

The Construction of Investment Portfolios under the Minimization of Variance

—Taking the Hong Kong Insurance Market as an Example

Yunjia Wang^{1,a,*}

¹*Shanghai Lixin University of Accounting and Finance, Shanghai, China*

a. bigwolf1234@tzc.edu.cn

**corresponding author*

Abstract: In the context of a decline in the Hang Sheng Index, this paper intended to study the different effect between using Markowitz Model and Index Model to invest insurance industry's stock in Hongkong, especially for those who are risk-averse. A recent ten years of adj close for 9 typical insurance stocks from Hongkong as well as the Heng sheng index has been chosen as the target data. The result illustrates that there is no significant difference between these two models. This study intends to give some reference for those who focus on Hong Kong insurance industry's investment and hate taking risk. This study find that the constraint's minimal variance is the smallest at 16.832(The index model of constraint 2) and the constraint's highest sharp ratio is around 0.26(The index model of constraint 4). The findings of this paper provide the investors empirical evidence on how to construct optimal investment portfolio in Hong Kong insurance market.

Keywords: The Markowitz Model, Index Model, insurance industry.

1. Introduction

Heng Sheng index's adj. close is 23,340.05 in January 2th 2014 and 16,788.55 in January 2th 2024. Decreasing about 28%. However, the stock market of Hong Kong is developing positively recently. The reverse of Hong Kong Stocks is not the bounce but the beginning [1]. This indicates there are some opportunities for investors to get some money from the market. As for the insurance industry in Hong Kong, it is one of the most developed insurance markets in the world.[2] During the year 2023, insurance industry of Hong Kong showing an upward trend [2]. Therefore, it is beneficial for investors to focus on stocks of this field. In today's modern world, many investment decisions were put forward. Among those, Markowitz model and index model is famous. So, this paper in order to figure out which model is more suitable for the Hong Kong insurance market. What's more, different investors have different demands. For those who are keen on profits, studying maximal return is the most appropriate. For those who are risk-averse, studying minimal variance is the most suitable. For those who are risk-neutral, studying max sharp ratio is the most suitable. This paper studied how to construct minimal variance portfolio. In other words, this study focusses on those who are risk-averse, optimize investment portfolios for investors as well as digging the insurance markets' investment value. So this paper choose adj close for 9 typical insurance stocks from Hongkong as well as the

Heng sheng index from April 6th, 2014 to April 6th 2024 to study, finding which model is more suitable for those who are risk-averse.

Using Markowitz Model to match the best ratio of insurance funds investor in the securities market can find that there is some difference between the theoretical value and actual value, which due to the complexity of the security markets, but the theoretical value can be seen as the reference of actual value[3]. Utilizing the Markowitz Model, simulating the selected A shares on the Shenzhen Stock Exchange can find that the riskier the combination, the more diversified the investment, while the higher the return on investment, the fewer the number of stocks in the portfolio[4]. Gao et al. analyzed three representative stocks in China's traditional Chinese medicine securities market and concluded that if risk investors invest in traditional Chinese medicine stocks with known expected returns, they can construct an optimal investment portfolio using the Markowitz model to thereby reduce risk and achieve risk diversification[5]. Liu studied the single index model on A-share market and find that it has efficiency in investment application in the securities market and this model is relatively simple, suitable for most investors to easily understand and apply[6].

2. Method

2.1. Markowitz Model

In 1952, Harry Markowitz formally published portfolio selection model including decentralize principle. The core principle of the risk assert efficient frontier is that for any risk level, people would only focus on the highest expected return. This model is combined management's first step, confirming the risk assert efficient frontier. The second step is introducing risk-free assert, but before doing this, people should consider the constraints. The point where the capital allocation line intersects with the efficient frontier is the optimal risk portfolio [7].

2.2. Index Model

However, during the process of using MM model, investors find that there are some drawbacks. One is that this model require a lot of estimates to fill the covariance matrix. The other is that it do not offer any handbook to find the useful estimates of these covariances or risk premium. Therefore, the Index Model was introduced. It symbolize the estimates of covariance matrix as well as enhancing the analysis of security risk premiums. It was first proposed by Sharpe and provide insights into portfolio diversification[7].

2.3. Constraints

1. This additional optimization constraint is designed to simulate some arbitrary “box” constraints on weights, which may be provided by the client [8], $|w_i| \leq 1$, for $\forall i$

2. A “free” problem, without any additional optimization constraints, to illustrate how the area of permissible portfolios in general and the efficient frontier in particular look like if you have no constraints[8];

3. This additional optimization constraint is designed to simulate the typical limitations existing in the U.S. mutual fund industry: a U.S. open-ended mutual fund is not allowed to have any short positions, for details see the Investment Company Act of 1940, Section 12(a)(3)[8], $w_i \geq 0$, for $\forall i$

4. Lastly, we would like to see if the inclusion of the broad index into our portfolio has positive or negative effect, for that we would like to consider an additional optimization constraint[8], $w_I = 0$

2.4. Data source

The stock data comes from Yahoo Finance. This paper chooses the historical close price as 3 month Hong Kong government bond yield as risk free rate, originating from investing.com. Assume there is 252 trading days per year.

This paper also analysis how the effective frontier changed if add constant constraints, thus helping the result to become closer to the real world.

2.5. Sample

The wind global industry classification system has the features that conform to international industry classification and adapt to China security study. The industry classification it do is the system which was launched by MSCI and S&P. Besides, it has the most widespread user of the security market. Therefore, this essay pick the top ten stocks(except the Sunshine Insurance Group Company Limited, China Reinsurance (Group) Corporation and ZhongAn Online P & C Insurance Co., Ltd. for the sake of its short listing date) in the second Hong Kong insurance industry which was proposed by Wind as well as the HIS. As to make the appear more convenient, this paper assigning abbreviations to all stocks: designing HANG SENG INDEX as HSI, Ping An Insurance (Group) Company of China, Ltd. as Ping, AIA Group Limited as AIA, China Life Insurance Company Limited as Life, PICC Property and Casualty Company Limited as PICC, Prudential plc as Chen, China Pacific Insurance (Group) Co., Ltd. as TaiB, The People's Insurance Company (Group) of China Limited as People's, New China Life Insurance Company Ltd. as Xin, Yunfeng Financial Group Limited as Yun, China Taiping Insurance Holdings Company Limited as TaiP.

3. Results

3.1. correlation analysis

Firstly, after acquiring the data, using workday and vlookup function to get the 5-week daily data during the specific ten years. Then calculate the notional free rate, starting with 1000. Thereafter, perform screening on these data by using judgmental statement. This step's purpose it to transform the daily data to the monthly data. And then calculate all the returns(divided by previous one minus one) as well as excess return(return minus risk free rate). After that, all the preliminary preparation for calculating the statistical data has been done. So it is able to get the table 1.

Table 1: return and risk performance of HIS and 10 stocks

	HSI	Ping	AIA	Life	PICC	Chen	TaiB	People's	Xin	Yun	TaiP
Annualized Average Return	-1.98%	7.00%	6.91%	-1.79%	10.69%	-4.56%	1.11%	3.07%	3.32%	4.03%	0.94%
Annualized StDev	20.38%	33.53%	25.01%	32.17%	27.75%	31.00%	32.42%	26.31%	37.46%	63.00%	38.68%
beta	100.00%	137.04%	97.79%	123.15%	65.59%	96.75%	113.70%	86.55%	128.51%	107.73%	136.74%
alpha	0.00%	9.71%	8.84%	0.65%	11.99%	-2.64%	3.36%	4.78%	5.86%	6.17%	3.64%
residual Stdev	0.00%	18.57%	15.11%	20.13%	24.32%	23.92%	22.68%	19.52%	26.78%	59.05%	26.83%

The correlation metric is the table 2. The most relevant ones are Ping and HSI, at 83.27%. This may be due to the fact that this company is one of the industry who has the most comprehensive

financial license and the widest range of service. However, the least relevant ones are Yun and Chen, at 18.5%. this may be due to Yun is an independent investment company and Chen is a big global finance serving industry. Their property is not very similar.

Table 2: correlations

correlations	HSI	Ping	AIA	Life	PICC	Chen	TaiB	People's	Xin	Yun	TaiP
HSI	100.00%	83.27%	79.68%	78.00%	48.16%	63.60%	71.46%	67.03%	69.91%	34.84%	72.03%
Ping	83.27%	100.00%	65.73%	80.49%	44.99%	56.11%	80.99%	68.75%	76.89%	27.50%	77.56%
AIA	79.68%	65.73%	100.00%	63.05%	41.75%	67.24%	56.27%	49.54%	56.58%	25.30%	56.62%
Life	78.00%	80.49%	63.05%	100.00%	55.82%	58.08%	76.96%	75.29%	81.74%	25.09%	80.02%
PICC	48.16%	44.99%	41.75%	55.82%	100.00%	37.90%	49.39%	72.37%	55.58%	18.74%	49.32%
Chen	63.60%	56.11%	67.24%	58.08%	37.90%	100.00%	46.34%	42.06%	50.45%	18.50%	62.73%
TaiB	71.46%	80.99%	56.27%	76.96%	49.39%	46.34%	100.00%	63.60%	75.66%	23.64%	74.69%
People's	67.03%	68.75%	49.54%	75.29%	72.37%	42.06%	63.60%	100.00%	70.52%	24.87%	68.04%
Xin	69.91%	76.89%	56.58%	81.74%	55.58%	50.45%	75.66%	70.52%	100.00%	27.14%	80.83%
Yun	34.84%	27.50%	25.30%	25.09%	18.74%	18.50%	23.64%	24.87%	27.14%	100.00%	25.65%
TaiP	72.03%	77.56%	56.62%	80.02%	49.32%	62.73%	74.69%	68.04%	80.83%	25.65%	100.00%

3.2. results of portfolio

All the preliminary preparation for calculating the Markowitz Model and Index Model has been done. the result of Markowitz Model is shown in table 3.

Table 3: the result of Markowitz Model

	HSI	Ping	AIA	Life	PICC	Chen	TaiB	People's	Xin	Yun	TaiP	Return	StDev	Sharp
weights	15.0%	4.9%	9.0%	-0.8%	-11.9%	46.5%	23.9%	3.0%	-10.0%	5.3%	15.1%	-2.3%	26.7%	-8.7%

The result of Index Model is shown in table 4.

Table 4: the result of Index Model

	HSI	Ping	AIA	Life	PICC	Chen	TaiB	People's	Xin	Yun	TaiP	Return	StDev	Sharp
weights	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.5	1.9%	28.4%	0.065

Standard deviation in a particular portfolio is a measure of how spread out or volatile the returns, helping the investor to understand the level of risk people are faced. In other words, it calculate the money which investor may gain or lose beyond what their expect[3]. For comparing the two model,

this essay calculates each of the minimal variance after adding various constraints as to provide investment advice for risk-averse individuals.

Constraint 1's computation is table 5 and table 6. By using solver instrument inside the Excel. Setting two constraints, one is all the weights are less than or equal to one, the other is all the weights are bigger than or equal to -1. In constraint 1, the Markowitz Model is better as it has higher sharp ratio.

Table 5: results of the Markowitz Model under constraint 1

HSI	Ping	AIA	Life	PICC	Chen	TaiB	People's	Xin	Yun	TaiP	Return	StDev	Sharp
100.0%	- 30.4%	1.6%	-16.5%	14.6%	10.7%	14.9%	24.3%	-4.8%	- 0.7%	-13.7%	-2.0%	17.3%	-11.8%

Table 6: results of the Index Model under constraint 1

	HSI	Ping	AIA	Life	PICC	Chen	TaiB	People's	Xin	Yun	TaiP	Return	StDev	Sharp
weights	-0.00	-1	-1	0.84	-1	0.79	0.98	0.99	0.40	-0.99	0.9	-27.3%	89.5%	-0.30

Constraint 2's computation is table 7 and table 8. It's a free problem without any additional optimization constraints. So the solver Parameters' subject to the constraints column don't need to be placed any thing. In constraint 2, the Markowitz Model is better as it has higher sharp ratio.

Table 7: results of the Markowitz Model under constraint 2

	HSI	Ping	AIA	Life	PICC	Chen	TaiB	People's	Xin	Yun	TaiP	Return	StDev	Sharp
weights	108.7%	-32.0%	-2.0%	-17.0%	14.4%	10.0%	14.2%	22.5%	-4.2%	-1.0%	-13.4%	-2.6%	17.3%	-15.1%

Table 8: results of the Index Model under constraint 2

	HSI	Ping	AIA	Life	PICC	Chen	TaiB	People's	Xin	Yun	TaiP	Return	StDev	Sharp
weights	1.4967 92	- 0.304 42	0.0274 46	- 0.161 88	0.1648 78	0.0161 15	- 0.075 45	0.1000 22	- 0.112 62	- 0.006 28	- 0.144 6	- 3.236 %	16.83 2%	- 0.192 27

Constraint 3's computation is table 9 and table 10. That is add a constraint that all the weights are bigger or equal to zero. In constraint 3, the Index Model is better as it has higher sharp ratio.

Table 9: results of the Markowitz Model under constraint 3

	HSI	Ping	AIA	Life	PICC	Chen	TaiB	People's	Xin	Yun	TaiP	Return	StDev	Sharp
weights	86.7%	0.0%	0.0%	0.0%	20.6%	9.4%	0.0%	10.6%	0.0%	0.0%	-27.3%	0.1%	18.4%	0.7%

Table 10: results of the Index Model under constraint 3

	HSI	Ping	AIA	Life	PICC	Chen	TaiB	People's	Xin	Yun	TaiP	Return	StDev	Sharp
weights	0.7969	0	0.034	0	0.20	0.02	0	0.12	0	0	-0.179	0.96%	18.7%	0.051

Constraint 4's computation is table 11 and table 12. The HSI's weight need to be set as 0. In constraint 4, the Index Model is better as it has higher sharp ratio.

Table 11: results of the Markowitz Model under constraint 4

	HSI	Ping	AIA	Life	PICC	Chen	TaiB	People's	Xin	Yun	TaiP	Return	StDev	Sharp
weights	0.0%	- 11.8%	43.7%	- 10.6%	17.0%	18.4%	22.5%	45.1%	- 11.0%	3.0%	- 16.1%	4.6%	20.3%	22.6%

Table 12: results of the Markowitz Model under constraint 4

	HSI	Ping	AIA	Life	PICC	Chen	TaiB	People's	Xin	Yun	TaiP	Return	StDev	Sharp
weights	0	- 0.126 85	0.359 51	0.0019 61	0.3134 27	0.1493 24	0.0604 84	0.3100 11	- 0.022 85	0.0144 02	- 0.059 42	5.208 %	19.889 %	0.2618 52

4. Conclusions

All constraints' expected value changed as the risk increase or decrease, and it partly verify a conclusion that if investor takes higher risk, his potential return may be higher as well. The minimal variance of the constraints is not equal to each other. The Markowitz Model constraint 1 and constraint 2's maximum sharp is higher than the Index model, indicating more wonderful ability of risk-adjusted return. The Index Model constraint 3 and constraint 4's maximum sharp is higher than the Index model, indicating more wonderful ability of risk-adjusted return. For the free constraint, Sharp ratio is higher than 1, illustrating the invest return is higher than risk free assert. At that case, it is likely that the Hong Kong insurance industry has the value of investment. All the constraint result of the minimal variance of the Markowitz Model and Index Model are similar to each other. Among all the results of Markowitz Model, the constraint 's minimal variance is the smallest at 17.3%. Among all the results, the constraint 's minimal variance is the smallest at 16.832(The index model of constraint 2) and its investment portfolio is relatively diversified compared to others. Among all the results, the constraint's highest sharp ratio is around 0.26(The index model of constraint 4), indicating that it may be better for the investors to avoid investing the Heng Sheng Index.

References

- [1] Shang Heng. The reversal of Hong Kong stocks has begun[J]. *Stock Market Dynamic Analysis*, 2024(05):29-30.
- [2] Mei Xuesong. "Pearl of the Orient" shines with new features- China Hong Kong's insurance industry under the new development pattern of "dual circulation" [J]. *The Banker*, 2023(12):78-79.
- [3] Wang Ying. *Empirical Study on the Investment of Insurance Funds in the Securities Market [D]*. Northese University of Finance and Economics, 2012.
- [4] Wu Weili, Zhao Liuyue. Research and Empirical Analysis of the Optimal Investment Portfolio of Shenzhen Stock Exchange Based on the Markowitz Investment Portfolio Model[J]. *Journal of Xuzhou Institute of Technology (Natural Science Edition)*, 2023, 38(02): 17-28. DOI: 10.15873/j.cnki.jxit.000511.

- [5] Gao Xiaoying, Huang Ying, Guo Simin, et al. *Analysis of the Traditional Chinese Medicine Stock Portfolio Based on the Markowitz Model—A Case Study of Pien Tze Huang, CR Sanjiu, and Yunnan Baiyao*[J]. *Modern Business*, 2020(35): 82-84. DOI: 10.14097/j.cnki.5392/2020.35.028.
- [6] Liu Minyue. *Value Analysis of an Optimal Investment Portfolio Based on the Single Index Model*[J]. *China Price*, 2022(10): 93-96.
- [7] Bodie Z. Kane A. Marcus A.J. 's *Investments* (10th edition), China Machine press, 2018.
- [8] Lin Q .*Constrained Portfolio Optimization:A Comparison of Markowitz Model and Single Index Model*[C]//Department of Economics, King's Business School, King's College London,Cardiff Business School, Business School.Proceedings of the 2nd International Conference on Business and Policy Studies(part13).Business School,Soochow University;,2023:11.DOI:10.26914/c.cnkihy.2023.096181.