

Comparative Analysis of Put-Call Ratio and Implied Volatility in the Multi-factor Model

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Abstract: During the process building multi-factor models to predict the excess return of the S&P 500 Index (SPX), this paper find that Implied Volatility (IV) and Put/Call trading volume (P/C) have different effects when acting as the fifth factor when the other four factors (The Capital Market Factor -CMA, the Profitability Factor-HML, the Relative Strength Index-RSI, and the Dollar Index-DXY) are already fixed in the model. To explain why Put/Call trading volume provides a better explanation for SPX excess return compared to Implied Volatility, this paper employed Granger Causality Tests and the Random-Walk Model by using daily data from August 1, 2014 to May 31, 2024. Statistical tests and robustness checks are also conducted to compare the explanatory power of these models. After the above series of tests, this study finds that the put-call ratio factor is more consistent with the other four factors in the model than the IV factor, and can explain more of the expected return of the S&P500. This study's innovation includes two perspectives, The initial point revolves around showcasing the enhanced forecasting capabilities of the Put/Call ratio in terms of trading volume compared to the Implied Volatility when it comes to predicting SPX returns. The second is highlighting Put/Call ratio's value in multi-factor models.

Keywords: Implied Volatility, Put-call ratio, Multi-factor model, S&P500, Asset Pricing.

1. Introduction

Fama-french five-factor model has always been a classic model for asset pricing and return forecasting in the international financial community[1], and has a wide range of application scenarios. However, in the practical application of the model by investors and the empirical test by scholars, there are often some deviations and errors. In 2017, Fama and French highlighted a pivotal flaw in the model, namely its inability to adequately account for the meager average yields of smaller equities, which mirror the performance of firms with low profitability that engage in robust investment strategies[2]. Mosoeu and Kodongo observed that the five-factor model falls short when applied to portfolios tailored to specific countries or those diversified across different regions, as evidenced by the disappointing outcomes of the Gibbons-Ross-Shanken (GRS) evaluations[3]. Additionally, Foye and Valentinčič discovered that alternative models yield substantial intercept values, and the five-factor model, at its best, contributes only marginally to enhancing the depiction of average LHS returns during their examination of the factor model's capacity to explain returns within an Indonesian

market portfolio[4]. Due to the shortcomings of the Fama-French five factors found above, there is still room for improvement in this model, and it is expected that a more convincing factor model to predict asset pricing can be found. Delving into the mechanisms of financial markets, particularly the analysis of capital asset valuation, has emerged as a prominent subject within the realm of contemporary finance. Fama and French created the classical 3-factors model in 1993[5]. In the past decades, many scholars tried to improve the factor models by adding other factors or replacing the original factors with the newly discovered factors.

On the basis of the vacent field of factor model research and improving ways mentioned above, this paper aims to construct the 5-factors model to get an advanced factor model, and the main way lies on the replacement of the 5-factors models. The HML factor, CMA factor, RSI factor, and DXY factor are selected to the model through regression tests. Further, An exhaustive strategy was implemented to delve into the utility of the IV or Put-call ratio as an additional metric for assessing the pricing mechanisms of underlying market assets. This entailed conducting a trio of distinct experiments consecutively, yielding a blend of outcomes across the empirical analyses documented in the paper. The applied significance of the study stands out as a notable feature. Given the influence of market variables, the conventional five-factor Fama-French model exhibits minimal explanatory power. In contrast, our Multivariate factor model is capable of offering investors valuable insights and a distinctive factor-based perspective for market investments. Additionally, investors can monitor the S&P 500 by leveraging our factor model to potentially achieve above-average returns.

The structure of the remaining portion of this investigation is outlined as follows: In Chapter 2, we delve into the pertinent scholarly works; Chapter 3 outlines the dataset and details our investigative approach; the analysis of the experimental outcomes is presented in Chapter 4, culminating with the summary of our findings.

2. Literature Review

Fama and French introduced the size premium (HML) and value premium factors (SMB) into the CAMP framework[1], revealing that the market beta alone is insufficient in accounting for the disparities in equity returns. They proposed that their enhanced model offers a superior gauge of the market's excess returns, known as the Fama-French three-factor model[5]. Subsequently, the Fama-French five-factor model[1] substantially bolsters the descriptive capabilities of asset valuation models with the inclusion of the profitability (RMW) and investment pattern factors (CMA). which help reveals the important role of company profitability and investment decisions on stock returns, equipping financiers with an expanded viewpoint to refine their investment selections.

Academics persistently endeavor to determine whether the elements within the Fama-French framework can be altered or augmented to enhance its explanatory power. Carhart explored the influence of momentum-based trading on equity performance[6]. Lam showcased the impact of market liquidity on share returns within the context of the Hong Kong equities exchange[7]. Chen and Gao delved into the influence of the MCB volatility risk element, derived from VIX and VIX3M, on asset pricing dynamics[8].

Historically, Information from the options market like IV and put-call ratio, is utilized to predict stock yield outcomes. Pan and Poteshman have demonstrated that the present put-call rate has a negative correlation with impending daily stock performance[9]. Chen and Liu have forecasted market yields based on an inferred volatility gauge derived from the bid and ask spreads of substantially out-of-the-money put options for the S&P 500 Index[10]. Zhou, Dai, and Wen have established a strong Granger causal link from the implied volatility of the stock market to actual stock price volatility[11]. Additionally, the out-of-sample analysis suggests that the stock market's implied volatility holds substantial predictive power for stock returns. Research conducted by Dumas and colleagues[12] as well as Bakshi and associates[13] demonstrate that IV exhibits distinct patterns,

such as the 'smirk' phenomenon, reflecting market sentiment and risk perceptions. Britten-Jones and Neuberger further illustrate how IV can be used to infer underlying stochastic volatility processes, highlighting its predictive power[14]. Some previous studies have pointed out some features of Put-Call ratio and IV. The research conducted by Blau and Brough revealed that the put-call ratios tend to trail behind spells of negative market returns[15]. However, the predictive power of returns that is seemingly embedded within put-call ratios is actually linked to the ratios subsequent to periods marked by positive returns. Jena, Tiwary, and Mitra[16] discovered that the Volume Put-call ratio serves as an effective forecaster of market returns within a concise time frame of 2.5 days, while the Open Interest Put-call ratio proved to be a reliable indicator over a span of 12 days.

Combining the respective characteristics of IV and put-call ratio, we would like to explore which metric is the more explanatory of the S&P 500. Many scholars have considered IV and put-call ratio separately for their effects on stock returns, while others have included both factors in fama-french analyses. Not only that, The researchers Bandopadhyaya and Jones employed the leftover values from a stochastic-process regression analysis of the S&P 500 index to depict the fluctuations in asset prices that remain unaccounted for by economic indicators[17]. They discovered that the PCR outperforms the VIX in capturing these elements, suggesting that the PCR is a superior indicator for gauging market sentiment. The uniqueness of our study lies in the fact that we added IV and Put-call ratios to a persuasive four-factor model of our own creation to illustrate the significance or otherwise of their effects. Our work adds to the existing body of knowledge by examining the influence of implied volatility and the open interest put-call ratio on the daily performance of the S&P 500 index, while also extending our examination beyond the traditional Fama-French framework.

3. Data and method

3.1. Data and variables

The current research encompasses a quintet of elements to construct a predictive factor model for the S&P 500 returns. It incorporates the CMA and HML components from the renowned Fama-French five-factor framework, pertaining to investment strategies and the equity's book-value to market-value ratio. Additionally, it utilizes the RSI metric, indicative of the asset's price volatility rate, and the DXY factor, which measures the movement of the dollar exchange rate. The latter two variables are the IV factor and the Put-call ratio factor, both of which are used to measure market sentiment. In this paper, they are tested to see who is more suitable to act as the fifth factor of the model.

Sample information is gathered from an assortment of data origins, such as freely accessible online repositories like Yahoo Finance, NASDAQ, and the Fama-French factor dataset portal, as well as research data from Bloomberg. Since data for the S&P500 on NASQ only goes back to 1 August 2014, We utilized information spanning from the specified date up until May 31st, 2024, and amassed figures pertaining to all elements of the Fama-French five-factor framework, in addition to the RSI and DXY metrics, throughout the said duration.

3.2. Methodology

Two distinct analytical frameworks, each incorporating multiple variables, were developed to scrutinize the additional profits from the S&P 500 Index. The first model introduces Implied Volatility (IV) as its additional fifth element, whereas the second framework integrates the Put-Call Ratio derived from Trading Volume. These analytical constructs employ the method of Ordinary Least Squares (OLS) for regression analysis to ascertain the correlations between the variables in question. The variable being predicted is the surplus return of the S&P 500, denoted as RI_{RF} . while the independent variables are CMA, HML, RSI, DXY, and either IV or the Put-Call Ratio.

The paper utilized Granger Causality Tests to determine the directional predictive relationship between the factors (IV and Put-Call Ratio) and the S&P 500 excess returns. This helped identify whether changes in these factors could predict changes in SPX returns.

To investigate the unexplained variance by the initial four factors (CMA, HML, RSI, DXY), this study estimated the residuals of a Random Walk Model that includes these four factors. Subsequently, this study tested whether the Put-Call Ratio or Implied Volatility better explains these residuals by separately adding them to the model. The impact of every element was assessed by contrasting the modified coefficient of determination and t-values derived from the formulated models.

Model-1:

$$SPX\ Excess\ Return_t = \alpha + \beta_1 \cdot IV_t + \beta_2 \cdot CMA_t + \beta_3 \cdot HML_t + \beta_4 \cdot RIS_t + \beta_5 \cdot DXY_t + \epsilon_t$$

Model-2:

$$SPX\ Excess\ Return_t = \alpha + \beta_1 \cdot Put/Call_t + \beta_2 \cdot CMA_t + \beta_3 \cdot HML_t + \beta_4 \cdot RIS_t + \beta_5 \cdot DXY_t + \epsilon_t$$

4. Empirical Results

4.1. Correlation Analysis

The study performs an analysis of the interrelation between six distinct variables, with the findings compiled in Table 1.

For RI_RF and CMA (-0.279546): A negatively inclined, moderately strong relationship exists between the additional return (RI_RF) and the difference between conservative and aggressive strategies (CMA), hinting at a trend where less risky investment approaches correlate with diminished excess returns.

Regarding HML and CMA (0.619000): A robust positive association is observed between the value-oriented factor (HML) and the measure of conservative versus aggressive behavior (CMA), indicating a tendency for these factors to align closely. This could suggest that companies with elevated book-to-market values frequently adopt more cautious investment tactics.

ADJ_CLOSE and PUT_CALL (-0.383718): A moderately negative correlation exists between the adjusted closing price (ADJ_CLOSE) and the put-call ratio (PUT_CALL), indicating that as the put-call ratio increases, the adjusted closing price tends to decrease.

LNRSI and CLOSE_IV (-0.429003): The natural logarithm of RSI (LNRSI) and implied volatility (CLOSE_IV) exhibit a moderately strong negative correlation. This suggests that as RSI increases, indicating stronger momentum, implied volatility tends to decrease, possibly reflecting reduced market uncertainty as sentiment improves.

The overall correlation matrix suggests that while there are some relationships between the variables, many of the correlations are weak, highlighting the potential for each factor to contribute uniquely to explaining variations in the dependent variable without significant multicollinearity concerns. However, the strong correlation between HML and CMA should be considered when using these variables together in a regression model, as multicollinearity could affect the stability of the coefficient estimates. "Although the HML and CMA factors exhibit multicollinearity due to their strong positive correlation, they each capture distinct dimensions of risk premia. In spite of the intercorrelation among variables, incorporating both elements into the model boosts its capacity to explain variations in asset returns across different segments, since each imparts distinct insights pertinent to the valuation of risk." [1]

Table 1: Correlation Analysis

| | RI-RF | LN(RSI) | ADJ_CLOSE | CLOSE_IV | P/C_RATIO_ TRADING_ VOLUME | HML | CMA |
|----------------------------------|---------|---------|-----------|----------|----------------------------------|---------|---------|
| RI-RF | 1.0000 | 0.2376 | -0.0136 | -0.1645 | -0.1857 | -0.0374 | -0.2795 |
| LN(RSI) | 0.2376 | 1.0000 | -0.1189 | -0.4290 | -0.1716 | -0.0018 | -0.0642 |
| ADJ_CLOSE | -0.0136 | -0.1189 | 1.0000 | 0.1404 | -0.3837 | 0.0000 | -0.0154 |
| CLOSE_IV | -0.1645 | -0.4290 | 0.1404 | 1.0000 | -0.0188 | -0.0263 | 0.0493 |
| P/C_RATIO_ TRADING_ VOLUME | -0.1857 | -0.1716 | -0.3837 | -0.0188 | 1.0000 | -0.0158 | 0.0509 |
| HML | -0.0374 | -0.0018 | 0.0000 | -0.0263 | -0.0158 | 1.0000 | 0.6190 |
| CMA | -0.2795 | -0.0642 | -0.0154 | 0.0493 | 0.0509 | 0.6190 | 1.0000 |

4.2. Granger Causality Tests

The present study performs Granger Causality Analyses on the Put-Call ratio, with the findings presented in Table 2.

Upon executing the Granger causality analyses, it was observed that the S&P 500 yield (RI_RF) acts as a Granger cause for the implied volatility, whereas the implied volatility fails to serve as a Granger cause for the S&P 500 yield. Conversely, the trading volume of the put/call ratio has been identified as a Granger cause for the S&P 500 yield, exhibiting considerable predictive capability. However, the S&P 500 yield does not act as a Granger cause for the trading volume of the put/call ratio. This evidence underscores the predictive strength of the put/call ratio trading volume in forecasting the S&P 500 yield, positioning it as a superior explanatory factor in comparison to implied volatility.

Table 2: Granger Causality Tests of P/C ratio

| Null Hypothesis: | Obs. | F-Statistic | Prob. |
|---|------|-------------|----------|
| PUT_CALL_RATIO_TRADING_VOLUME does not Granger Cause RI_RF | 2471 | 34.3975 | 2.00E-15 |
| RI_RF does not Granger Cause PUT_CALL_RATIO_TRADING_VOLUME | | 0.0999 | 0.9050 |
| CLOSE_IV does not Granger Cause RI_RF | 2471 | 0.7490 | 0.4730 |
| RI_RF does not Granger Cause CLOSE_IV | | 3.5127 | 0.0300 |

4.3. Random-Walk Model

The present study employs the Random-walk approach, with the findings collated in Table 3. The model's residuals reflect the discrepancies that remain unaccounted for by the aforementioned four variables.

In order to ascertain whether the P/C transaction volume or the Implied Volatility (IV) offers a superior explanation for the residuals derived from Equation (1), the research proceeds to estimate Equations (2) and (3).

The outcomes of the estimations for Equations (2) and (3) are depicted in Exhibits 6 and 7, respectively. The data suggests that both the P/R ratio and the IV exhibit a significant correlation with the residuals. Moreover, their respective coefficients bear the expected negative signs, suggesting that elevated levels of these indices correspond to a decline in the S&P 500 index.

Upon comparing the empirical outcomes of Equations (2) and (3), it is observed that the P/C Trading Volume demonstrates a stronger capacity for explanation than the IV. The adjusted R-squared value for Equation (2), which incorporates the P/C Trading Volume, exceeds that of Equation (3) by 0.0193 points. Despite both the P/C Trading Volume and IV coefficients displaying p-values of zero, the P/C Trading Volume coefficient boasts a higher t-statistic compared to the IV coefficient. (-6.9547 versus -3.37431).

$$\text{SPX return} - \text{risk free rate} = a + b(P/C) + e \quad (1)$$

$$\text{Res} = a + b(P/C) + e \quad (2)$$

$$\text{Res} = a + b(VIX) + e \quad (3)$$

Table 3: results of the Random-walk model

| Variable | (1) | (2) | (3) |
|-----------------|--------------|-------------|---------------|
| P/C | | -0.00435*** | |
| | | -6.95470*** | |
| IV | | | -9.70E-0.5*** |
| | | | -3.37431*** |
| Constant | -0.00672*** | 0.00730*** | 0.00177*** |
| | -8.78900 *** | -6.81900*** | 3.13053*** |

5. Discussion

This paper explains the reason behind why the P/C ratio of Trading Volume serves as a better model from three angles.

The Put/Call Volume Ratio could potentially serve as a superior gauge for market mood and impending price shifts compared to Implied Volatility. This ratio is commonly employed as a barometer of market attitude. An elevated Put/Call Ratio often signifies a negative outlook among investors regarding the market, while the opposite is true for lower ratios. Researchers Benjamin M. Blau and Tyler J. Brough discovered that the put-call ratios tend to trail periods marked by negative returns[15]. Nonetheless, the predictive power of returns apparent in put-call ratios stems from those ratios subsequent to periods of favorable returns.

Furthermore, the study provides insights into the P/C ratio's relationship with the borrowing element. The theoretical framework suggests that if borrowing expenses escalate, knowledgeable investors tend to shift towards the options market. M. Blau and Tyler J. Brough's analytical findings consistently indicate a direct correlation between put-call ratios and indicators of the equity loan availability[15]. Consequently, when there is a surplus in loan availability, borrowing costs decrease, prompting investors to shift their focus away from the options market, potentially moving towards the equities market, which aids in elucidating the fluctuations in stock returns.

Indeed, changes in P/C ratio capture investors' immediate reaction to information like macroeconomic events, corporate earnings reports. The timeliness and effectiveness of this

information transfer can provide a more accurate basis for SPX return forecasting. Upon monitoring stock returns over the course of several weeks, the degree of foreseeability steadily diminishes, suggesting that the insights embedded within the option trading volumes are ultimately integrated into the fundamental values of the stocks. Thus, our main point lies on the timeliness reflected by the P/C ratio, it reflects changes in market participants' expectations and sentiments about future market movements, and is able to capture shifts in market sentiment more quickly and directly, and thus performs better in predicting market returns. Implied volatility, while also containing information on market expectations, can lag in its response and is a more appropriate measure of volatility and risk.

6. Conclusion

This paper constructs a five-factor model to predict the return of the S&P 500, and tries to test whether IV or the Put-call ratio is more suitable to be the fifth factor of the model. The results shows that the factor models with the Put-call ratio as a factor in it has stronger explanatory power and persuasiveness of the S&P 500 return, since the Put-call ratio obtained by Granger test is a Granger Cause for the return rate of S&P 500, and the Put-call ratio has a higher R-square in the Random-walk model test.

Following the advent of Implied Volatility, a plethora of investigations has emerged concerning its derivatives and the efficacy of its pricing mechanisms. Notably, Bandopadhyaya and Jones conducted a study employing the residuals from a regression analysis of the S&P 500 index based on a random walk model to evaluate its utility, and they contrasted this with the Put/Call ratio's effectiveness [17]. Despite its role as an indicator of market risk sentiment, there is a paucity of research into the role of IV within multi-factor asset pricing models. Furthermore, there is a gap in scholarly inquiry regarding the comparative effectiveness of IV versus the Put-call ratio as components within factor models.

Nonetheless, the inception of Implied Volatility has spurred a plethora of investigations into its derivatives and the efficacy of pricing strategies. Despite its role as an indicator of market risk sentiment, minimal exploration has been conducted regarding the role of IV or the Put/Call ratio as components within multi-factor asset pricing models, and little has been done on the question whether the IV or the Put-call ratio more effective as a factor in factor models. Therefore, for future studies, researchers can consider putting IV factor or Put-call ratio factor into the model with more factors from multiple dimensions to test their respective effects and compare the explanatory power of the model.

Authors Contribution

All the authors contributed equally and their names were listed in alphabetical order.

References

- [1] Fama EF, French KR (2015) A five-factor asset pricing model. *Journal of Financial Economics* 116:1–22.
- [2] Fama EF, French KR (2017) International tests of a five-factor asset pricing model. *Journal of Financial Economics* 123:441–463.
- [3] Mosoeu S, Kodongo O (2022) The Fama-French five-factor model and emerging market equity returns. *The Quarterly Review of Economics and Finance*
- [4] Foye J, Valentinčič A (2020) Testing factor models in Indonesia. *Emerging Markets Review* 42:100628.
- [5] Fama EF, French KR (1993) Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics* 33:3–56.
- [6] Carhart MM (1997) On Persistence in Mutual Fund Performance. *The Journal of Finance* 52:57–82.
- [7] Lam KSK, Tam LHK (2011) Liquidity and asset pricing: Evidence from the Hong Kong stock market. *Journal of Banking & Finance* 35:2217–2230.

- [8] Chen X, Gao NR-W (2020) Revisiting Fama–French’s asset pricing model with an MCB volatility risk factor. *The Journal of Risk Finance* 21:233–251.
- [9] Pan J, Poteshman AM (2006) The Information in Option Volume for Future Stock Prices. *Rev Financ Stud* 19:871–908.
- [10] Chen J, Liu Y (2020) Bid and ask prices of index put options: Which predicts the underlying stock returns? *Journal of Futures Markets* 40:1337–1353.
- [11] Dai Z, Zhou H, Wen F, He S (2020) Efficient predictability of stock return volatility: The role of stock market implied volatility. *The North American Journal of Economics and Finance* 52:101174.
- [12] Dumas B, Fleming J, Whaley RE (1998) Implied Volatility Functions: Empirical Tests. *The Journal of Finance* 53:2059–2106.
- [13] Bakshi G, Cao C, Chen Z (1997) Empirical Performance of Alternative Option Pricing Models. *The Journal of Finance* 52:2003–2049.
- [14] Britten-Jones M, Neuberger A (2000) Option Prices, Implied Price Processes, and Stochastic Volatility. *The Journal of Finance* 55:839–866.
- [15] Blau BM, Brough TJ (2015) Are put-call ratios a substitute for short sales? *Rev Deriv Res* 18:51–73.
- [16] Jena SK, Tiwari AK, Mitra A (2019) Put–Call Ratio Volume vs. Open Interest in Predicting Market Return: A Frequency Domain Rolling Causality Analysis. *Economies* 7:24.
- [17] Bandopadhyaya A, Jones AL (2011) Measures Of Investor Sentiment: A Comparative Analysis Put-Call Ratio Vs. Volatility Index. *JBER* 6:.