Markowitz Model and Index Model: A Comparative Study of Constructing Optimal Portfolios

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Abstract: In the field of investment, how to strike a balance between high returns and low risks has always been a challenge for investors. With the rise of modern portfolio theory (MPT) and index models, investors have more options for constructing optimal portfolios. This paper takes the US stock market as the research object, selects the S&P 500 index and six stocks representing different industries, uses the Markowitz model and the index model to optimize, and analyzes the optimal portfolio allocation, return, risk and Sharpe ratio of the two models under different constraints. It is found that the Markowitz model is more advantageous in risk diversification, while the index model relies more on the allocation of a single stock, especially the S&P 500 index. Both models perform similarly in terms of risk-adjusted returns, but the Markowitz model is slightly superior in terms of Sharpe ratio. In addition, there are differences in the portfolio allocation and risk-return characteristics of the two models under different constraints. The results of this study can help investors better understand the advantages and disadvantages of the two portfolio optimization models, and choose the appropriate model according to their own risk preferences and market environment to construct the optimal portfolio and achieve their investment objectives.

Keywords: Markowitz Model, Index Model, Optimal Portfolio, Risk Diversification, Sharpe Ratio.

1. Introduction

When making investment decisions, it is necessary to analyze securities and portfolio management to help investors choose good investments. For a long time, investors analyzed stocks solely on the basis of their fundamentals and recent earnings performance. However, the market environment and conditions are constantly changing, and investors need to adjust their investment strategies in time according to new information. So, in the current volatile market environment, there has been a shift away from a single focus on individual stocks to a broader consideration of the risk and return profile of different portfolios. This is where modern portfolio theory (MPT) comes into play, providing a framework for investors to allocate capital across various assets and construct optimal portfolios [1-2]. Since investors face great uncertainty in selecting stocks, risk preferences depend on each individual investor and are usually represented in modern portfolio theory by setting constraints. Therefore, figuring out how to balance high returns with low risks is definitely a key area of focus in financial research.

Over the decade many researchers have conducted research on portfolio and diversification. For instance, Fischer and Robert introduced the global CAPM balance model and discussed how to apply the global asset allocation model to the actual investment decision to solve the two major problems in the traditional model that the expected return is hard to estimate, and the optimal portfolios are too sensitive to expected return assumptions [3]. Philippe took the international bond portfolio as an example; a simulation method is proposed to evaluate the reliability of mean-variance analysis in portfolio optimization by making a comparison between the performance of the ex-post optimal portfolio and the world bond index and the US bond index [4]. Myles presented Markowitz's contributions to modern portfolio theory from a simple perspective and provided an in-depth look at the complex mathematical/statistical models commonly associated with discussions of the theory [5]. Niranjan used the Sharpe single index model to construct the optimal portfolio and compared it with Markowitz's mean-variance model [6]. However, most of the previous studies compared the differences between the two models in theory, but rarely applied the Markowitz model and the index model to select assets and construct portfolios for optimal planning simulation.

Given the description mentioned above, this article aims to carry out a real-world examination of the contrast between improving the Markowitz investment portfolio and enhancing the Index investment portfolio. Based on the stock assets of the US stock market, this paper will go through the process of data processing, model building, and data visualization, and compare the effects of the two models in optimizing the portfolio with the specific result data.

2. Data

The research collated approximately two decades of S&P 500 returns, spanning from May 1, 2004, to June 1, 2024. This dataset was augmented with data for six individual stocks, each drawn from three distinct sectors, as well as an instrument representing the risk-free rate, the 5-year treasury yield. The data were sourced from Yahoo Finance. The S&P 500 index was chosen as a proxy for market performance due to its status as a comprehensive, diversified, and universally acknowledged indicator. The subsequent Table 1 presents the selected six stocks and their respective details.

Table 1: Stock ticker symbols in portfolios

	Portfolio	Full name	Sector
1	AAPL	Apple Inc.	Technology
2	NVDA	NVIDIA Corporation	Technology
3	AMZ	Amazon.com,Inc	Consumer cyclical
4	F	Ford Motor Company	Consumer cyclical
5	BAC	Bank of American Corporation	Financial service
6	JPM	JPMorgan Chase & Co.	Financial service

Then the daily data were consolidated into monthly observations to mitigate the influence of non-Gaussian effects and reduce the time of calculation. The monthly return was computed by difference return.

Additionally, the residual return was calculated, which equates to the return minus the sum of the risk-free rate the product of the beta coefficient, and the difference between the market return and the risk-free rate. Subsequently, the annual average return, annual standard deviation, beta, and alpha were computed with the Excel-inserted functions based on the excess return, and the residual standard deviation was derived from the residual return. With these computations in place, the correlation matrix among the six stocks and the index was calculated using an Excel function.

The Table 2 shows that Apple and NVIDIA have the highest annual average return during the 20 years, at 34.01% and 41.85% respectively. The return volatility of all stocks is greater than the volatility of the Sharpe index, and the largest volatility is for NVIDIA and Ford.

BAC and JPM have the strongest correlation with the Sharpe index, which may mean that the stock performance of financial services companies is closely related to the performance of the market. Meanwhile, there is a strong positive correlation between Bank of America Corporation (BAC) and JPMorgan Chase & Co. (JPM) with a correlation coefficient of 0.804329. This means that when BAC's stock price rises, JPM's stock price is likely to follow suit, and vice versa (See Table 3).

Table 2: Descriptive statistics of the seven assets

	SPX	AAPL	NVDA	AMZN	F	BAC	JPM
Annual Average Return	6.57%	34.01%	41.85%	25.85%	10.30%	8.21%	12.62%
Annual StDev	14.99%	32.65%	48.83%	36.68%	48.59%	40.56%	26.92%
Beta	1	1.24	1.78	1.23	1.78	1.72	1.20

Table 3: Correlation matrix of the seven assets

	SPX	AAPL	NVDA	AMZN	F	BAC	JPM
SPX	1	0.5678	0.5450	0.5045	0.5493	0.6345	0.6693
AAPL	0.5678	1	0.4350	0.3795	0.2660	0.2457	0.2193
NVDA	0.5450	0.4350	1	0.3758	0.2836	0.2341	0.2366
AMZN	0.5045	0.3795	0.3758	1	0.2359	0.1573	0.2003
F	0.5493	0.2660	0.2836	0.2359	1	0.4533	0.4875
BAC	0.6345	0.2457	0.2341	0.1273	0.4533	1	0.8043
JPM	0.6693	0.2193	0.2366	0.2003	0.4875	0.8043	1

3.   Method

In this section, there will be a delineation of two methodologies employed in venture capital: the Markowitz approach and the Index model. These frameworks facilitate the assembly of diverse investment portfolios, catering to the varying objectives of investors, including the construction of minimum variance portfolios and maximum Sharpe ratio portfolios.

The following key concepts are used in the model:

Expected value of asset return: Investors' expectations of future returns.

Variance and standard deviation of asset returns: A statistical measure of asset risk; the smaller the variance, the lower the risk.

Correlation between assets: correlation between returns of different assets, low correlation can diversify risk.

Efficient frontier of the portfolio: The frontier formed by all possible portfolios in the risk-return plane, representing the maximum possible return for different levels of risk.

Capital Market Line (CML): denotes the existence of a portfolio between the market portfolio and the risk-free asset that provides the highest expected return for each level of risk [7].

Sharpe Ratio: Measures the ratio of the excess return (the portion of the portfolio that exceeds the risk-free rate) to the overall risk (usually expressed in standard deviation).

3.1.   Markowitz Model

Harry Markowitz introduced what is known as the Markowitz model, or Modern Portfolio Theory (MPT), in the year 1952. The model provides a quantitative method for investors to construct and

evaluate portfolios with the aim of maximizing expected returns for a given level of risk or minimizing risk for a given expected return.

The core idea of the Markowitz model is to treat the risk and return of a portfolio as a weighted average of the risks of the assets in the entire portfolio. It hypothesizes that investors are risk-averse, in the sense that they tend to choose portfolios that are less risky for the same expected return [8-9].

Drawing upon the foundational assumptions, the Markowitz model is capable of accurately determining the desired outcomes for investors through the employment of two key equations:

$$E(R_p) = \sum_{i=1}^{n} wiE(Ri) \tag{1}$$

where:
E(Rp) is the expected return of this portfolio
wi is the weight of asset i
E(Ri) is the expected return of asset i

$$\sigma_p{}^2 = \sum_{i=1}^{n} w_i{}^2 \sigma_i{}^2 + \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} w_i \cdot w_j \cdot \sigma_{ij} \tag{2}$$

where:
$\sigma_p{}^2$ is the variance of the portfolio
wi is the weight of asset i
$\sigma_i{}^2$ is the variance of asset i
σ_{ij} is the covariance of asset i and asset j

The advantage of the Markowitz model lies in its comprehensive consideration of the risk and return of each asset in the portfolio, as well as their mutual relationship, so as to achieve risk diversification and return maximization more effectively. Compared with other models, it provides a more comprehensive and personalized framework, enabling investors to construct tailored optimal investment strategies according to their own risk preferences.

3.2. Index Model

Following the establishment of the Markowitz portfolio analysis model, its shortcomings became apparent. A significant limitation is the complexity and unwieldiness of its computation, which is further encumbered by the extensive number of estimations required, thereby diminishing computational efficiency and precision. To address these issues, Professor William Sharpe developed an alternative analytical model that simplifies the analysis of security portfolios while maintaining user-friendliness. This model is the Single Index Mode, usually referred to as the Capital Asset Pricing Model (CAPM). It was developed independently in the 1960s by William Sharpe, John Lintner, and Jan Mossin. The model provides a simplified way to evaluate the expected return of a security or portfolio in comparison to its systematic risk [10].

The core idea of CAPM is that the expected return of a security or portfolio is related to the expected return of the market as a whole and is proportional to its sensitivity to market risk. This sensitivity is often referred to as beta (β), which is a way to measure the volatility of a security or portfolio relative to the entire market. The basic formula of CAPM is as follows:

$$E(R_i) = R_f + \beta_i \times [E(R_m) - R_f] \tag{3}$$

Where:
E(Ri) is the expected return on a security or portfolio.

Rf is the risk-free rate.

βi is the beta of the security or portfolio

$E(Rm)$ is the expected return of the market portfolio

The advantages of the index model for constructing an optimal portfolio lie in its simplicity, low cost, and wide diversification, which enables investors to track market performance with low fees and achieve stable returns over the long term. Compared with the complexity and the need for active management of the Markowitz model, the index model provides a more straightforward and easy-to-implement strategy for investors who seek average market performance and moderate risk appetite.

4. Result

In this research, five constraints are also added to obtain the portfolio's optimal solution in various cases. Using these five constraints, the policies and regulations of different national markets and the investment preferences of most investors in the real world can be well simulated. In this scenario, the parallel examination and the subsequent differentiation between the two models exhibit greater harmony and feasibility.

1. This extra optimisation constraint is intended to mimic the rules of Regulation T set by FINRA. The stockbroker is permitted to enable their clients to have positions, which at least 50% financed by the equity in the client's account:

$$\sum_{i=1}^{11} |w_i| \leq 2 \tag{4}$$

2. This accessional constraint is formulated to mimic certain predefined "box" constraints on the allocation of weights, as specified by the client.

$$|w_i| \leq 1, \ for \ \forall i \tag{5}$$

3. A "free" problem, devoid of any extra optimization restrictions, is presented to demonstrate the general shape of the feasible portfolio region and the efficient frontier.

4. The supplementary optimization restrain is purposefully crafted to replicate the standard limitations prevalent in the United States' mutual fund sector, which includes the ban on short selling for American open-ended mutual funds.

$$w_i \geq 0, for \ \forall i \tag{6}$$

5. Lastly, to investigate if incorporating the comprehensive index into the portfolio influences it favorably or unfavorably, it introduces another optimization constraint:

$$w_i = 0 \tag{7}$$

4.1. Markowitz Model

By using the programming solver function of Excel, each asset weight and return rate, standard deviation, and Sharpe ratio data of the optimal portfolio obtained by the minVar strategy and maxSharpe strategy under five different constraints are found (See Table 4).

Table 4: Minimum variance portfolio and maximum shape portfolio in MM under five constraints

Constraint 1	SPX	AAPL	NVDA	AMZN	F	BAC	JPM	return	stDev	sharpe
minVar	0.00%	8.20%	6.53%	13.95%	26.30%	31.69%	13.33%	16.12%	10.68%	1.51
maxSharpe	77.32%	0.00%	0.00%	0.00%	7.73%	49.54%	-34.59%	5.58%	5.83%	0.96
Constraint 2	SPX	AAPL	NVDA	AMZN	F	BAC	JPM	return	stDev	sharpe
minVar	0.00%	10.03%	0.52%	15.89%	27.96%	32.30%	13.30%	14.95%	10.11%	1.48
maxSharpe	80.08%	0.00%	0.00%	0.00%	8.38%	43.95%	-32.41%	5.64%	5.84%	0.97
Constraint 3	SPX	AAPL	NVDA	AMZN	F	BAC	JPM	return	stDev	sharpe
minVar	0.00%	8.20%	6.53%	13.95%	26.30%	31.69%	13.33%	16.12%	10.68%	1.51
maxSharpe	77.32%	0.00%	0.00%	0.00%	7.73%	49.54%	-34.59%	5.58%	5.83%	0.96
Constraint 4	SPX	AAPL	NVDA	AMZN	F	BAC	JPM	return	stDev	sharpe
minVar	0.00%	8.21%	6.53%	13.97%	26.32%	31.55%	13.42%	16.13%	10.69%	1.51
maxSharpe	74.75%	0.00%	0.00%	0.00%	5.32%	19.92%	0.00%	7.10%	6.41%	1.11
Constraint 5	SPX	AAPL	NVDA	AMZN	F	BAC	JPM	return	stDev	sharpe
minVar	0.00%	8.20%	6.53%	13.95%	26.30%	31.69%	13.33%	16.12%	10.68%	1.51
maxSharpe	0.00%	0.00%	0.00%	3.13%	31.18%	97.77%	-32.08%	8.00%	7.23%	1.11

The information presented in the table leads to the following conclusion:

1. In the minVar strategy in all constraints, the weight of the SPX index is zero. However, in the max Sharpe strategy, the weight of SPX is the highest under all constraints (except the fifth constraint, which requires SPX weight to be 0), indicating that it is a very effective asset in terms of risk-adjusted return and can significantly improve the Sharpe ratio of the portfolio

2. Under the condition of allowing short selling, the weight of JPM in the max Sharpe strategy is in the range of -34.59% to -32.08%, which means that the price of JPM is expected to decline and the operation of shorting JPM is taken

3. In the minVar strategy under all constraints, the weight of BAC is the highest (31.55%-32.3%), and the weight of F is the second highest (26.3%-27.69%). By checking the correlation coefficient

matrix, it is found that this is because the correlation between these two stocks and other assets is low. It can play a better role in diversifying risks in the portfolio, thus reducing the overall variance

4. The returns and standard deviations of the minVar strategy under the five constraints are relatively stable, with Sharpe ratios ranging from 147.85% to 150.94%. The returns and standard deviations of the max Sharpe strategy fluctuate widely, with Sharpe ratios ranging from 95.68% to 110.76%.

Then, the CAL was drawn by the return and standard deviation on constraint 3. Through running the solver table, a figure of the Minimal Variance Frontier, Efficient Frontier, and Inefficient Frontier of the portfolio under the five constraints was drawn (See Figure 1).
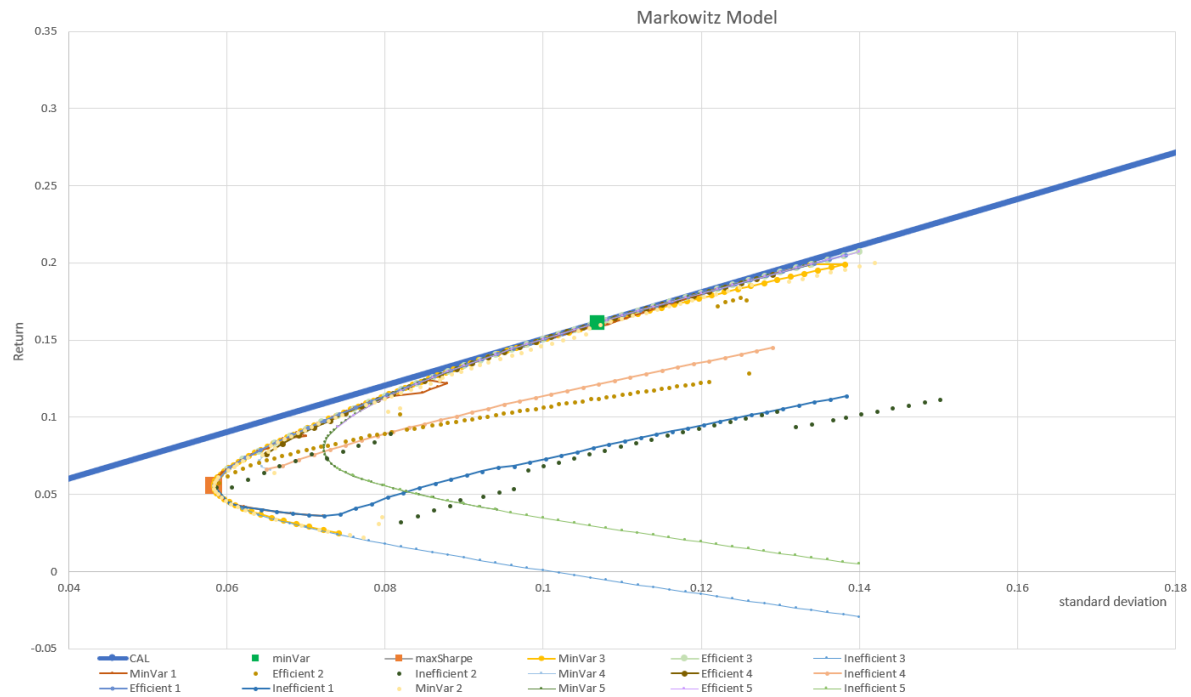

Figure 1: CAL and Minimal Variance Frontier, Efficient Frontier and Inefficient Frontier in MM under five constraints

4.2. Index Model

By using the programming solver function of Excel, each asset weight and return rate, standard deviation, and Sharpe ratio data of the optimal portfolio obtained by the minVar strategy and maxSharpe strategy under five different constraints are found.

The information presented in the table leads to the following conclusion:

1. SPX occupies the main weight in the minimum variance (minVar) strategy and the maximum Sharpe ratio (maxSharpe) strategy under most constraints, especially in the minVar strategy in constraints 1 to 4, the weight is close to or more than 95%. While the weight in the maxSharpe strategy is close to or more than 100%.

2. The weight of AAPL, NVDA and AMZN in constraint 5 increases significantly

3. JPM has a negative weight in the maxSharpe strategy under all constraints, especially in constraints 1 and 3 with a weight of -2.27% and in constraint 2 with a weight of -1.56%, showing short operation.

4. The return and standard deviation of minVar strategy under constraints 1 to 4 are relatively stable, with the return between 6.74% and 6.76%, the standard deviation around 6.71%, and the Sharpe ratio between 100.55% and 100.62%. The return and standard deviation in constraint 5

increase significantly, with a return of 10.26%, a standard deviation of 15.25%, and a Sharpe ratio of 67.30%.

The maxSharpe strategy has fewer volatile returns and standard deviations under constraints 1 to 4, with returns ranging from 6.53% to 6.63%, standard deviations ranging from 6.55% to 6.63%, and Sharpe ratios ranging from 99.63% to 100.01%. The return and standard deviation in constraint 5 also increase, and the return is 9.81%, the standard deviation is 14.91%, and the Sharpe ratio is 65.82% (See Table 5).

Table 5: Minimum variance portfolio and maximum shape portfolio in IM under five constraints

Constraint 1	SPX	AAPL	NVDA	AMZN	F	BAC	JPM	return	stDev	sharpe
minVar	95.43%	1.97%	0.93%	0.99%	0.00%	0.00%	0.69%	6.76%	6.71%	100.62%
maxSharpe	102.27%	0.00%	0.00%	0.00%	0.00%	0.00%	-2.27%	6.53%	6.55%	99.63%
Constraint 2	SPX	AAPL	NVDA	AMZN	F	BAC	JPM	return	stDev	sharpe
minVar	95.78%	1.97%	0.72%	0.41%	0.31%	0.12%	0.68%	6.74%	6.71%	100.55%
maxSharpe	100.00%	0.00%	0.72%	0.41%	0.31%	0.12%	-1.56%	6.63%	6.63%	100.01%
Constraint 3	SPX	AAPL	NVDA	AMZN	F	BAC	JPM	return	stDev	sharpe
minVar	95.43%	1.97%	0.93%	0.99%	0.00%	0.00%	0.69%	6.76%	6.71%	100.62%
maxSharpe	102.27%	0.00%	0.00%	0.00%	0.00%	0.00%	-2.27%	6.53%	6.55%	99.63%
Constraint 4	SPX	AAPL	NVDA	AMZN	F	BAC	JPM	return	stDev	sharpe
minVar	95.43%	1.97%	0.93%	0.99%	0.00%	0.00%	0.69%	6.76%	6.71%	100.62%
maxSharpe	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	6.57%	6.57%	100.00%
Constraint 5	SPX	AAPL	NVDA	AMZN	F	BAC	JPM	return	stDev	sharpe
minVar	0.00%	21.69%	11.81%	14.26%	8.70%	13.48%	30.06%	10.26%	15.25%	67.30%
maxSharpe	0.00%	20.61%	6.57%	15.05%	6.68%	11.30%	39.79%	9.81%	14.91%	65.82%

Then, the CAL was drawn by the return and standard deviation on constraint 3. Through running the solver table, a figure of the Minimal Variance Frontier, Efficient Frontier, and Inefficient Frontier of the portfolio under the five constraints was drawn (See Figure 2).
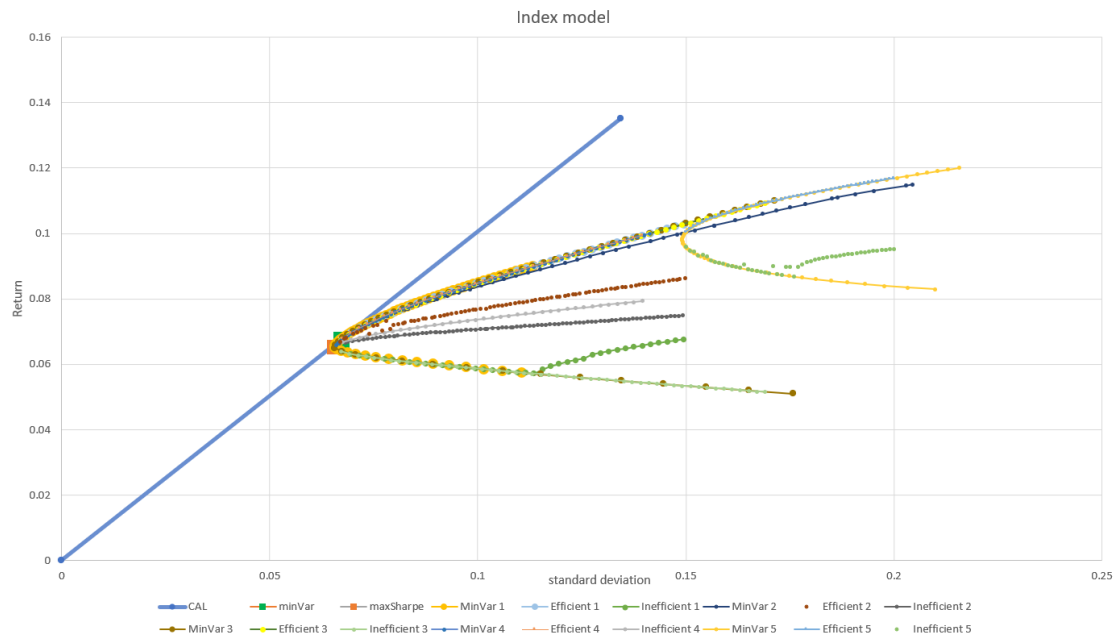
Figure 2: CAL and Minimal Variance Frontier, Efficient Frontier and Inefficient Frontier in IM under five constraints

By comparing the conclusions of using the Markowitz model and the index model to construct the optimal portfolio, this study finds that the weight distribution of different stocks is more balanced under the Markowitz model, especially under the minimum variance strategy, the effect of risk diversification is more significant. The index model, on the other hand, relies more on the weight allocation of a single stock, especially on the SPX, which is heavily weighted under most of the constraints, probably due to the high representativity and stability of the SPX in the market. Under both models, portfolio returns, and standard deviations fluctuate with different constraints, but in general, the Sharpe ratio is higher for the Markowitz model, indicating that its risk-adjusted returns are higher. However, the portfolio returns, and standard deviation of the index model are relatively stable, but they fluctuate greatly under constraint 5, and the Sharpe ratio is slightly lower than that of the Markowitz model.

5.   Conclusion

The results of this paper show that both Markowitz model and index model have advantages and disadvantages in portfolio optimization, and investors should pick the applicable model that consistent with their own risk preference and market environment. The Markowitz model is suitable for investors who pursue risk diversification and personalized investment strategies, while the index model is more suitable for investors who pursue average market returns and low costs. In addition, there are differences in the portfolio allocation and risk-return characteristics of the two models under different constraints, so investors should choose and adjust according to their own situation and market environment.

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