Proceedings of the 8th International Conference on Economic Management and Green Development
DOI: 10.54254/2754-1169/2024.17736

Application of GARCH Model in the Field of Finance

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Abstract: This paper reviews the fundamental principles, mathematical formulation, advantages, and disadvantages of the GARCH model and its extensive applications in finance. Initially, this paper introduces the mathematical structure and fundamental properties of the GARCH model and analyzes the advantages and disadvantages of the model in practical applications. Furthermore, this paper applies the GARCH model in financial market forecasting, risk management, and asset pricing, illustrating these through specific cases. In particular, the GARCH model is employed to forecast the volatility of the electricity and stock markets, demonstrating its efficacy in handling high volatility and time series data. In risk management, the GARCH model assists investors in assessing potential losses and formulating risk management strategies by predicting future volatility. Additionally, the GARCH model and its variants enhance the precision of asset pricing by considering the skewness and kurtosis of the stock market. Furthermore, the current research frontiers and future development trends are also discussed. Research indicates that future enhancements to the GARCH model will primarily concentrate on capturing nonlinear market characteristics, developing multivariate GARCH models, and incorporating long memory characteristics into the model. As blockchain technology develops and cryptocurrency markets expand, it is anticipated that the GARCH model will become increasingly prevalent in these emerging markets. The integration of machine learning and artificial intelligence will further enhance the precision and reliability of the GARCH model, providing more accurate and robust tools for financial market analysis and forecasting.

Keywords: GARCH model, Market forecast, Risk management, Asset pricing.

1. Introduction

One of the most salient features of financial markets is their volatility, which is difficult to be effectively captured by traditional time series models. As markets develop, investors and researchers require models that can accurately describe and predict market fluctuations. Bollerslev proposed an important generalization of the ARCH model, called the GARCH model, which is used to deal with heteroscedasticity in time series data, thus improving the application scope and effect of the model [1]. This paper aims to review the rationale, mathematical expression, strengths, and weaknesses of the GARCH model and their wide application in finance using a literature review and case study. Firstly, the existing literature is sorted out and analyzed, and the basic principles, mathematical expression, and advantages and disadvantages of the GARCH model are introduced. Secondly, specific cases are analyzed to show the practical application of GARCH models in financial market

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forecasting, risk management, and asset pricing. Finally, the current research hotspots and future development trends are considered in order to identify the improvement direction and application prospect of the GARCH model. By reviewing and analyzing the theory and application of the GARCH model, researchers and practitioners can gain a deeper understanding of the model and improve the accuracy of financial market analysis and prediction. Furthermore, this paper discusses the improvement direction and future application prospect of the GARCH model, which provides a reference point for subsequent research.

2. The Basis of the GARCH Model

2.1. BasicPrinciples of the Model

The volatility characteristics of time series are mainly manifested in three aspects. Volatility Clustering: The volatility of financial time series is not uniformly distributed but exhibits clustering phenomena, with periods of high volatility and low volatility alternating [2]. Leverage Effect: In the stock market, there is a negative correlation between volatility and returns, meaning that volatility increases when the market falls and decreases when the market rises [3]. The GARCH model is a generalized autoregressive conditional heteroskedasticity model, proposed by Bollerslevbased on the ARCH model [1]. The fundamental concept is to utilize past error terms and past variances to predict the current variance, which allows for better capture of the volatility characteristics of time series data.

2.2. Mathematical Expression of the Model

The general form of the model is GARCH(p, q), where p represents the past p variance terms and q represents the past q residual terms. The model is expressed as follows:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} a_i \epsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{1}$$

$$y_t = \mu + \epsilon_t \tag{2}$$

$$\epsilon_t = \sigma_t z_t \tag{3}$$

The time series value is represented by y_t, while the mean is denoted by μ. The residual, ϵ_t, is the error term, and the conditional standard deviation is represented by σ_t. Finally, z_t is an independent and identically distributed standard normal random variable. The model parameters are α_0, α_i, and β_j, while α_0 represents the underlying volatility in the absence of any shocks and the influence of past volatility. It is the minimum of the conditional variance. The parameter α_i represents the effect of the squared term of the past residuals on the current conditional variance. A larger value of α_i indicates that past shocks have a greater effect on the current volatility. The parameter β_j represents the effect of the past conditional variance on the current conditional variance. A larger value of β_j indicates that the past volatility level has a greater effect on the persistence of the current volatility.

3. Analysis of Advantages and Disadvantages of GARCH Model

3.1. Advantages of the Model

The term $\sum_{i=1}^{q} a_i \epsilon_{t-i}^2$ of above formulas demonstrate that the current conditional variances σ_t^2 are depend on the prophase residual square ϵ_{t-i}^2. This dependence reflects the phenomenon of volatility clustering in the time series, which is defined as the alternation of periods of high and low volatility. The term $\beta_j \sigma_{t-j}^2$ was previously defined as the current conditional variance of σ_t^2, which is itself a

DOI: 10.54254/2754-1169/2024.17736

function of the early conditional variance σ_{t-j}^2. The introduction of this term enables the GARCH model to reflect the characteristics of volatility over time and effectively address the heteroscedasticity problem. The incorporation of the current squared residuals, ϵ_{t-i}^2 , into the prediction formula enables the model to respond expeditiously to novel information and market shocks, thereby enhancing the sensitivity and precision of the prediction. At the same time, the GARCH model can address the common phenomenon of fat-tailed distribution with high kurtosis in financial time series. This phenomenon implies that the probability of extreme values is higher than that predicted by the normal distribution, thereby enhancing the accuracy of risk assessment [4]. The model is founded upon robust statistical theory and is equipped with a comprehensive array of methods for parameter estimation and model verification. These include the maximum likelihood estimation method, which ensures the scientific and reliable nature of the model.

3.2. Disadvantages of the Model

The parameters to be estimated in the GARCH model include the underlying volatility, the influence of the squared term of the past residual on the current conditional variance, and the influence of the past conditional variance on the current conditional variance. These parameters are typically estimated using the maximum likelihood estimation method, which entails a more complex calculation process, particularly when the data size is large or the model order is high. Moreover, the parameter estimation process is sensitive to the initial values, and different initial values may lead to different convergence results. In order to obtain stable and accurate estimation results, many iterations and debugging are usually required. According to the formula, the computational complexity is high because the conditional variance calculation at each step depends on the previous data. This means that when dealing with large-scale data sets, the amount of computation increases significantly as the size of the data increases. Moreover, although the GARCH model is capable of addressing heteroscedasticity, it is inadequate for handling complex nonlinear features in time series. Consequently, it is necessary to integrate other models , such as EGARCH and TGARCH, or implement model enhancements [5, 6]. In certain instances, such as during extreme market conditions or unexpected events, the predictive capacity of the GARCH model may be constrained.

4. Application of GARCH Model in the Field of Finance

4.1. Market Forecast

In their 2005 study, Garcia et al. employed a GARCH model to predict the day-ahead electricity price in Spain and California [7]. Their findings indicate that GARCH models are effective in addressing high volatility and time series data. The GARCH model was demonstrated to effectively capture price fluctuations and spikes in the electricity market. Specifically, the model forecasts future prices by considering the conditional variance of the price series, which is better able to handle the volatility and heteroscedasticity of the data than the traditional ARIMA model. The results demonstrate that the average prediction error of the GARCH model in the electricity market is low, particularly during periods of severe price fluctuations [7]. Furthermore, the study by Garcia et al. also demonstrates the adaptability and flexibility of the GARCH model in diverse market environments. Utilizing an empirical analysis of electricity markets in Spain and California, the study validates the efficacy of the GARCH model under varying market structures and volatility characteristics. This attribute renders the GARCH model not only a viable approach for electricity market prediction but also provides a theoretical and practical foundation for its extensive applicability in financial markets [7].

The GARCH model is employed to forecast the volatility of the stock market and to elucidate the underlying causes of stock return fluctuations. This model can more accurately predict future

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volatility by considering past volatility and residuals. The volatility of the stock market is typically characterized by periods of high volatility and low volatility, which persist for an extended period. Based on the empirical analysis of NVIDIA stock, the study finds that the GARCH model can effectively capture and predict its volatility. The predictive ability of this model is verified in Monte Carlo simulation, and the simulation results demonstrate that the model can replicate the observed volatility clustering phenomenon [8]. GARCH model permits investors to conduct scenario analysis and stress testing, enabling them to assess the level of risk under different market conditions.

4.2. Risk Management

The return rate of NVIDIA stock exhibits a significant volatility clustering phenomenon, where volatility increases during periods of market turbulence and decreases during periods of market stability, providing empirical evidence supporting the application of the GARCH model [8]. The GARCH(1,1) model is employed to forecast the volatility of NVIDIA stock, and it is evident that the model is capable of fitting the data well and is utilized to predict future conditional variance (volatility). The GARCH model is employed to forecast future volatility and calculate Value-at-Risk(VaR) and expected loss(EL) to assist investors in assessing potential losses under extreme market conditions and in developing more effective risk management strategies.

Marchese et al. underscored the significance of multivariate GARCH models in anticipating fluctuations and correlations in the prices of crude oil and refined oil products. In particular, the authors compared short-memory and long-memory GARCH models, demonstrating the substantial advantages of the latter in risk management [9]. The multivariate GARCH model employing long-memory characteristics has been demonstrated to yield more accurate predictions of the conditional covariance matrix and associated risk quantities, particularly in volatility forecasting and VaR calculation. The study revealed the existence of significant long-memory decay and leverage effects in volatility, as well as time-varying autocorrelation, by analyzing spot return data for West Texas Intermediate crude and two major refined oil products, conventional gasoline and heating oil in New York Harbor. The long-memory multivariate GARCH model demonstrated superior performance in dealing with these complex volatility and correlation dynamics, significantly outperforming the traditional short-memory model [9]. To assess the predictive accuracy of the model, the study employed the Superior Predictive Ability test and Model Confidence Set method. The results demonstrated that the long-memory GARCH model exhibited superior predictive ability in several predictive indicators. This evidence supports the potential of the GARCH model for practical application, particularly in risk management operations within complex financial market environments. The model's ability to provide more accurate volatility prediction and risk assessment suggests its suitability for risk management in such environments. This study demonstrates the effective application of the GARCH model and its variants in real financial risk management. The scientific nature and reliability of risk management are enhanced through more accurate volatility and risk forecasts. This has important implications for refiners, physical oil traders, non-commercial oil traders, and other energy market participants in their hedging and risk management operations.

4.3. Asset Pricing

This study examines the application of the GARCH model and its variants, including EGARCH, IGARCH, etc., in different indices of the Kuala Lumpur Stock Exchange, Malaysia [10]. Additionally, it explores the application of these models in asset pricing. The study demonstrates that although the EGARCH model is not the optimal choice in terms of goodness-of-fit statistics, it exhibits superior performance in describing the skewness commonly observed in stock market indices, as well as in out-of-sample, one-step forecasting, scenarios. In contrast, the IGARCH model demonstrates the least

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favorable performance in both of these respects. Additionally, the empirical distribution of stock returns has been found to deviate significantly from the normal distribution, exhibiting high kurtosis and skewness. Although the GARCH model effectively addresses the issue of high kurtosis, it does not adequately address skewness. In contrast, the EGARCH model effectively addresses this issue by considering the distribution of symmetry. When using the maximum likelihood estimation method for joint estimation of the model parameters and variances, the GARCH-M model performs the best in terms of goodness-of-fit statistics. However, the EGARCH model performs better in terms of out-of-sample forecasting. The results indicate that the EGARCH model is the optimal choice for asset pricing due to its superior performance in addressing skewness and out-of-sample forecasting. Conversely, the IGARCH model is not a suitable option for forecasting due to its limitations.

5. Research Frontiers and Future Prospects

5.1. Current Research Trends

The research also combines GARCH models, such as EGARCH and TGARCH, to capture market characteristics. Furthermore, it entails systematically comparing the performance of various GARCH models across diverse market conditions, to identify the optimal model for a specific scenario. For example, a variety of GARCH models are compared to study their performance in oil price volatility and VaR prediction [11]. GARCH models and their variants, such as FIGARCH and HYGARCH, can capture long memory and asymmetric effects in financial time series, thereby providing a more accurate description of oil price volatility [11]. A novel hybrid memory GARCH (MMGARCH) model is proposed, which combines GARCH and FIGARCH to enhance forecasting performance by dynamically adjusting the volatility level and memory structure [11]. The findings indicate that the MMGARCH model performs better than other GARCH variants in forecasting oil price volatility and calculating VaR, particularly in managing varying volatility levels and shock persistence. By accurately anticipating oil price volatility, MMGARCH model can assist companies and investors in more effectively navigating risks and mitigating potential losses in extreme market conditions.

5.2. Expectations for Future Development

The improvement direction of the GARCH model primarily concerns the nonlinear feature capture, the application of the multivariate GARCH model, and the introduction of long-memory features into the market. Specifically, future research will focus on developing methodologies for more effectively capturing nonlinear features in the market, developing multivariate GARCH models that can accommodate volatility and correlation across multiple assets, and introducing long-memory features into GARCH models in a manner that is both efficient and accurate. Additionally, GARCH models have the potential for novel applications and technological integration. The development of blockchain technology and the rise of the cryptocurrency market will result in a greater prevalence of the GARCH model in this emerging market. Furthermore, the integration of GARCH models with machine learning and artificial intelligence technologies may enhance the model's intelligence and automation, while the study of the impact of climate change and environmental risks on financial markets in environmental finance represents another potential avenue for future research. There is still considerable scope for developing GARCH models in high-frequency data applications, model improvements, and applications in emerging fields, and it is anticipated that further breakthroughs and innovations will emerge in the future.

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6. Conclusion

This literature review examines the application of GARCH model in finance, including, but not limited to, market forecasting, risk management, and asset pricing. GARCH model is effective in capturing the volatility characteristics of financial time series, such as volatility clustering, sharp peaks and fat tails, and leverage effects. Although the GARCH model effectively addresses the issue of high kurtosis, it is insufficient in handling skewness. It is recommended that the GARCH model algorithm be optimized to reduce computational complexity, enhance the processing efficiency of large-scale datasets, and guarantee the stability of parameter estimation. Additionally, further investigation into the application of the multivariate GARCH model is necessary to enhance its applicability in complex financial markets. Furthermore, the integration of machine learning and artificial intelligence technology is essential to elevate the level of model intelligence and automation. Finally, the exploration of the GARCH model's potential applications in emerging markets, such as cryptocurrency and environmental finance, is crucial to expand its scope and prospects.Although this paper provides a comprehensive examination of the application of the GARCH model in the financial field, it is evident that some things could still be improved. The selection of application examples is relatively limited, and the analysis of other financial markets, such as the bond market and foreign exchange market, is lacking. Moreover, the literature needs to adequately discuss the performance of GARCH models in dealing with extreme market conditions or contingencies.

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