

An Empirical Study of the Markowitz Mean-Variance Model on the Nasdaq Stock Market

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Abstract: In the vast global investment market, balancing risk and return is a pressing issue for investors. A significant part is played by the Markowitz model, which provides investors with a means of calculating risk and return. This paper focuses on analyzing portfolios using the Markowitz mean-variance model by applying empirical daily stock data from 11 different companies (AAL, NYA, IXIC, HSI, GDAX, KS11, SSMI, SBUX, KDP, NSRGY, and SJM) over the period from January 2014 to April 2014. The study also optimizes investments by screening for short-selling opportunities. NYA, IXIC, HSIA, and SJM were the four stocks that were eliminated following four rounds of computation. As a result, the paper identifies the optimal portfolio strategy and concludes that KS11 stocks exhibit the best behavior. The Markowitz model allows investors to efficiently evaluate risk and return while increasing their income. In actual circumstances, the Markowitz model is quite useful and practical.

Keywords: Markowitz Mean-Variance Model, Portfolio Theory, Nasdaq Stock Market, Short-Selling Strategy.

1. Introduction

Investment portfolios rely on portfolio construction principles, with the Markowitz mean-variance model playing a crucial role among these models. This model provides investors with a mathematical framework for calculating risk and returns, and for selecting an optimal portfolio. In this paper, we focus on analyzing portfolios using the Markowitz model and demonstrating its application with empirical data.

Among the approximately 60 large stock markets worldwide, about 16 have a market value exceeding one trillion dollars. For investors, measuring returns and risks is a critical indicator of the potential for higher returns. The mean-variance model, which earned the Nobel Prize in Economics in 1990, effectively addresses the challenge of risk-return measurement.

2. Literature Review

The Markowitz model has been extensively studied in the literature. Sarker [1] applied the mean-variance (MV) model to the monthly closing prices of 164 companies listed on the Dhaka Stock Exchange (DSE) and the DSE All Share Price Index for the period from July 2007 to June 2012. Pfiffelmann et al. [2] compared the mean-variance model with the behavioral portfolio model by analyzing stock prices from the CRSP database for the period 1995–2011. Lee et al. [3] assessed the

degree of adaptation of the MV model to the performance of Malaysian equity investments. Iqbal et al. [4] compared artificial neural networks (ANNs) with the Markowitz model by applying these models to approximate the returns of 100 companies listed on the Pakistan Stock Exchange (PSX). They measured portfolio value using the Sharpe ratio and the information ratio, finding that the ANN framework outperformed the MV model in both long and short positions, with significantly higher portfolio returns. Najafabadi et al. [5] integrated time series prediction methods, including autoregressive models, autoregressive moving average models, and artificial neural networks, with the mean-variance model. Applying this combined model to data from nine major stock exchange indices selected over a three-year period (2015 to 2018), they found that this integration enhanced the efficiency of the Markowitz model. Safitri [6] used the Markowitz model to analyze the stock prices of five companies from the IDX LQ45 index from January 2014 to July 2018 to make optimal portfolio decisions. Putra and Dana [7] compared the Single Index Model with the Markowitz Model using stocks from the LQ45 index on the Indonesia Stock Exchange for the period February 2017 to January 2020. Shadabfar and Cheng [8] employed the Markowitz model and Monte Carlo sampling methods to establish a probabilistic criterion for optimization problems. They used data from seven major Shanghai stocks (2014–2018) to calculate the probability for each portfolio. Hanif et al. [9] examined the performance of the MV model in stock risk prediction during the COVID-19 pandemic. They applied the MV model to data from all companies in the Liquid-45 (LQ-45) index listed on the Indonesia Stock Exchange (BEI) for 2019-2020, finding that BBKA and BRPT were consistent with the Markowitz model. Bai et al. [10] addressed the multidimensional investment problem of the MV model by introducing a spectrally corrected method to adjust the influence of covariance. By comparing the Markowitz model with the spectrally corrected MV model, they analyzed the top 500 stocks of the S&P 500 index from January 1, 2004, to December 31, 2013. They concluded that the Markowitz model combined with the spectrally corrected method exhibited higher accuracy and better performance in terms of risk-return. Mba et al. [11] conducted research using data from two markets: 108 stocks from the Johannesburg Stock Exchange (JSE) from January 2007 to December 2009, and 95 Forex data pairs (USD against 95 other currencies) from August 2007 to April 2008. By applying the behavioral MV (BMV) and copula behavioral MV (CBMV) models to these markets, they found that the BMV outperformed the CBMV based on the Sharpe ratio in nearly all cases, suggesting that the behavioral BMV approach in the Forex market is more promising and could add value to investors' portfolios. Rasoulzadeh et al. [12] aimed to optimize intuitionistic fuzzy investment by combining the Markowitz model with the DEA model (a cost-efficiency model) and applying the NSGA II algorithm. They used data from 50 active enterprises on the Tehran Stock Exchange from 2016 to 2019, and compared fuzzy returns and intuitionistic fuzzy returns. They concluded that while portfolio efficiency significantly increased under intuitionistic conditions, the risk also increased substantially, despite a small increase in returns. Chaweewanchon and Chaysiri [13] investigated the efficiency gains from combining a machine learning stock price prediction model with the Markowitz model. They compared the Markowitz model and the equal-weight portfolio (1/N) model with each machine learning model (LSTM, BiLSTM, and CNN-BiLSTM).

After combining, the advantages of the Markowitz model were highlighted through comparison. Historical data from the Stock Exchange of Thailand 50 Index (SET50) was used as the benchmark for calculations, and it was concluded that the combined model improved the efficiency of the Markowitz model in stock prediction.

This paper considers the daily closing prices of 11 companies and the all-share price index for the period from January 2014 to April 2014. We used secondary data as it pertains to the historical analysis of reported financial data. The mean-variance model is presented below:

3. Methodology

This paper examines the daily closing prices of 11 companies—AAL, NYA, IXIC, HSI, GDAX, KS11, SSMI, SBUX, KDP, NSRGY, and SJM—and the all-share price index for the period from January 2014 to April 2014. We used secondary data for its relevance to the historical analysis of reported financial data. The mean-variance model is outlined below:

$$\min_{\{w\}} \frac{1}{2} \mathbf{w}^T \mathbf{V} \mathbf{w}$$

$$\text{s.t. } \mathbf{w}^T \mathbf{1} = 1 \text{ and}$$

$$\mathbf{w}^T \mathbf{e} = E[\tilde{r}_p]$$

where w is the vector of weights, e is the vector of expected daily rates of return, V is the variance-covariance matrix, and 1 is the vector of ones.

And the calculation processes are shown below:

$$A = \mathbf{1}^T \mathbf{V}^{-1} \mathbf{e} = \mathbf{e}^T \mathbf{V}^{-1} \mathbf{1}$$

$$B = \mathbf{e}^T \mathbf{V}^{-1} \mathbf{e}$$

$$C = \mathbf{1}^T \mathbf{V}^{-1} \mathbf{1}$$

$$D = BC - A^2$$

$$g = \frac{1}{D} [B(\mathbf{V}^{-1} \mathbf{1}) - A(\mathbf{V}^{-1} \mathbf{e})]$$

$$h = \frac{1}{D} [C(\mathbf{V}^{-1} \mathbf{e}) - A(\mathbf{V}^{-1} \mathbf{1})]$$

$$w_p = g + hE[\tilde{r}_p]$$

4. Result

First, consider the initial 11 stocks and set the expected return rate $E[\tilde{r}_p] = 0.1\%$. The results are as follows:

Table 1: Frontier Portfolio of First Turn with $E[\tilde{r}_p] = 0.1\%$

	AAL	NYA	IXIC	HSI	GDAXI	KS11	SSMI	SBUX	KDP	NSRGY	SJM
weight	0.05783522	0.538499899	-0.503045697	-0.130262012	0.023641493	0.440576147	0.159460271	0.098415188	0.124532998	0.141210326	0.049136168

According to the analysis, the investment weights for IXIC and HSI are negative, so investments in these two stocks are canceled. The revised investment strategy is shown in Table 2:

Table 2: Frontier Portfolio of Second Turn with $E[\tilde{r}_p] = 0.1\%$

	AAL	NYA	IX IC	H SI	GDAX I	KS11	SSMI	SBUX	KDP	NSRG Y	SJM
wei ght	0.0653 53461	0.0138 49306	0	0	0.0194 87991	0.3478 70403	0.1759 37798	0.0162 65787	0.2094 58619	0.1672 70854	- 0.0154 9422

From the re-analysis, the investment weight for SJM is also negative, so we have chosen not to invest in this stock. The updated investment strategy is shown in Table 3:

Table 3: Frontier Portfolio of Third Turn with $E[\tilde{r}_p] = 0.1\%$

	AAL	NYA	IX IC	H SI	GDAXI	KS11	SSMI	SBUX	KDP	NSRGY	SJ M
wei ght	0.07089 1485	- 0.0188 1597	0	0	0.01946 4328	0.35310 8002	0.18292 2275	0.01832 4543	0.21079 1304	0.16331 4033	0

The final analysis shows that the investment weight for NYA is negative, so we have decided not to invest in this stock. The final investment strategy is presented in Table 4:

Table 4: Frontier Portfolio of Final Turn with $E[\tilde{r}_p] = 0.1\%$

	AAL	NY A	IXI C	H SI	GDAXI	KS11	SSMI	SBUX	KDP	NSRGY	SJ M
wei ght	0.06967 545	0	0	0	0.01954 3746	0.35007 7064	0.18012 7084	0.01312 0179	0.20844 7112	0.15900 9366	0

5. Discussion

In the mean-variance model, the frontier portfolio calculated based on expected returns and risk performance of stocks differs from that obtained using the expectation maximization model. In practice, stock market performance can be unbalanced. Without a comprehensive stock evaluation strategy, the frontier portfolio's calculation results may be negatively impacted by poorly performing stocks. Therefore, this issue should be considered when applying the mean-variance model in practice. This paper employs a short-selling selection method to perform the unconstrained frontier portfolio calculation iteratively. By analyzing the performance of each stock, we screen and eliminate underperforming stocks based on actual short-selling conditions. The frontier portfolio calculation is carried out iteratively, ultimately resulting in a frontier portfolio strategy involving 7 stocks out of an initial set of 11. This approach provides an optimal solution for the current market environment with a daily return of 0.1%. Additionally, investors can adjust the expected return level according to their risk aversion to obtain the optimal solution for the corresponding daily rate of return.

6. Conclusion

Based on the Markowitz theoretical model, this paper addresses the issue of risk investment in financial markets. We analyze the principles of the mean-variance model, review related research findings, and derive the calculation formula for the frontier portfolio. By collecting real stock price data, we conduct empirical research to determine and calculate the optimal investment strategy under these conditions. The practical value and significance of this method are demonstrated. For a broad range of investors, the Markowitz theory offers a robust framework for portfolio research that is well-suited to the risk investment market. Further research on this theory can enhance investment returns and contribute to the advancement of the financial market.

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