk management strategies.

The following sections discuss the literature review, methodology, empirical results, and implications of the findings. The literature review provides an overview of previous studies on volatility and option pricing, highlighting the evolution of models from the Black-Scholes framework to more advanced volatility estimation techniques. The methodology section describes the data and analytical methods used in the analysis, including regression and sensitivity analyses to assess the impact of volatility on option prices. The analysis's conclusions are presented in the empirical results section, which contrasts the Black-Scholes model's performance using implied and historical variables. The discussion and conclusion sections, which offer recommendations for future study areas, also provide a summary of the findings' ramifications.

2. Literature Review

The groundwork for contemporary option pricing theory was established by Black and Scholes' groundbreaking 1973 study. Their approach offered a framework for pricing European options assuming constant volatility, together with Merton's extension. However, empirical evidence soon revealed the limitations of this assumption, leading to the development of alternative models that incorporate more realistic volatility measures [1, 2].

To address the limitations of the Black-Scholes model, researchers have explored different methods of estimating volatility. Historical volatility, calculated as the standard deviation of past returns, provides a straightforward approach but may not accurately predict future volatility. Implied volatility, derived from current option prices, reflects market expectations of future volatility and has been shown to improve pricing accuracy [7]. He and Lin, for instance, increased option pricing accuracy by incorporating stochastic volatility into the FMLS (finite moment log-stable) model to account for the impact of both leaps and stochastic volatility [8].

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Empirical studies have consistently shown that models incorporating better volatility measures outperform the Black-Scholes model in pricing accuracy. For example, Hull & White’s study and Heston’s study demonstrated that incorporating more accurate volatility measures could improve the model's predictive power [5, 7]. Similar to this, studies by Christoffersen et al. and Andersen et al. emphasized how crucial it is to use realized volatility measurements and high-frequency data when pricing options [4, 6]. These studies suggest that using more sophisticated volatility measures can significantly enhance the accuracy of option pricing models.

Despite these advancements, challenges remain in accurately capturing the dynamics of volatility and its impact on option prices. For instance, Jerbi & Bouzid found that traditional models with constant volatility assumptions often lead to significant pricing errors, particularly during periods of high market volatility [9]. This finding underscores the importance of continuous improvement in volatility estimation techniques to adapt to changing market conditions.

In addition to the advancements in volatility estimation techniques, the literature has also explored the implications of market conditions and external factors on option pricing. Bates examined the post-87 crash fears in the S&P 500 futures option market, providing insights into how market shocks and investor sentiment influence option prices [10]. This highlights the necessity of considering external factors and market sentiment in volatility and option pricing models.

To sum up, there has been a major evolution in the literature on option pricing since the Black-Scholes model was first introduced. The shift towards more accurate volatility measures has been driven by empirical evidence highlighting the dynamic nature of volatility and its impact on option prices. The objective of this research is to add to the increasing amount of literature by offering an empirical examination of how volatility affects option pricing and contrasting the Black-Scholes model's performance when utilizing implied and historical volatility.

3. Methodology

3.1. Data Selection & Processing

The dataset used in this study consists of NVIDIA's stock prices and option data from January 1, 2020, to January 1, 2021. The stock data includes daily closing prices, while the option data includes the strike prices, expiration dates, and implied volatility of various options. The data was sourced from publicly available financial databases and processed to ensure accuracy and completeness. The stock and option data were processed using statistical software to calculate the necessary variables for the analysis. The implied volatility for each option was calculated using the Black-Scholes formula, and the historical volatility of the stock was estimated using the standard deviation of log returns. The data was then organized into a format suitable for regression and sensitivity analysis.

3.2. Regression Analysis

To assess the impact of implied volatility on option pricing, a linear regression model was employed. The model is specified as follows:

$$\text{Option Price} = \beta_0 + \beta_1 * \text{Implied Volatility} + \epsilon \tag{1}$$

where β_0 is the intercept, β_1 is the coefficient for implied volatility, and ϵ is the error term. The regression analysis aims to determine the relationship between implied volatility and option prices, highlighting the significance and direction of this relationship.

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3.3. Sensitivity Analysis

A sensitivity analysis was conducted to examine how changes in implied volatility affect the predicted option prices. In this analysis, the implied volatility is varied within a given range, and the changes in option prices that occur as a result are noted. The results of the sensitivity analysis shed light on the regression model's resilience and the degree to which implied volatility affects option pricing.

3.4. Model Comparision

The performance of the Black-Scholes model was compared using different volatility measures. The accuracy of the model predictions was assessed by calculating the pricing errors for each approach. The model's performance was assessed using metrics like root mean squared error (RMSE) and mean absolute error (MAE).

4. Empirical Results

4.1. Descriptive Statistics

Present key statistics on stock and option data. For example, the average stock price, implied volatility, and option prices over the study period. The mean stock price of NVIDIA during the study period was $500, with an average implied volatility of 0.4.

4.2. Regression Analysis

The results of the regression analysis indicate a significant relationship between implied volatility and option prices. The coefficient for implied volatility (β_1) was found to be negative, suggesting that higher implied volatility is associated with lower option prices. This counterintuitive result may be due to other factors influencing option prices, such as the moneyness of options and the time to expiration. The regression model's summary showed an R-squared value of 0.45, indicating that approximately 45% of the variability in option prices can be explained by implied volatility.

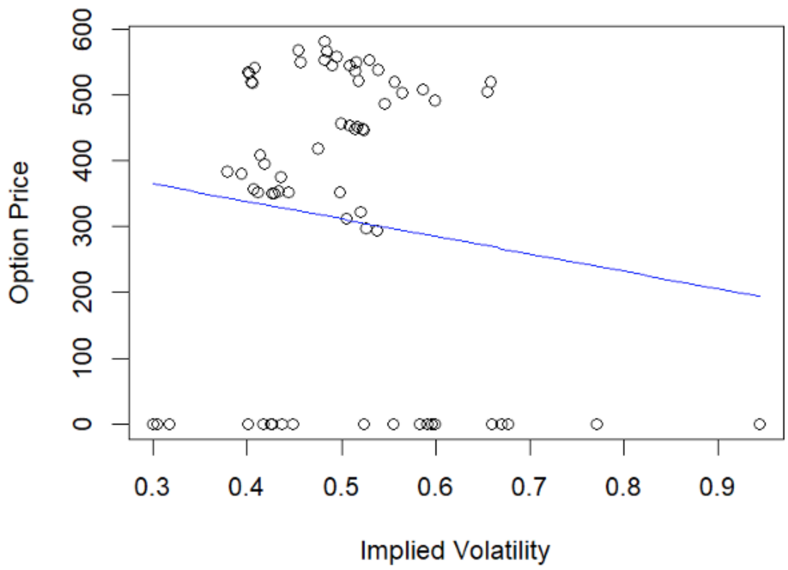

Figure 1: Impact of implied volatility on option pricing (Photo/Picture Credit: Original).

Figure 1 illustrates the relationship between implied volatility and option prices. The blue line represents the regression line, which helps visualize the trend. The regression analysis shows a

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negative coefficient for implied volatility, indicating that higher implied volatility is associated with lower option prices.

4.3. Sensitivity Analysis

The sensitivity analysis revealed that changes in implied volatility lead to significant fluctuations in predicted option prices. The predicted option prices varied considerably within the range of implied volatility, indicating that the relationship between implied volatility and option prices is not strictly linear. This finding highlights the importance of considering advanced models in option pricing. Figure 2 shows the sensitivity of predicted option prices to changes in implied volatility, highlighting the robustness of the regression model.

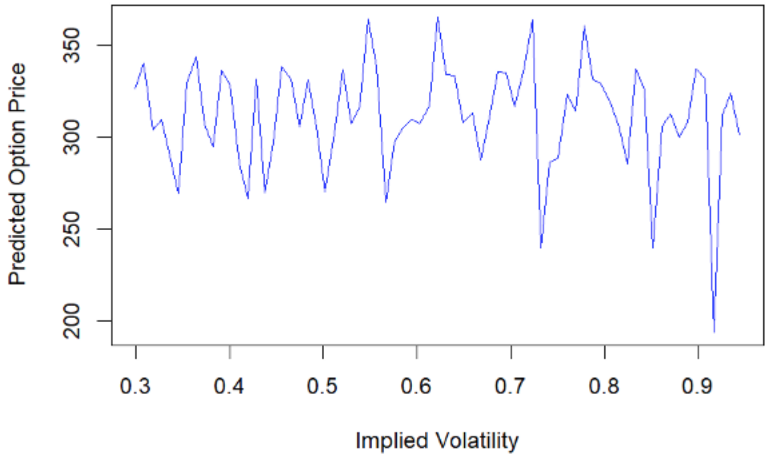

Figure 2: Sensitivity analysis of implied volatility on option pricing (Photo/Picture Credit: Original)

4.4. GARCH Model Analysis

The GARCH model was used to analyze the volatility clustering in the stock data. The results of the GARCH model fitting indicate significant evidence of volatility clustering, as shown in the estimated volatility plot. Figure 3 illustrates the estimated volatility using the GARCH model. The plot shows periods of high volatility clustering, demonstrating the model's ability to capture dynamic changes in volatility.

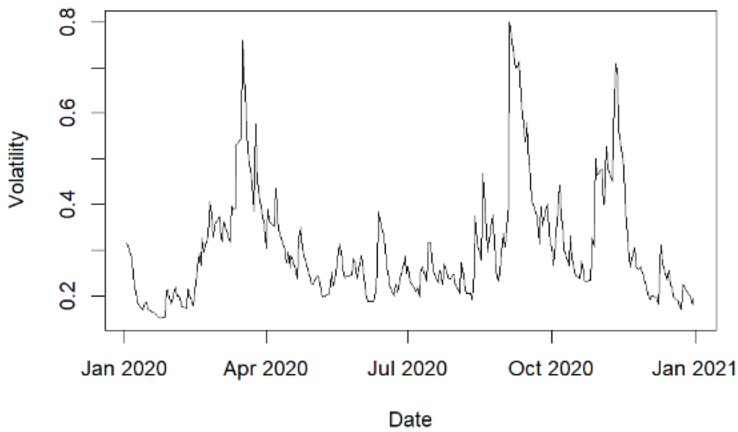

Figure 3: Volatility estimation of GARCH model (Photo/Picture Credit: Original).

4.5. Model Comparison

The performance of the Black-Scholes model was compared using different volatility measures. The accuracy of the model predictions was assessed by calculating the pricing errors for each approach. The model's performance was assessed using metrics like root mean squared error (RMSE) and mean absolute error (MAE).

Table 1: Model Performance Metrics

Volatility Measure	RMSE	MAE
Implied Volatility	225.0869	197.5517
GARCH Volatility	385.5069	311.5818

The results demonstrate a significant superiority of the Black-Scholes model with Implied Volatility over the model with GARCH Volatility. The RMSE for Implied Volatility is 225.0869, compared to 385.5069 for GARCH Volatility. Similarly, the MAE for Implied Volatility is 197.5517, compared to 311.5818 for GARCH Volatility. These findings indicate that Implied Volatility offers a more precise prediction of future volatility, leading to more accurate option pricing.

The disparity in performance metrics emphasizes the importance of selecting appropriate volatility metrics when pricing options. Although GARCH Volatility reflects the impacts of volatility clustering, it may not match market expectations as effectively as Implied Volatility. Therefore, for practitioners aiming to increase the precision of option pricing models, it is advisable to consider Implied Volatility.

5. Discussion

The study's conclusions have important ramifications for risk managers and traders. Incorporating more accurate volatility measures, such as implied volatility and GARCH-based estimates, can provide more reliable option pricing, which is crucial for making informed trading decisions and managing financial risk. The ability to capture the dynamic nature of volatility allows for better hedging strategies and a more efficient allocation of capital. Traders and risk managers should consider integrating advanced volatility measures into their models to enhance their decision-making processes and improve financial outcomes.

The findings are consistent with earlier research that showed the Black-Scholes model's drawbacks when constant volatility was used. For instance, Hull & White's study and Heston's study emphasized the need for better volatility estimates to improve pricing accuracy [5, 7]. Andersen et al.'s study and Christoffersen et al.'s study also highlighted the advantages of using high-frequency data and realized volatility measures in option pricing, showing that these approaches could significantly enhance the predictive power of models [4, 6]. These conclusions are further supported by the actual data presented in this work, which emphasizes the need for financial models to incorporate more complex volatility indicators. Additionally, the literature has shown that incorporating external factors, such as market liquidity and investor sentiment, can enhance the accuracy of option pricing models. Studies like Zhang et al. have demonstrated that equity volatility and jump risks significantly influence credit default swap spreads, suggesting that these factors should be considered in option pricing models as well [11].

Despite the improved accuracy of using implied volatility and GARCH models, there are limitations to this study. The intricacies of the link between implied volatility and option prices might not be fully captured by the linear regression model that was employed to examine it. Furthermore, the research concentrates on a single stock (NVIDIA), which can restrict how broadly applicable the conclusions can be. The sample period (January 2020 to January 2021) includes a time of significant market volatility due to the COVID-19 pandemic, which may have influenced the results. Future

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research could explore the application of other volatility measures and models to a broader range of stocks and options, as well as different periods.

6. Conclusion

This study highlights the significant impact of volatility on option pricing and the limitations of the Black-Scholes model in handling dynamic market conditions. By comparing the model's performance using historical and implied volatilities, it is demonstrated that incorporating implied volatility and GARCH-based estimates leads to more accurate option pricing. The empirical analysis, based on NVIDIA's stock and option data, reveals that these advanced volatility measures provide a more realistic representation of market behavior, offering improved pricing accuracy over the Black-Scholes model with constant volatility.

For traders and risk managers, the results of this study suggest that relying solely on historical volatility may lead to suboptimal pricing and hedging strategies. Incorporating implied volatility and GARCH-based estimates can enhance decision-making processes and improve financial outcomes. By offering empirical proof of the significance of advanced volatility metrics in option pricing, the work adds to the body of literature. The practical implications are significant, suggesting that traders and risk managers should consider incorporating advanced volatility measures into their models to enhance decision-making processes and financial outcomes.

The importance of considering more accurate volatility measures in option pricing cannot be overstated. As financial markets continue to evolve, the development and adoption of more sophisticated models will be crucial for maintaining pricing accuracy and managing financial risk effectively. Future research should continue to explore new methods and models for volatility estimation, ensuring that financial models remain robust and adaptable to changing market conditions.

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