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Comparative Analysis and Reconstruction of the Fama-French Three-Factor, Five-Factor and Carhart Four-Factor Models on U.S. Consumer Discretionary Market

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Abstract: The stock market has been studied and analyzed by scholars and investors for a long time. This paper aims to compare the applicability of the Fama-French three-factor (FF3F), five-factor (FF5F), and Carhart four-factor models (Carhart 4F) in the U.S. consumer discretionary sector. For the market return, data is obtained from the U.S. Consumer Discretionary Sector ETF (VCR), covering a period from 2014 to 2024, comprising 120 months. Regression results indicate that the FF3F model performs the best, explaining 89.41% of the returns. Although the FF5F and Carhart 4F models show slight improvements, additional factors (WML, RMW, CMA) are statistically insignificant. A restructured model that contains market premium (Rm-Rf), size (SMB), and investment (CMA) factors is proved to be more effective, with market premium and small-cap stocks, as well as high investment firms, driving higher returns. This research provides experiential evidence on the efficacy of factor models in the U.S. consumer discretionary sector's stock market.

Keywords: Fama-French model, Carhart four-factor model, New asset pricing model, U.S. consumer discretionary market.

1. Introduction

The U.S. consumer discretionary market typically encompasses industries that are highly sensitive to economic cycles and volatility, including automotive, luxury goods, hospitality, durable goods, retail, and services [1]. The returns in these sectors are heavily dependent on disposable income and consumer spending patterns. Over the past decade, although the COVID-19 pandemic had a significant influence on energy, real estate and utilities sectors, the consumer discretionary sector, including hospitality and luxury goods, was also adversely affected [2]. Special events such as earthquakes, health crises, and terrorist attacks have further exacerbated the volatility of these industries by inducing behavioral changes in consumers, such as herd mentality and panic buying [3]. Consequently, accurately predicting returns in the consumer discretionary market holds substantial research value.

In modern finance, forecasting stock market returns has been a widely explored topic among investors and financial scholars. The classic Capital Asset Pricing Model (CAPM) links the expected return of an asset with the market's expected return, laying the groundwork for this field [4]. However, it has faced extensive criticism due to its unrealistic single-factor assumptions. The three-factor model

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model (FF3F), a notable extension of the CAPM, established by Fama and French in 1993 [5]. Although this model has been validated as a strong explanatory tool for the U.S. market in the long term testing [6], it still fails to capture several significant market anomalies, such as liquidity and asset growth [7][8]. The momentum issue was addressed by Carhart in 1997 [9], who introduced it and add into FF3F model, resulting in the Carhart four-factor model (Carhart 4F). Awwaliyah and Husodo found that Carhart 4F model outperformed the FF3F model in explaining excess returns of U.S. stock portfolios [10]. Moreover, Fama and French found that momentum factors were present in all other regions studied except Japan. With further research [11], Fama and French introduced the five-factor model (FF5F) [12]. However, the outcome of the FF5F model in predicting stock returns in emerging markets remains contentious, with studies indicating suboptimal results in Asian markets [13].

Although these models have been widely utilized and validated across different countries and regions, research specific to particular industries remains relatively scarce. Therefore, further exploration is needed regarding the performance of the FF3F, FF5F models, and the Carhart 4F model in the U.S. consumer discretionary sector. Additionally, given the unique characteristics of individual markets, this study plans to seek the most applicable factors for the U.S. consumer discretionary market using stepwise regression and develop a new asset pricing model based on the three aforementioned models. The study will utilize monthly returns from the Vanguard Consumer Discretionary ETF (VCR) to represent the returns of the U.S. consumer discretionary market.

The remaining of this paper will introduce the data and research method in Section 2, followed by data analysis and regression results in Section 3. Section 4 will discuss the regression results in conjunction with the market, and Section 5 will give a conclusion of the research.

2. Data and Methodology

2.1. Data

Table 1 summarizes the names and descriptions of all variables used in the study's models, with data calculated on a monthly basis covering the period from January 2014 to January 2024 (a total of ten years, or 120 months) [14].

Table 1: Variables and Descriptions

Variables	Descriptions
R_i	Total return of Consumer discretionary ETF (VCR).
Rf	Risk free rate of return .
Rm	Total market portfolio return.
SMB	Size premium, small-firms return minus big-firms return.
HML	Value premium, high book-to-market equity minus low book-to-market equity.
WML	Volume premium, the weighted average returns of the winning portfolio minus weighted average returns of the losing portfolio.
RMW	Profitability factor (Robust Minus Weak), portfolios return of high profitability stocks minus portfolio returns of low profitability stocks.
CMA	Investment factor (Conservative Minus Aggressive), portfolios returns of low-investment companies minus portfolio returns of high-investment companies.

The Vanguard Consumer Discretionary ETF (VCR) is selected to represent the U.S. consumer discretionary market, as it offers a more diversified exposure to the industry compared to the ETF(XLY), which dominants the sector. VCR focuses on providing exposure to the U.S. consumer

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discretionary sector, encompassing 303 stocks, covering apparel, hotels, cruise companies, automobile manufacturers and so on [15]. The data of VCR are downloaded from the yahoo finance website [16]. Other factor definitions are detailed in the table 1.

2.2. Methodology

2.2.1. Model Setup

To better explain the expected return of assets (especially stocks), Fama and French extended the single-factor Capital Asset Pricing Model (CAPM), which introduced in 1960s [4], by adding two additional risk factors to the market risk factor (Rm-Rf): the size factor (SMB) and the value factor (HML) [5]. Forming the Fama-French three-factor model (FF3F), which is as follows in Eq.(1).

$$R_{it} - Rf_t = \alpha_i + \beta_i\left(Rm_t - Rf_t\right) + s_i SMB_t + h_i HML_t + \varepsilon_{it} \tag{1}$$

Although the FF3F model always performed well in predicting stock returns, it still suffers from under-prediction in some specific types of stocks [17]. For this reason, Carhart introduced a fourth factor, the momentum factor (WML) in 1997 [9]. This factor indicates that stocks that have performed well (winners) or not (losers) in the past will continue to maintain their performance in the future. Carhart four-factor model (Carhart 4F) is shown in Eq. (2).

$$R_{it} - Rf_t = \alpha_i + \beta_i\left(Rm_t - Rf_t\right) + s_i SMB_t + h_i HML_t + w_i WML_t + \varepsilon_{it} \tag{2}$$

Research has proven that a company's profitability and investment model have a correlation on stock returns [18]. In order to make the model more responsive to the complexities in the market and thus improve the accuracy of stock return forecasting, Fama and French supplemented the FF3F model again in 2015. The final Fama-French five-factor model (FF5F) [12], which takes into account the profitability and investment factor (RMW & CMA) of the asset, is as follows in Eq.(3).

$$R_{it} - Rf_t = \alpha_i + \beta_i\left(Rm_t - Rf_t\right) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + \varepsilon_{it} \tag{3}$$

The factors utilized in this study are derived from the entire U.S. stock market, including data from the NYSE, AMEX, and NASDAQ [14]. In order to explore the validity of these models in specific industries, the excess expected returns of the U.S. Consumer Discretionary ETF (VCR) between January 2014 and January 2024 will first be analysed in a regression using each of these three capital asset pricing models above. Subsequently, in order to better explain the expected returns of the ETF (VCR), this paper will determine the optimal asset pricing model for this market by combining the existing factors and applying a stepwise regression method (backward regression) to the following Eq.(4).

$$R_{it} - Rf_t = \alpha_i + \beta_i\left(Rm_t - Rf_t\right) + s_i SMB_t + h_i HML_t + w_i WHL + r_i RMW_t + c_i CMA_t + \varepsilon_{it} \tag{4}$$

2.2.2. Regression Method

Both ordinary least squares (OLS) regression and stepwise regression will be used in this study. Stepwise regression works by gradually introducing (forward selection) or removing (backward elimination) predictor variables based on their significance, continuing until any introduced or

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excluded variable no longer significantly improves the R-squared of model or a predetermined stopping criterion is reached [19]. This method reduces multicollinearity by selecting the most important variables, thereby improving the stability and accuracy of model parameter estimates. In this research, the backward elimination method will be used and the stopping criterion is set at a p-value greater than 0.05.

3. Results

3.1. Descriptive Statistics of Variables

Table 2 shows the summary statistics for each variable. The average return for the U.S. consumer discretionary market was about 1%, with high volatility reaching 6% and a return range between -17.99% and 21.93% during the sample period, exceeding even the overall market volatility of 4.53%. Over this ten-year period, the market's average return was approximately 1.02%, while the risk-free rate remained relatively stable at around 0.1%. The average returns for SMB and value HML were -0.15% and -0.16%, respectively, both of which were negative. The average value of the WML was 0.12%, confirming the existence of the momentum effect, though its standard deviation of 3.96% indicates relatively high volatility. The average returns of RMW and CMA were 0.36% and -0.03%, respectively, both showing moderate levels of volatility.

Table 2: Descriptive Statistics of Variables

Variables	Obs	Mean	Std. dev.	Min	Max
R_i	121	1.0019	6.0019	-17.9879	21.9264
Rf	121	0.1003	0.1316	0.0000	0.4700
Rm	121	1.0176	4.5312	-13.2500	13.6500
SMB	121	-0.1490	2.7723	-5.9500	7.3700
HML	121	-0.1557	3.7210	-13.8300	12.8800
WML	121	0.1217	3.9614	-16.0200	9.9800
RMW	121	0.3636	2.1323	-4.7600	7.2000
CMA	121	-0.0320	2.3485	-6.8100	7.7400

3.2. Correlation

From the correlation matrix presented in Table 3, it can be observed that HML and CMA, as well as between RMW and SMB, is correlated strongly, with correlations of 0.6559 and -0.4907, respectively. Although these factors show high correlations, the correlation coefficients are still below the widely used threshold of 0.8, which demonstrates that there is no serious problem of multicollinearity. Additionally, the correlation values between most other factors are below 0.3, further supporting the view that the data does not have the significant multicollinearity issue.

Table 3: Correlation between Explanatory Variables

	Rm-Rf	SMB	HML	WML	RMW	CMA
Rm-Rf	1.0000					
SMB	0.2997	1.0000				
HML	0.0292	0.0684	1.0000			
WML	-0.4129	-0.3143	-0.3029	1.0000		
RMW	0.0293	-0.4907	0.1449	-0.0549	1.0000	
CMA	-0.2047	-0.1266	0.6559	0.0050	0.1849	1.0000

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3.3. Regression analysis

The paper begins with Ordinary Least Squares regressions on Equation(1), (2)and(3) respectively. The results (Table4) show that in the U.S. consumer discretionary market, all three factors in the FF3F model (M1) show strong significance (5% level), especially for the factors Rm-Rf and HML, with p-values of less than 0.001. Moreover, the Adjusted R-squared of the model reaches 89.41% .

Regression results of Carhart 4F model (Eq. (2), M2) demonstrated that when momentum factor was introduced into the FF3F model, the performance of the model was only slightly improved (Adjusted R-squared increase from 89.41% to 89.45%). The initial introduction of three factors is still significant but the momentum factor is not. Therefore, it can be judged that, in the U.S. consumer discretionary market, the Carhart 4F model did not increase the explanatory power of FF3F model .

After regressing of the FF5F model, although the model Adjusted R-squared improves to 89.70%, all the factors are statistically insignificant except for the Rm-Rf and SMB. Based on the 65.59% correlation between the HML and CMA (Table 3), it is inferred that there may be a slight collinearity between the two factors which displaces the explanatory power of HML.

To address the minor multicollinearity among factors and to identify the optimal capital asset pricing model for the U.S. consumer discretionary market, this study will use stepwise regression to filter and eliminate variables from the six-factor model, Eq.(4). The results (M4) indicate that the factors Rm-Rf, SMB and CMA are retained, all p-values of these factors are less than 0.05. Their coefficients are 1.18, 0.14, and -0.3075, respectively, that are economically meaningful. Therefore, compared to the FF3F model, the new model (M4) is better suited for the U.S. consumer discretionary market, although the Adjusted R-squared (89.46%) shows only a marginal improvement.

Table 4: Regression Results

	M 1	M2	M3	M4
	Fama-French three-factor Model	Carhart four-factor Model	Fama-French five-factor Model	New capital asset pricing Model
Rm-Rf	1.2141***	1.1943***	1.1816***	1.1835***
	(29.42)	(26.99)	(27.44)	(28.30)
SMB	0.1764*	0.1585*	0.2185**	0.1413*
	(2.61)	(2.30)	(2.75)	(2.09)
HML	-0.1861***	-0.2055***	-0.1161	
	(-3.87)	(-4.07)	(-1.78)	
WML		-0.0653		
		(-1.23)		
RMW			0.1431	
			(1.44)	
CMA			-0.2000	-0.3075***
			(-1.90)	(-3.96)
_cons	-0.2148	-0.1944	-0.2262	-0.1728
	(-1.18)	(-1.06)	(-1.24)	(-0.95)
N	121	121	121	121
Adj. R^2	0.8941	0.8945	0.8970	0.8946

t statistics in parentheses
* p<0.05, ** p<0.01, *** p<0.001

The regression analyses of the four models reveal that the market premium consistently explained best for asset returns and the coefficient is stable in all models at about 1.9. This indicates that the

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market factor dominant role in explaining asset returns. In spite of the relatively low coefficient of the SMB, this factor shows significance in all models, which proves its importance in the model.

In the FF3F model, the value premium factor (HML) is highly significant (p-value less than 0.001). However, with the inclusion of factor CMA, the role of HML can be replaced by CMA. As both factors show strong significance in the regression, but the coefficient of CMA in the M4 is -0.3075 which is higher than the coefficient of HML -0.1861 (M1). Suggesting that the CMA captures a similar risk premium as the HML and provides more information in explaining asset returns. Therefore, the introduction of CMA further enhances the model explanatory power.

4. Discussion

In the New pricing model (M4), the market risk factor (Rm-Rf) has a coefficient of 1.1835, signifying a substantial positive influence on the U.S. consumer discretionary market returns. Compared to essential goods, companies in the consumer discretionary market are more sensitive to economic cycles. During periods of economic expansion, increased consumer income boosts demand for discretionary goods, thereby enhancing the returns of related companies. Conversely, during economic downturns, consumers tend to reduce spending on non-essential items, prioritizing essential goods instead. Therefore, the returns in this sector are strongly dependent on the overall economic conditions.

The positive size premium factor (SMB) coefficient of 0.1413 suggests demonstrate that the return premium of small-cap stocks plays a more important status in describing returns in the consumer discretionary market. Over the past ten years, as demand for leisure and entertainment has grown and the consumer discretionary market has expanded across various sectors such as retail, automotive, sales, entertainment, and hospitality, the number of small firms has increased in this market. Consequently, the rising returns of small-cap stocks in the U.S. market have further driven the growth of returns for small-cap stocks within the consumer discretionary sector, thereby influencing the market returns.

The regression coefficient of the investment factor (CMA) at -0.3075 shows that in the consumer discretionary market, high-investment companies outperform low-investment companies in terms of returns. In highly segmented industries like hospitality and tourism, high-investment firms expand through strategic initiatives, such as opening new branches or entering international markets, which helps them capture market share and enhance brand influence, leading to higher returns. Additionally, the negative mean of CMA shown in Table 2 further suggests that the investment patterns in the consumer discretionary market align with those of the broader U.S. market. Overall, high-investment strategies typically yield higher returns during economic expansion periods, a trend that is similarly reflected in the consumer discretionary market.

5. Conclusion

Regression analysis of the U.S. consumer discretionary ETF (VCR) over the past decade indicates that the FF3F model performs the best, explaining 89.41% of the returns. Although the FF5F and Carhart 4F models slightly improve the adjusted R-squared value, the additional factors—WML, RMW and CMA—are not significant. Thus, the inclusion of these extra factors does not provide substantial additional value.

In the FF3F model, while all factors are significant at the 5% level, only the Rm-Rf and SMB consistently exhibit significance and stability across all models. Stepwise regression analysis reveals that, due to multicollinearity issues, the factor HML can be substituted by the CMA in the regression, with CMA offering more explanatory power for asset returns.

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Overall, this study finds that, compared to the FF3F model, the new model (M4) comprising the factors Rm-Rf, SMB and CMA is more suitable for the U.S. consumer discretionary market. In the new model, the market premium emerges as the dominant factor, while small-cap stocks and high-investment companies deliver higher asset returns. This provides investors with effective strategic insights.

Future studies could explore how these models perform across different market conditions, particularly across various stages of the economic cycle. Additionally, incorporating macroeconomic variables or industry-specific indicators could improve the models' ability to explain the data. As the U.S. consumer discretionary market continues to evolve, revisiting and updating factor models will contribute to maintaining the research's relevance and applicability.

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